Logic, Game Theory, and the Real World

Arnis Vilks[1]
Leipzig Graduate School of Management (HHL)
VILKS@HHL.DE

Introduction

Both Logic and Game Theory are fascinating fields. I for one have always been intrigued by the perennial philosophical puzzle of how reasoning works and which kind of reasoning is sound. How reasoning human agents interact with each other, and how they contribute to each other's well-being, and why they often harm each other, is a similar perennial puzzle, but for some reason its more explicit investigation has had to wait for the genius of John von Neumann. I am quite confident, however, that in the history of thought bright men or women before von Neumann must have thought about the "I think that you think that he thinks that I think"-loops that are so typical for game theoretic reasoning. In fact, this little observation about game-theoretic reasoning already makes clear why game theory cannot really live without including logic into its field of inquiry.

Many years ago, when I was a student at Hamburg University, I was taught that Logic is the theory of *valid* or *correct* reasoning. One might perhaps use "rational" as opposed to factual reasoning as a third near-synonym. My logic professors some 30 years ago made much of the point that logic is not psychology. The latter may try to investigate how people *actually* develop, i.e., modify and extend their belief systems, even if no external information arrives. That is, psychology may look at how people *actually* think. But Logic, I was told, is rather a *normative* discipline. It tells you, how you ideally *ought* to reason, what you are allowed or required to infer from given premises. Thus, it seems that Logic needs to prove that the inferences it singles out as correct, cannot lead you astray. As – I assume - many other serious students of logic, I was somewhat puzzled by the fact that in proving the theorems of logic one frequently makes use of just those very modes of reasoning the validity of which one is proving - but of course the distinction between formal and informal proofs, between object- and metalanguage made that puzzle much less worrying.

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Some years later, I was taught, and many economics students all over the world are still taught today, that Game Theory studies the interaction of rational agents, where “rationality” has a double meaning. It covers both rationality of choice, and rationality of reasoning. Rationality of choice means that relative to the information you happen to have, you choose what you prefer most among the options you consider possible. If I observe you sitting and listening, I may thus infer by the rationality of choice principle that, however annoyed you may be by what I am saying, your observable choice reveals that right now you prefer sitting and listening to getting up and leaving. In making this inference, I certainly take for granted that you consider it possible to leave, but given this assumption, observed behaviour implies certain things about preferences.

The second part of Game Theory’s standard rationality assumption is more or less just the assumption that the players’ *reasoning is correct*, and that *no relevant inference fails to be made*. This is sometimes expressed by saying that game theory assumes players to be perfect logicians. Actually, they are often required not only to be logicians, but to know a good deal about higher layers of mathematics as well, but most game theorists don’t know much about the distinction between logic and mathematics.

Now, unlike the rationality of choice principle, logical omniscience seems a pretty bold assumption to make. After all, game theory is meant to elucidate *actual* behaviour of *human beings* in situations where they interact, and *actually* reason about each others’ reasoning.

This brings me closer to one of the main points I would like to make. Game theory has become the most important tool kit for economists, and it is sometimes also used in political science, because it seems to elucidate real-world interactions. The world is of course full of interaction between humans. Every organisation or institution is brimming with interaction, every market is essentially interaction, not to mention international politics like the current scary escalation of violence between Islamic extremists and the George-W.-Bush-led Western alliances.

Of course, as most of you know, game theory can also be applied to the interaction of machines and software agents, and John Maynard Smith has inspired the development of a whole new sub-discipline - evolutionary game theory - by showing that many game theoretic ideas can be applied to the interaction of animals – without of course assuming them to be perfect mathematicians. Unlike its rationalistic ancestor, evolutionary game theory does not look at interaction between a few, who consciously choose their respective strategies, but it looks at whole populations of agents who are genetically programmed to play particular strategies, and who may or may not have offspring with the same genes – depending on how “fit” their strategy makes them to survive within the current population’s “mix” of strategies.

Of course, with certain modifications and reinterpretations, this non-rationalistic branch of game theory can also be applied to the interaction among populations of machines or computer programmes – and it can also be applied to that particular species of animal that likes to think of itself as very different from all other species, because it has free will to choose and because it a mind to reason.

I am getting even closer to the point I want to make. There are conflicts and interactions between humans in the real world out there that arguably cost many thousands of human lives each day. Nevertheless, the reasoning that apparently informs the most far-reaching strategic decisions of the political and business leaders in the real world, appears to be often so innocent of any decision-theoretic or game-theoretic wisdom, that one cannot but marvel at the lack of interaction between us thinkers, and those who move and often shake the world.

Now, it is always easy to blame others, and intellectuals tend to underestimate the amount and complexity of reasoning that quite a few of the people in authority, actually do carry out before making their strategic choices. But if I am right that many important decisions “out there” lack all the insights that logic and game theory can provide, I think we intellectuals have to blame ourselves, too. I have the impression that both fields we are talking about today, both logic and game theory, often tend to ignore the important problems, and waste much time and effort in defining precisely, and then solving problems - which nobody had before they were invented by the logicians or game theorists.

Even the terminologies used within our fields tend to be so idiosyncratic that members of one research group sometimes have great difficulties to even understand what is written by a member of another research group.

Now, I am not, of course, urging the Amsterdam GloriClass to forego research on foundational issues, or to write popular expositions of logic and game theory for the politician. Actually, I try not to urge anybody to do anything. But I kindly invite you to consider the research you are doing in the context of the problems out there which urgently await if not solutions then at least informed advice on the soundness of reasoning and on the rationality of choice.

A simple game of some importance

To illustrate, let me consider a very simple example. Assume there are two players, the rich civilization R, and the poor one P. Both have two strategies: To play “aggressive” A, or to play “meek” M. If both play meek, the rich civilization gets almost the whole “pie”, which I take to be 22 t USD, but the poor can improve a little against this allocation by playing aggressive. If the rich play aggressive they hurt the poor, but against a meek strategy by the poor, the rich loose only little. If both play aggressive – well, humankind may not survive the ensuing nuclear war.

A first distinction can be illustrated here, the one between decision-theoretic rationality and game-theoretic rationality. While it is clearly the dominant strategy for the rich to play meek, the poor civilization’s best response depends on its opponent’s choice. If the poor do not know about the character of the rich, and therefore regard both strategies of the rich as equiprobable, the poor will be indifferent between its two strategies, and hence may well choose to play meek. Many other beliefs of the poor, represented by a probability distribution over its opponent’s strategies, are of course just as consistent with playing meek. However, considering that the rich opponent is rational, and knowing about the structure of the game, the poor must conclude that the rich will not play aggressive, but meek. Therefore game theory *recommends* the aggressive strategy to the poor.

P R	A	M
A	0 0	1 18
M	3 17	2 20

Obviously, the strategy combination (M,A), where the first component denotes the rich player’s strategy, is the only Nash equilibrium for this little game, and one might think this all one can say.

Not so. Do you really think (M,A) is rational? As I just mentioned, there are different notions of rationality. Decision-theoretic rationality does not quite coincide with game-theoretic rationality. One is more “demanding” than the other, but both are treated as normative concepts in the sense I indicated by saying that game theory “recommends” M to R and A to P.

But the more thoughtful game-theorists such as Harsanyi have been very aware of the fact that there is of course a still more demanding notion of rationality – what one may call moral, ethical, or fully normative rationality. Assume for a moment that both civilizations have the same number of people, and that each poor life counts just as much as a rich one. Is there an outcome that is ethically rational? You think ethics is neither logic nor game-theory? But as long as both disciplines end up making recommendations, and tell you what you *should* infer from given premises, and what the *rational* strategy choice is, both disciplines *are* normative. So why stop at the conventional wisdom of a Nash equilibrium?

You may say, none of the outcomes looks particularly “ethical”.
Right, I agree. So, what about inventing a new strategy, call it the innovative one.

P R	A	M
A	0 0	1 18
M	3 17	2 20
I	3 17	11 11

You might say, well that’s a different game. Of course it is. Within the academic world of the game theorist, it may be considered “cheating” if, in the middle of an analysis, you all of a sudden “add” a new strategy. But imagine we are modelling a real world conflict. How do we know which strategies are “available”? What does it mean to claim that a real human in a particular situation has options A, B, C and no others? Both George W. Bush and Usama bin Laden may have thought that, in order to achieve their respective goals, they had no better option but to ask their followers to go to faraway countries, and to kill themselves and many others. But maybe they were just lacking creativity. I think it is a very relevant (and essentially open) question of logic in game theory what we take an “option” to be.

(I do think it is easier to see the problem in the context of a serious application than in an Alice-and-Bob-type toy example, but I did not necessarily mean to suggest that my model of the "clash of civilizations" is capturing all the essentials of that global problem. There is a folk meta-theorem of game-theoretic modelling to the effect that for any given result there is a set of game-theoretic assumptions from which the result follows. It goes without saying that if I had wanted to praise the US president here, I could have designed a different little game than the one I just presented – a game which would yield what I wanted to show: *Not* invading Iraq must lead to disaster.)

Eight important logical problems of game theory

I guess I have messed up things a little. So let me try to disentangle some of the logical problems of game theory which I see as really important. I will briefly explain eight such issues.

1. The distinction between a normative and a predictive (or descriptive) use of theory is of utmost importance in applications of game theory. Are the solutions we define in game theory what we *ought* to *recommend*? Or are they just some kind of "predictions"? Maybe this is more of a philosophical or methodological issue than a logical one, but it seems to be also very relevant to logic itself. I do not have to tell you about the plethora of logical systems which have been analysed in the learned monographs and journals. If that is not just playing around with symbols, than what is the status of those systems? "Should" we use classical or intuitionistic logic when we reason about games? Should we use a different one when we reason about quantum information? A Leipzig colleague of mine, Piero La Mura has recently generalized Aumann's concept of correlated equilibrium to allow for correlation by means of possibly entangled q-bits. Which logic should we use in reasoning about this? To ask the question again: what is the logical status of Logic and of GT? Normative, predictive, or what? Johan van Bentham has recently written that "discussions of 'normative' versus 'predictive' views of logic have become predictable and boring". That may be so, but it is an issue which is can hardly be avoided unless one is happy to confine oneself to the narrow world of "pure", academic, discourse. To my mind, game theory today should help to avoid World War III.

2. Of course one of the strategies academics use in order to answer a question consists of making distinctions. Of course one can begin to define normativity1 as opposed to normativity2 as distinct from rationality 4. I know full well, of course, that one often needs to make the right distinctions, if one wants to convince others. But even more important than making clever distinctions, and leave it at that, is to make a

difference. I very much hope, and I trust, that the research that is going to be done in the GloriClass is going to make a difference, but if it made a difference not only to some other members of GloriClass, that would be splendid. In the world of Business today, where game theory is used and applied, the importance of ethics is widely acknowledged. Scandalous behaviour by Top-Managers as exemplified by, e.g., the Enron and Worldcom cases, may have been the main reason for this, but the fact remains that Business Ethics is increasingly seen as necessary by quite sober-minded business people. There are of course applications of logic to ethics and law, which might well be reconsidered in the context of games. Or to put this as a question: What is the logic of “fully normative” or “ethical” recommendations?

3. More specifically, the logical relation between standard game-theoretic rationality and whatever one takes ethical rationality to be, needs to be addressed. Just as game-theoretic rationality is a special case of decision-theoretic rationality, one would expect moral or ethical rationality to be a special case of game-theoretic rationality. But look at my 3*2 example again. You may have noticed that inventing the innovative strategy did not really help at first glance. The extended game still has just one Nash equilibrium – the one we had before. Can we expect anything else than (M,A) in the extended game? Maybe not, if game theory is interpreted as predictive, but if the model was used in normative sense, one might - perhaps - think up and defend an ethical justification for the outcome (11,11). Should we then recommend I to R, rather than M? As far as I am aware, such questions are not among the “standard” questions logicians are trained to think about. But who else if not logicians - or maybe philosophers with a good understanding of logic and game-theory - should be able to address them?

4. The notion of an option or “feasible choice” that is presupposed by game theory has puzzled me a lot. I made a suggestion before my 5-year excursion into the world of business to the effect that one can interpret the “dual” of the epistemic-logic “knowledge” operator, ¬K¬, to express what a player subjectively “considers possible”. I still think that was a good idea. But when I presented it at a LOFT conference with Bob Aumann in the audience, he almost got angry with me, and repeated a couple of times that the “feasible” actions in a game are not those whereof one does not yet know that one will not carry them out. With his amazing mathematical intuition he had apparently deduced on the spot that all “real” decision problems would be reduced to trivial ones within his preferred framework of S5. The situation is quite close to the one in Fitch’s paradox: If it is the case that I will choose M, but still consider it possible that I will not choose M, one would like it to be possible to know

this. In one of those most useful coffee breaks after my talk, Aumann very kindly approached me to say he was sorry about getting angry, and explained that he thought the feasible options in a game must be the objectively given, *physically possible* actions. Well, I like and respect Bob Aumann very much, he no doubt deserved the Nobel prize he got last year, but I find the idea that the feasible moves in a game are all the *physically* possible ones, outright grotesque. We never know all the physically possible actions we might carry out at any given instant, and if we did, and moreover could somehow “specify” all of them, there would just be one big “game of life” – or Savage’s “grand world decision problem” - left to consider, and any attempt to “apply” the tiny little textbook games we think are useful to analyse real-world conflicts and cooperation would at best be reduced to training material for students. So I challenge you logicians to think about what exactly turns a hypothetical action into one that has to be included as a move in the relevant game.

5. There is another problem that links game theory with logic. It is the problem of choosing the optimal logic. To take just epistemic logic as an example, most of you know that one can define somewhat different systems even within the narrow confines of Kripkean, or, say, neighbourhood, semantics. There are the systems K, KT, S4, S5, and many others. Of course, in a relaxed research environment there may be little need to “choose” one instead of the others. But in applications one would very much want to know which model of knowledge is the most appropriate one. In principle, decision or game theory should be able at least to give structure to the problem of rational choice between alternative “logical” options. But again - the devil is in the detail, and as soon as one looks closely at the basics of decision theory, the dividing line between logic and decision theory seems elusive. I mentioned the principle of rational choice in my introductory remarks. Let me repeat what it says. “If one chooses an action, it must have been considered feasible by the agent, and there cannot be another feasible one that is preferred to the chosen one.” Depending, of course, on a number of details about the notions of action and preference, non-logicians have criticized this principle of rational choice as an “empty tautology”. At least on a slightly naïve account of logic, it seems “logically impossible” to simultaneously claim that a certain behaviour is an intentionally chosen action, and that in the very moment of choosing this action - the chooser considers an alternative action feasible which he actually prefers to the chosen one. So is there a logic that connects actions, preference, and feasibility in such a way that the principle is valid? If so, there might be a grand unified theory of logic and games.

6. The next issue I would like to mention, is the notion of “information”, and its relation to the foundations of mathematics. Johan van Bentham has, I think quite rightly, emphasized that the notion of information may be a more basic notion for an adequate understanding of the world than the notions we have inherited from the logicians of 100 years ago. It may well be that I am just revealing my ignorance here, but I have not seen any theory of information that could aspire to be the foundation for, say, most of the mathematics applied in the sciences. I think it is fair to say that most applications of mathematics today take it for granted that mathematics rests on a fairly safe basis, and that this basis is the one thought up by Zermelo and Fraenkel, and later shown by the Bourbaki group to produce virtually all of known useful mathematics. If one asks why this basis is pretty safe, one can probably just point to the fact that the mathematics that can be derived from it, has not only proved very useful in combination with all sorts of non-mathematical ideas, but it has apparently nowhere collided with observations or ideas which could claim to be just as “evident” as the axioms of Zermelo-Fraenkel set theory. Now, you are probably familiar with the Quine-Duhem thesis, which roughly says that one can “refute by observation” only the conceptual apparatus used “as a whole”, but hardly ever a particular segment of that conceptual apparatus. Thus, it is essentially a matter of convention what is taken to be the infallible “hard core” of ones world view. It seems clear to me that taking set theory to be “beyond reasonable doubt” is such a convention. And conventions can and should be modified, if there are convincing alternatives one can choose - and good reasons to actually do choose some modification.

Now, again, I may just be revealing my personal ignorance when I say I have not seen an alternative to standard set theory that could at least plausibly hold the promise to clear away some of the conceptual riddles in fields like logic and game theory - where in spite of the aforementioned zoo of logical systems, at the meta-level standard mathematical logic and set theory are treated almost as god-given. But there are such riddles, and somehow many of them seem to confirm that information is more basic than sets. Let me mention one from a field I know very little about, and explain another one that I know much better.

6.1. The field I know next to nothing about except that it seems to rely heavily on the right way of modelling information, is quantum mechanics. Recently, Conway and Kochen have discovered and proved a theorem that goes by the name of “free-will theorem”. It says, apparently, that if an experimenter has some free will, sub-atomic particles also have some free will. One of the assumptions in the theorem is that there is a finite upper bound on the speed with which information can be transmitted.

Although the speed of light seems a natural candidate for this upper bound, it seems that it is experimentally not that clear that nothing can travel faster than light. And what is there in the theory of information that tells us what the carrier of information must be? Is it absurd to say that I sometimes transmit information to myself? Is the speed of thought limited by the speed of light? Can we choose what to think next? Can we choose to focus on one thought, and suppress any new ideas for a while? The famous physicist Roger Penrose has argued forcefully that human thought is creative and therefore not algorithmic. But as far as I know, the "physics of mind" is still unknown.

6.2. There is another conceptual riddle that I am quite sure has not been answered in the learned literature. To me, it strongly suggests that the logical foundations of mathematics need modification if we want a satisfactory theory of reasoning about choice. Imagine one of the simplest games there is – a one-player game with two strategies. Assume the strategies are actually given by their payoffs. Let these be monetary payoffs, and assume our agent prefers more to less money. Now, he is offered the choice between alternative payoffs which would be given to him as gifts. Either he can have 3^7 or 7^4. Our agent, like most of us, gets out his pocket calculator, types in the required calculations, and ends up with the information $3^7 = 2187$ and $7^4 = 2101$. According to the principle of rational choice, he chooses 3^7 instead of 7^4. So far, so good. What I need to tell you, perhaps, is that I cheated again. Actually, $7^4=2\mathit{4}01$. The situation I had in mind is one where our agent is unaware of the fact that his pocket calculator has a defect. As it happens, the third digit from the right is always a "1". Now why is this a conceptual riddle? Well, standard predicate logic with equality has the "axiom of substitutivity", or "indiscernablility of identicals" which says

$$Y=X \rightarrow (P(Y)\rightarrow P(X))$$

for any predicate P. Now let P(X) be the predicate "the agent has the information that 2187>X". As I assumed in my little fictitious story, $P(7^4)$ is true. It is also true that $7^4 = 2401$. The axiom thus allows us to deduce P(2401). But it seems plausible to assume that our fictitious agent is neither an idiot nor otherwise out of his mind, and that P(2401) is false. It seems that if we did not choose by convention to treat standard mathematical logic as irrefutable, we would have a nice Popperian falsification of a logical axiom here.

7. I do not know, of course, how to deal satisfactorily with these and the many other problems that arise in thinking about speed of thought, choice, free will, or the so-called "propositional attitudes". It is certainly

easy to think up an ad-hoc logic that allows one to accommodate any one of these problems. But one certainly does not want to throw away the beloved, and essentially healthy, baby of mathematics with the bathwater of some stubborn anomalies. What seems needed is a theory - or logic - of information that is in accordance with our intuitions and observations about the nature of information, but which can still accommodate at least large parts of present-day mathematics. The problems of information, consciousness, free will, and choice seem to be so much at the intersection of quantum physics, logic, and decision theory that I for one would not be overly surprised if the next big step in the history of thought arose from a unified perspective on information and on how it is processed by the human mind.

I mentioned already that because of creativity, human thought is not algorithmic. A fortiori, it is not isomorphic to a formal system, as this term is commonly defined. It may well be that a suitable modification of the definition of “formal system” will do the trick of understanding how the human mind works, and will allow one to defend the correspondingly reinterpreted “strong AI” thesis. In order to allow for creativity, one might try to investigate open systems instead of the usual formal ones. What do I mean by an open system? Well, it might look much like a standard formal system – except that neither the alphabet nor the axioms are fixed once and for all. Thus, we might say, let O be a system which has an alphabet a,b,c,…, and possibly also greek, Arabic, Chinese, or still other characters. These may be introduced if required, but not without giving reasons. We might specify a certain number of assumptions or “axioms” and rules of inference, but allow for other assumptions to be introduced if required – but again not without mentioning that the formula or sentence written down is meant to be a new, additional, assumption. There is, of course, a notion of derived or inferred formula in O. A formula F is derived in O, if it is the last one of a finite sequence such that each element of the sequence is either an assumption or derived from previous elements. At any given moment, one can “freeze” an open system into the formal one that has as its alphabet and axioms *only* the signs and assumptions introduced so far. There may not be many other interesting things one can say about *all* open systems. But are there many interesting things one can say about all formal systems? In order to say something interesting, one needs to get more specific, but the only point I want to make here, is this: The notion of “formal system” is not one that cannot be modified in interesting ways. Certainly, it seems quite straightforward to develop standard mathematics within an open system – I cannot see that anything would need to get lost. But of course, the associated philosophy of mathematics would be closer to Lakatos’ Proofs and Refutations than to Bourbaki’s Théorie des Ensembles.

One would hope that logic and mathematics are capable of progress, and maybe the time is ripe for a step forward after 100 years of essentially refining the received view of Frege, Russell, Zermelo and Fraenkel. Why not think about how to do this big step in the Amsterdam GloriClass?

8. The eigths and last issue I would like to try and disentangle from the others hidden in my little clash-of-civilizations game, probably needs to be addressed jointly with the issue of what constitutes information. It is the issue of how one *interprets* some given information. There is of course much learned literature about both formal and informal semantics, but the basic idea seems to be that any given information typically admits of different interpretations. I think I do not overly stretch the ordinary usage of the English language when I say that the information we have about the “clash” can be *interpreted* as an indication that if the escalation continues, the American president may one day decide to sacrifice the people who crucified Jesus. Both George W. Bush and Usama bin Laden are known to interpret their respective holy books in a very literal - if not naïve – way. One of the more obvious urgent things logicians should do, is telling the world out there that for any given information or book, no fallible human knows the only correct “reading” or interpretation. Educated men and women should know that both the Bible and the Qu’ran have content that *cannot* be interpreted “literally”, even if one tries. The very concept of a God who is beyond human understanding makes the word “god” - a metaphor.

Conclusion

Now it seems, I have “come full circle”, as they say. To conclude, I suggest that logicians working on game theory should neither fall prey to the ever-present temptation of misplaced modesty in the choice of topics and issues, nor succumb to the pressure of publishing *many* instead of *good* papers. The LPU or “least publishable unit” of wisdom – like other very small particles - may well be subject to Heisenberg’s uncertainty principle: As soon as its content has been “observed” by careful reading, it has already sped into the forgotten prehistory of human thinking. And if you try to keep it in the present, its position reduces to an item in your list of publications that is not read or understood by anybody.

Wisdom and courage are virtues that must not be separated by the societal division of labour – wisdom and cowardice for the scholars and academics, as it were, and thoughtless bravery for those in authority. If nothing else, the need to convince those who allocate research funds should make logic in game theory be aware of how relevant both fields are, or at any rate how relevant both fields might again become – relevant both to quite down-to-earth issues of economics and international politics, and to seemingly remote academic disciplines like quantum physics or theology.

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