"The game of billiards has destroyed my naturally sweet disposition."
— Mark Twain, April 24, 1906

Conditionals, Causality and Conditional Probability

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R. van Rooij & K. Schulz,
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Concluding Conference "The many What Ifs"

TOPIC

Indicative conditionals

Haegeman [2003]: “event conditionals”

Dancygier [1998]: “predictive conditionals”

MOTIVATION/CONTEXT

MISSING LINK CONDITIONALS

(1) *If Arsenal wins next year's Champions League finals, Great Britain will join the European Union again.* [adapted from Douven, 2017]

"I am inclined to reject (1) as being false, where this inclination is independent of what I believe about Arsenal's chances of winning next year's Champions League finals as well as of my belief about the possibility of Great Britain [joining] the European Union [again].
Even if Arsenal does win next year's Champions League finals and Great Britain does [join] the European Union [again], (1) seems defective: it seems to assert the existence of a link between the two events that — we are highly confident — does not exist." [Douven, 2017, p. 1542]

MOTIVATION/CONTEXT
MISSING LINK CONDITIONALS

(1) If Arsenal wins next year's Champions League finals, Great Britain will join the European Union again. [adapted from Douven, 2017]

"While it is widely acknowledged that conditionals whose antecedent and consequent are not internally connected … tend to strike us as odd, the felt oddness is, according to modern semantic theorising, to be explained along pragmatic lines.

Broadly, the idea is that the assertion of a conditional generates the implicature that there is an internal connection between antecedent and consequent." [Douven, 2017, p. 1542]

➡ Skovgaard-Olsen et al. 2017 as a recent pragmatic approach

INFERENTIALISM

KRZYŻANOWSKA, WENMACKERS, DOUVEN (2014)

Definition 1 *A speaker S's utterance of "If p, q" is true iff*

(i) *q is a consequence — be it deductive, abductive, inductive, or mixed — of p in conjunction with S'd background knowledge,*

(ii) *q is not a consequence — whether deductive, abductive, inductive, or mixed — of S's background knowledge alone but not of p on its own, and*

(iii) *p is deductively consistent with S's background knowledge or q is a consequence (in the broad sense) of p alone.*
[Krzyżanowska et al. 2014, p. 5]

INFERENTIALISM

KRZYŻANOWSKA, WENMACKERS, DOUVEN (2014)

The antecedent is necessary and sufficient to **infer** the consequent, be it deductively, abductively, inductively, or a mixed inference.

Why?

- ➡ The consequent becomes true *by virtue of* the antecedent.
- ➡ a causal analysis?

THE STORYLINE

ACT 1

- Introduce a probabilistic measure for the acceptability of conditionals
- Provide a causal explanation

ACT 2

- **Challenge 1:** the data point to a conditional probability approach

- **Challenge 2:** what about diagnostic conditionals?

- **Challenge 3:** what about epistemic conditionals?

ACT 3

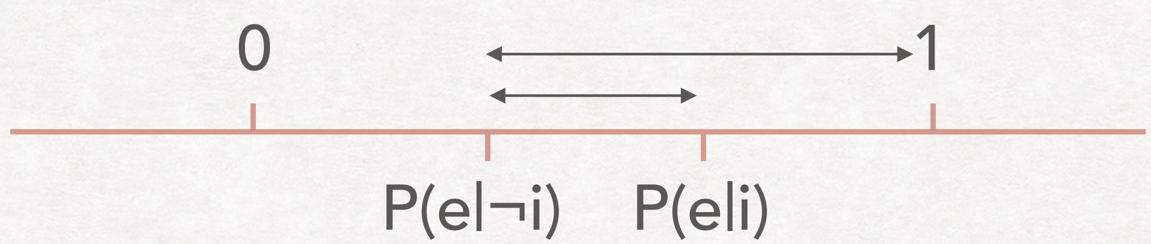
- Summary
- Two extensions: biscuit conditionals and generics

AN ALTERNATIVE APPROACH

MEASURE ACCEPTABILITY IN TERMS OF RELATIVE DIFFERENCE

Contingency: $\Delta P_i^e = P(e|i) - P(e|\neg i)$ [Shanks 1995]

"relative difference": $\boxed{\Delta^* P_i^e = \frac{P(e|i) - P(e|\neg i)}{1 - P(e|\neg i)}}$ [Shep 1958]



$$\Delta^* P_i^e = \frac{P(e|i) - P(e|\neg i)}{1 - P(e|\neg i)} = \frac{\Delta P_i^e}{1 - P(e|\neg i)} = \frac{P(e|i) - P(e)}{P(\neg e \wedge \neg i)}$$

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A conditional is **acceptable** iff the relative difference of the consequent given the antecedent is high.

The **meaning** of a conditional can be equated with the conditions under which we would **learn** the expressed generalisation.

AN ALTERNATIVE APPROACH

MEASURE ACCEPTABILITY IN TERMS OF RELATIVE DIFFERENCE

Contingency: $\Delta P^e_i = P(e|i) - P(e|\neg i)$ [Shanks 1995]

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A conditional is **acceptable** iff the relative difference of the consequent given the antecedent is high.

A conditional is **acceptable** in case there is a causal relation between antecedent and consequent.

$\Delta^* P^e_i$ can be understood as a measure of the causal power of i to cause e.

A CAUSAL EXPLANATION

CHENG 1997

The causal power of i to bring about e

p_{ie}

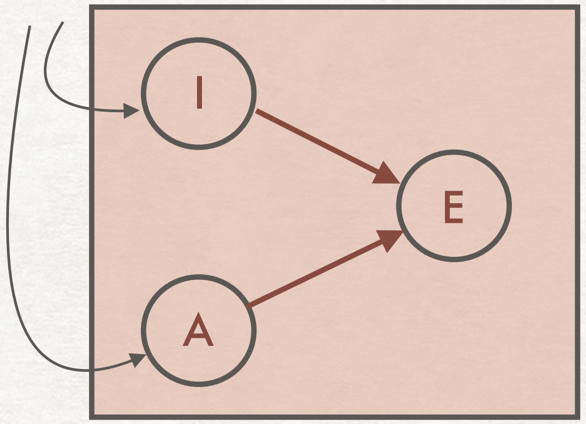
- also between 0 and 1
- but $p_{ie} \neq P(e|i)$!
- Cartwright's capacities
- Popper's propensity analysis of probability

A CAUSAL EXPLANATION

CHENG 1997

ASSUMPTION

Independent of each other



$$P(e) = P(i) \times p_{ie} + P(a) \times p_{ae} - (P(i \wedge a) \times p_{ie} \times p_{ae})$$

$$\begin{aligned}\Delta P_i^e &= P(e|i) - P(e|\neg i)\\ &= p_{ie} + (P(a|i) \times p_{ae}) - (p_{ie} \times P(a|i) \times p_{ae}) - (P(a|\neg i) \times p_{ae})\\ &= [1 - (P(a|i) \times p_{ae})] \times p_{ie} + [P(a|i) - P(a|\neg i)] \times p_{ae}\end{aligned}$$

$$p_{ie} = \frac{\Delta P_i^e - [P(a|i) - P(a|\neg i)] \times p_{ae}}{1 - P(a|i) \times p_{ae}} = \frac{\Delta P_i^e}{1 - P(a|i) \times p_{ae}} = \frac{\Delta P_i^e}{1 - P(e|\neg i)} = \Delta^* P_i^e$$

RESULT

$$p_{ie} = \Delta^* P_i^e$$

AN ALTERNATIVE APPROACH

A conditional is **acceptable** in case there is a causal relation between antecedent and consequent.

$\Delta^*P_i^e$ measures the presence of such a causal relation.

A conditional is **acceptable** iff the relative difference of the consequent given the antecedent is high.

$$\Delta^*P_i^e = \frac{P(e|i) - P(e|\neg i)}{1 - P(e|\neg i)}$$

CHALLENGE 1: BUT THE DATA!

VAN ROOIJ & SCHULZ 2018

Various empirical studies argue that conditional sentences are accepted just in case P(eli) is high.

➡ *Neither a analysis in terms of relative difference, nor in terms of causal power is correct.*

Not necessarily!

In certain circumstances causal power comes down to P(eli).

➡ *The causal approach can be defended.*

STILL A CAUSAL EXPLANATION

VAN ROOIJ & SCHULZ 2018

ASSUMPTION

E has only one cause: I

$$I \rightarrow E$$

$P(e) = P(i) \times p_{ie}$

$P(e|i) = p_{ie}$

$P(e|\neg i) = 0$

$$\Delta^* P_i^e = \frac{P(e|i) - P(e|\neg i)}{1 - P(e|\neg i)} = \frac{P(e|i) - 0}{1 - 0} = P(e|i)$$

RESULT

$$p_{ie} = \Delta^* P_i^e = P(e|i)$$

STILL A CAUSAL EXPLANATION

VAN ROOIJ & SCHULZ 2018

ASSUMPTION

E has only
one cause: I

$$I \rightarrow E$$

Is this a natural assumption?

Yes, it is!

When people's attention is drawn to one possible cause, they tend to overlook the possible existence of alternative causes.

cf. Koehler 1991, Brem & Rips 2000

Conditional perfection

RESULT

$$p_{ie} = \Delta^* P^e_i = P(e|i)$$

CHALLENGE 1: BUT THE DATA!

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CHALLENGE 2: DIAGNOSTIC CONDITIONALS

(1) If John is nervous, he smokes.
(2) If fire, then smoke.

But what about:

(3) If John smokes, he is nervous.
(4) If smoke, then fire.

CHALLENGE 2: DIAGNOSTIC CONDITIONALS

There are also conditionals that express an evidential or diagnostic dependency.

➡ *A causal power doesn't work in this case.*

That's not true!

We can explain the appropriateness of diagnostic conditionals in terms of relative difference.

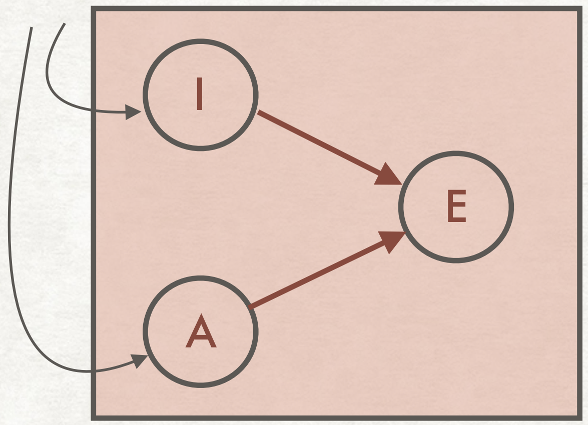
➡ *The causal approach can be defended.*

ALSO STILL A CAUSAL EXPLANATION

CHENG ET AL. 2007

ASSUMPTION

Independent of each other



(3) If John smokes, he is nervous.

(4) If smoke, then fire.

What's the likelihood that in case we have evidence for e it was caused by i, i.e. what's $P(i \rightsquigarrow e|e)$?

$$P(e) = P(i) \times p_{ie} + P(a) \times p_{ae} + (P(i \wedge a) \times p_{ie} \times p_{ae})$$

$$P(i \rightsquigarrow e|e) = \frac{P(i) \times p_{ie}}{P(i) \times p_{ie} + P(a) \times p_{ae} + (P(i \wedge a) \times p_{ie} \times p_{ae})} = \frac{P(i) \times p_{ie}}{P(e)}$$

Compare with Bayes' Law!

DEFINITION

$$P(i \rightsquigarrow e|e) = \frac{P(i) \times p_{ie}}{P(e)}$$

$$P(i|e) = \frac{P(i) \times P(e|i)}{P(e)}$$

ALSO STILL A CAUSAL EXPLANATION

VAN ROOIJ & SCHULZ 2018

ASSUMPTION

Independent
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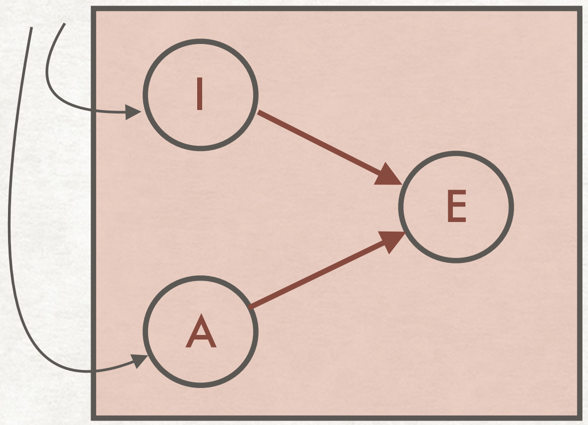

$$P(i \rightsquigarrow e|e) = \frac{P(i) \times p_{ie}}{P(e)}$$

$$\Delta^* P_i^e = \frac{P(e|i) - P(e|\neg i)}{1 - P(e|\neg i)} = \frac{P(e|i) - P(e)}{P(\neg e \wedge \neg i)}$$

$$\Delta^* P_e^i = \frac{P(i|e) - P(i)}{P(\neg i \wedge \neg e)}$$

$$\Delta^* P_i^e = \frac{P(e)}{P(i)} \Delta^* P_e^i = \frac{P(e)}{P(i)} p_{ie} = P(i \rightsquigarrow e|e)$$

$$p_{ie} = \Delta^* P_i^e$$

RESULT

$$P(i \rightsquigarrow e|e) = \Delta^* P_i^e$$

CHALLENGE 2: DIAGNOSTIC CONDITIONALS

VAN ROOIJ & SCHULZ 2018

There are also conditionals the express an evidential or diagnostic dependency.

➡ *A causal power doesn't work in this case.*

That's not true!

We can explain the appropriateness of diagnostic conditionals in terms of relative difference.

➡ *The causal approach can be defended.*

CHALLENGE 3: EPISTEMIC CONDITIONALS

But what about:

(5) If it wasn't the butler, then it was the gardener.

CHALLENGE 3: EPISTEMIC CONDITIONALS

There are also conditionals that express purely epistemic reasoning.

➡ *A causal power doesn't work in this case.*

Not necessarily!

We can explain the appropriateness of epistemic conditionals in terms of causal powers as well.

Epistemic conditionals are about the causal power of the information in the antecedent to make us believe the consequent.

➡ *The causal approach can be defended.*

SUMMARY

AN ALTERNATIVE APPROACH

MISSING LINK CONDITIONALS

(1) If Arsenal wins next year's Champions League final, Great Britain will join the European Union again. [adapted from Douven, 2017]

A conditional is **acceptable** in case there is a causal relation between antecedent and consequent.

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AN ALTERNATIVE APPROACH

- **Challenge 1:** the data point to a conditional probability approach

- **Challenge 2:** what about diagnostic conditionals?

- **Challenge 3:** what about epistemic conditionals?

EXTENSION 1: BISCUIT CONDITIONALS

VAN ROOIJ & SCHULZ, SUBMITTED

Contingency:

$\Delta P^e_i = P(e|i) - P(e|\neg i)$

Relative difference

$$\boxed{\Delta^* P^e_i = \frac{\Delta P^e_i}{1 - P(e|\neg i)}}$$

Representativeness:

$\nabla P^e_i = P(e|i) \times V(e|i) - P(e|\neg i) \times V(e|\neg i)$

Measures the absolute value/ intensity of the consequent given the antecendet

(6) If you're hungry, there are biscuits on the table.

(7) Ashley will come, even if Harvey will come.

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Experiments in aversive (i.e. fear) conditioning paradigms: acquisition and strength of association increases with the intensity of the stimulus.

(6) If you're hungry, there are biscuits on the table.

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Relative difference+

$$\boxed{\Delta^* P_i^e = \frac{\nabla P_i^e}{1 - P(e|\neg i)}}$$

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Relative difference

$$\boxed{\Delta^* P_i^e = \frac{\Delta P_i^e}{1 - P(e|\neg i)}}$$

Relative difference+

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VAN ROOIJ & SCHULZ, SUBMITTED

Representativeness:

$$\nabla P_i^e = P(e|i) \times V(e|i) - P(e|\neg i) \times V(e|\neg i)$$

$=$ $\neq$

Relative difference+

$$\boxed{\Delta^* P_i^e = \frac{\nabla P_i^e}{1 - P(e|\neg i)}}$$

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EXTENSION 2: GENERICS

VAN ROOIJ & SCHULZ, SUBMITTED

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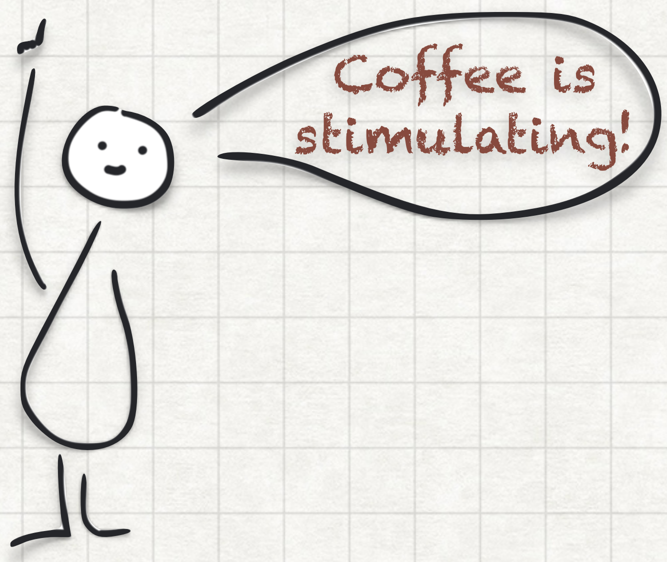
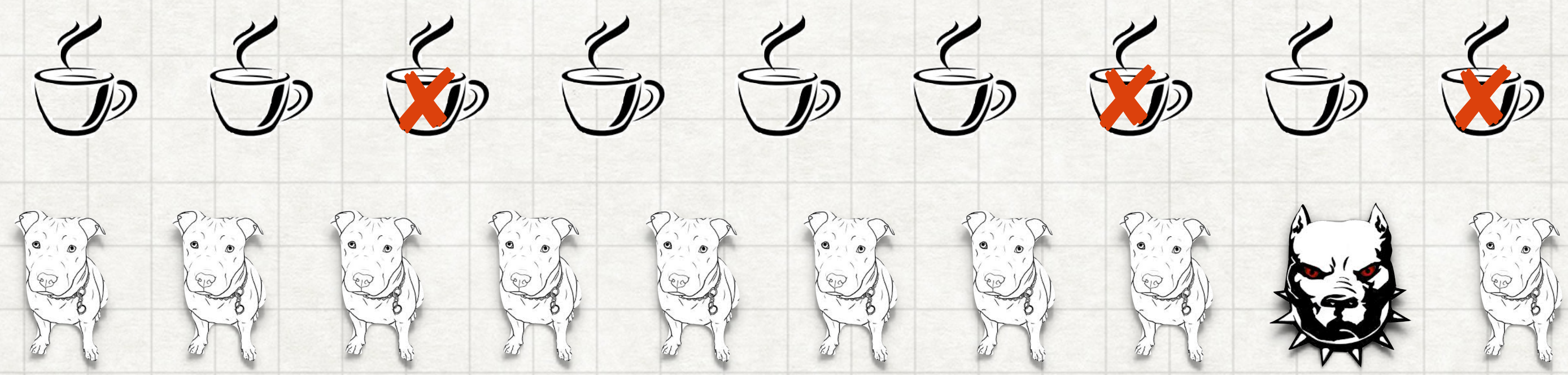
GENERIC SENTENCES $\xrightarrow{\text{express}}$ GENERALISATIONS



GENERIC SENTENCES $\xrightarrow{\text{express}}$ GENERALISATIONS

?

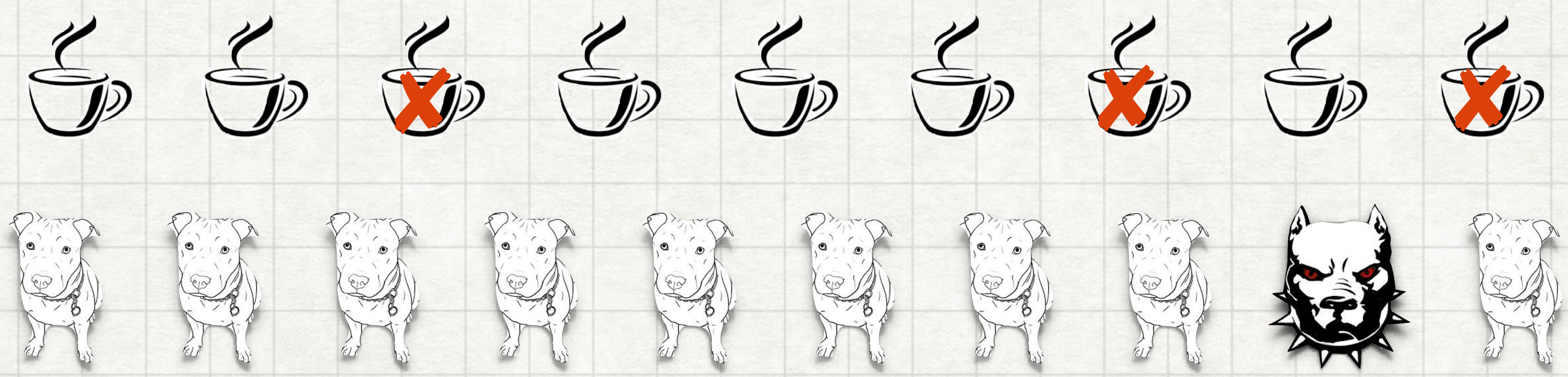
Reality/Observation



GENERIC SENTENCES $\xrightarrow{\text{express}}$ GENERALISATIONS

Meaning ?

Reality/Observation

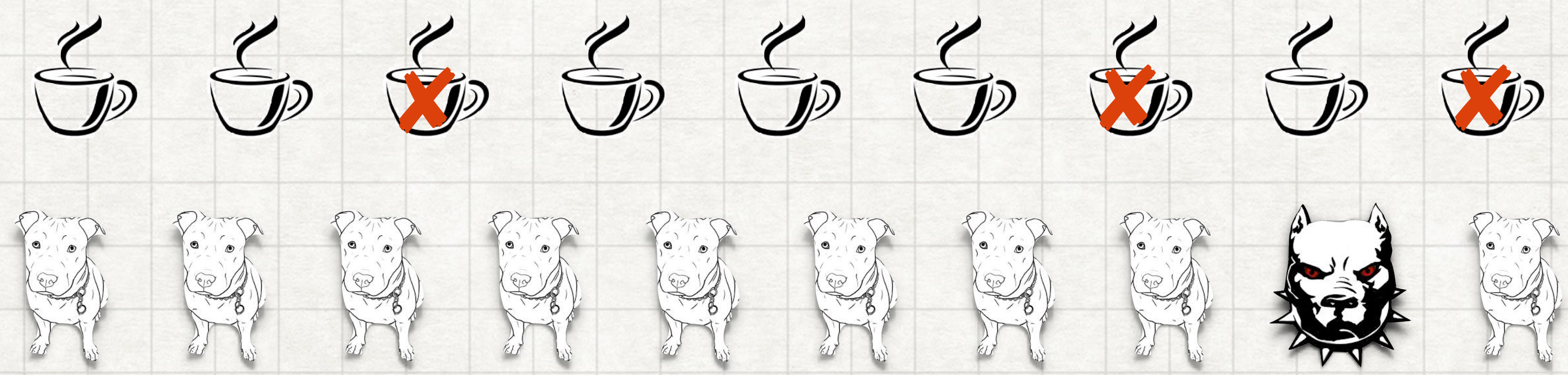


Generic Sentences $\xrightarrow{\text{express}}$ Generalisations

Meaning ?

Learning!

Reality/Observation



Generic Sentences $\xrightarrow{\text{express}}$ Generalisations

Meaning = Learning

Relative difference+

$$\Delta^* P_i^e = \frac{\nabla P_i^e}{1 - P(e|\neg i)}$$

"Pit-bulls are dangerous."

EXTENSION 2: GENERICS

VAN ROOIJ & SCHULZ, SUBMITTED

[Cohen 1999]

<u>Absolute reading</u>

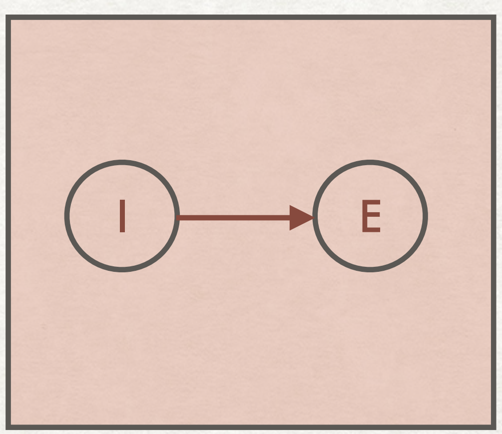
<u>Relative reading</u>

$P(e|i)$ ← **Special case** — $\Delta^* P_i^e$

[Van Rooij & Schulz]

<u>ASSUMPTION</u>

E has only
one cause: I



(8) Humans are mortal.

➡ *Humans are mortal by virtue of their being human.*

"The game of billiards has destroyed my naturally sweet disposition."

— Mark Twain, April 24, 1906

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BIBLIOGRAPHY

- Douven (2017). How to account for the oddness of missing-link conditionals. Synthese 194:1541-1554.
- Krzyżanowska et al. (2014). Rethinking Gibbard's Riverboa Argument. Studia Logica: An internation journal for symbolic logic, 102(4): 771-792.
- Skovgaard-Olsen (2017). Relevance differently affects the truth, acceptability and probability evaluations of 'and', 'but', therefore', and 'if then'. Cognition, 150:26-36.