Twelve Angry Men: A Study on the Fine-grain of Announcements

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"... your verdict must be unanimous."

"You've listened to the testimony, had the law interpreted as it applies in this case. It's now your duty to sit down and try and separate the facts from the fancy. One man is dead. Another man's life is at stake. If there's a reasonable doubt in your minds as to the guilt of the accused, a reasonable doubt, then you must bring me a verdict of not guilty. If there's no reasonable doubt, then you must, in good conscience, find the accused guilty. However you decide, your verdict must be ***unanimous.***"

Relevant notions of information

- *Explicit information*: information available without any further process.
- *Implicit information*: closure under logical consequence of explicit information.
- *Awareness of*: the agent's current language.

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The language

The language

Formulas ($\mathcal{L}_f$) and rules ($\mathcal{L}_r$), resp.:

$$\varphi ::= p \mid {}^{[i]}\boldsymbol{p} \mid \mathbf{Ac}_i\,\boldsymbol{\varphi} \mid \mathbf{R}_i\,\boldsymbol{\rho} \mid \neg\varphi \mid \varphi \vee \psi \mid \Box_i\varphi$$
$$\rho ::= (\{\varphi_1, \ldots, \varphi_{n_\rho}\}, \psi)$$

Extending ${}^{[k]}p$ to ${}^{[k]}\varphi$

$$\begin{aligned} {}^{[k]}(\mathrm{Ac}_i\,\varphi) &:= {}^{[k]}\varphi & {}^{[k]}(\varphi \vee \psi) &:= {}^{[k]}\varphi \wedge {}^{[k]}\psi \\ {}^{[k]}(\mathrm{R}_i\,\rho) &:= {}^{[k]}\,\mathrm{TR}(\rho) & {}^{[k]}(\Box_i\varphi) &:= {}^{[k]}\varphi \\ {}^{[k]}(\neg\varphi) &:= {}^{[k]}\varphi & {}^{[k]}({}^{[i]}\varphi) &:= {}^{[k]}\varphi \end{aligned}$$

The semantic model

Semantic model

A tuple $M = \langle W, R_i, V, PA_i, AC_i, R_i \rangle$ where:

- $\langle W, R_i, V \rangle$ is a Kripke model (R an equivalence relation).
- $PA_i : W \to \wp(\mathrm{P})$ is the *propositional availability function* ($PA_i(w)$ are the atoms at disposal for i at w).
- $AC_i : W \to \wp(\mathcal{L}_f)$ is the *access set function* ($AC_i(w)$ are the formulas i can access at w).
- $R_i : W \to \wp(\mathcal{L}_r)$ is the *rule set function* ($R_i(w)$ are the rules i can access at w).

M is the class of all semantic models.

The semantic interpretation

Semantic interpretation

$$
\begin{array}{lll}
(M,w) \models {}^{[i]}p & \text{iff} & p \in PA_i(w) \\
(M,w) \models \mathrm{Ac}_i\,\varphi & \text{iff} & \varphi \in AC_i(w) \\
(M,w) \models \mathrm{R}_i\,\rho & \text{iff} & \rho \in R_i(w)
\end{array}
$$

Axiom system: *S5*.

Awareness, implicit and explicit knowledge

i is ***aware of*** formula φ	$\mathrm{Aw}_i(\varphi) := \Box_i\, {}^{[i]}\boldsymbol{\varphi}$
i is ***aware of*** rule ρ	$\mathrm{Aw}_i(\rho) := \Box_i\, {}^{[i]}\,\mathbf{TR}(\boldsymbol{\rho})$
i knows φ ***implicitly***	$\mathrm{Im}_i(\varphi) := \Box_i({}^{[i]}\varphi \wedge \boldsymbol{\varphi})$
i knows ρ ***implicitly***	$\mathrm{Im}_i(\rho) := \Box_i({}^{[i]}\,\mathrm{TR}(\rho) \wedge \mathbf{TR}(\boldsymbol{\rho}))$
i knows φ ***explicitly***	$\mathrm{Ex}_i(\varphi) := \Box_i({}^{[i]}\varphi \wedge \varphi \wedge \mathbf{Ac}_i\,\boldsymbol{\varphi})$
i knows ρ ***explicitly***	$\mathrm{Ex}_i(\rho) := \Box_i({}^{[i]}\,\mathrm{TR}(\rho) \wedge \mathrm{TR}(\rho) \wedge \mathbf{R}_i\,\boldsymbol{\rho})$

Back to the Jury's room

A	B	C
$\Box_A \mathrm{TR}(sgns \Rightarrow gls)$	$\Box_B \mathrm{TR}(sgns \Rightarrow gls)$	$\Box_C \mathrm{TR}(sgns \Rightarrow gls)$
$\Box_A \mathrm{TR}(gls \Rightarrow ss)$	$\Box_B \mathrm{TR}(gls \Rightarrow ss)$	$\Box_C \mathrm{TR}(gls \Rightarrow ss)$
$\Box_A \mathrm{TR}(ss \Rightarrow \neg glt)$	$\Box_B \mathrm{TR}(ss \Rightarrow \neg glt)$	$\Box_C \mathrm{TR}(ss \Rightarrow \neg glt)$
		$\Box_C\, sgns$
$\Box_A \mathrm{R}_A\,(sgns \Rightarrow gls)$	$\Box_B \mathrm{R}_B\,(sgns \Rightarrow gls)$	$\Box_C \mathrm{R}_C\,(sgns \Rightarrow gls)$
$\Box_A \mathrm{R}_A\,(gls \Rightarrow ss)$	$\Box_B \mathrm{R}_B\,(gls \Rightarrow ss)$	$\Box_C \mathrm{R}_C\,(gls \Rightarrow ss)$
$\Box_A \mathrm{R}_A\,(ss \Rightarrow \neg glt)$	$\Box_B \mathrm{R}_B\,(ss \Rightarrow \neg glt)$	$\Box_C \mathrm{R}_C\,(ss \Rightarrow \neg glt)$
		$\Box_C \mathrm{Ac}_C\, sgns$
$\mathrm{Aw}_A(glt)$	$\mathrm{Aw}_B(glt)$	$\mathrm{Aw}_C(glt)$
$\mathrm{Aw}_A(ss)$		

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The intuition

- Agent j ***considers*** p.
- Agent j ***apply*** ρ ***to infer*** $\mathrm{conc}(\rho)$.
- Agent j ***announces*** φ

The operations

Given $M = \langle W, R_i, V, PA_i, AC_i, R_i \rangle$, we get

- $M^{\ulcorner q \urcorner;j} = \langle W, R_i, V, PA'_i, AC_i, R_i \rangle$, with

$$PA'_j(w) := PA_j(w) \cup \{q\}$$

- $M_{j;\sigma} = \langle W, R_i, V, PA_i, AC'_i, R_i \rangle$, with

$$AC'_j(w) := \begin{cases} AC_j(w) \cup \{\mathbf{conc}(\sigma)\} & \text{if } \sigma \in R_j(w) \ \& \ \mathbf{prem}(\sigma) \subseteq AC_j(w) \\ AC_j(w) & \text{otherwise} \end{cases}$$

- $M_{j;\chi!} = \langle W', R'_i, V', PA'_i, AC'_i, R'_i \rangle$, with W', R'_i, V' as in PAL and

$$PA'_i(w) := PA_i(w) \cup \mathbf{atom}(\chi) \qquad AC'_i(w) := AC_i(w) \cup \{\chi\}$$

Modalities for the actions

Preconditions

$$\mathrm{Pre}(j;\sigma) := \mathbf{Ex}_j(\sigma) \wedge \bigwedge_{\varphi \in \mathbf{prem}(\sigma)} \mathbf{Ex}_j(\varphi) \qquad \mathrm{Pre}(j;\chi!) := \mathbf{Ex}_j(\chi)$$

Semantic interpretation

$$\begin{array}{lll} (M,w) \models \langle \ulcorner q \urcorner; j \rangle \varphi & \text{iff} & (M_{\ulcorner q \urcorner; j}, w) \models \varphi \\ (M,w) \models \langle j;\sigma \rangle \varphi & \text{iff} & (M,w) \models \mathrm{Pre}(j;\sigma) \text{ and } (M_{j;\sigma}, w) \models \varphi \\ (M,w) \models \langle j;\chi! \rangle \varphi & \text{iff} & (M,w) \models \mathrm{Pre}(j;\chi!) \text{ and } (M_{j;\chi!}, w) \models \varphi \end{array}$$

Some properties of the actions

Proposition

- *After considering p, the agent is aware of p*:

$$[\ulcorner q \urcorner; j]\, \mathbf{Aw}_j(q)$$

- *After applying σ, the agent knows* conc(σ) *explicitly*:

$$[j; \sigma]\, \mathbf{Ex}_j(\mathbf{conc}(\sigma))$$

- For χ ***propositional***, *after the announcement of χ, all agents know χ explicitly*:

$$[j; \chi!]\, \mathbf{Ex}_i(\chi)$$

Rubbing your nose

A: ***Don't you feel well?***

$$\langle\ulcorner sgns\urcorner; A\rangle\langle\ulcorner gls\urcorner; A\rangle(\mathrm{Aw}_A(sgns) \wedge \mathrm{Aw}_A(gls) \wedge \mathrm{Aw}_A(sgns \Rightarrow gls) \wedge \mathrm{Aw}_A(gls \Rightarrow ss) \wedge \mathrm{Aw}_A(ss \Rightarrow \neg glt))$$

"... the woman [...] had those same marks ..."

A: ***Now, why were you rubbing your nose like that?***

D: ***If it's any of your business, I was rubbing it because it bothers me a little.***

A: ***Your eyeglasses made those two deep impressions on the sides of your nose.***

A: ***I hadn't noticed that before.***

A: ***The woman who testified that she saw the killing had those same marks on the sides of her nose.***

$$\langle A; \mathrm{Aw}_A(sgns)! \rangle \left(\mathrm{Aw}_{JURY}(sgns) \wedge \mathrm{Ex}_C(sgns) \wedge \langle A; sgns! \rangle \mathrm{Ex}_{JURY}(sgns) \right)$$

"Listen, he's right!"

C: ***Hey, listen. Listen, he's right. I saw them too. I was the closest one to her. She had these things on the side of her nose.***

$\langle C; sgns! \rangle \mathrm{Ex}_{JURY}(sgns)$

"... eyeglasses?"

D: ***What point are you makin'?***

D: ***She had dyed hair, marks on her nose. What does that mean?***

A: ***Could those marks be made by anything other than eyeglasses?***

$$\langle A; sgns \Rightarrow gls \rangle \left(\mathrm{Ex}_A(gls) \wedge \langle A; gls! \rangle \, \mathrm{Ex}_{JURY}(gls) \right)$$

"... the woman's eyesight is in question now!"

D: ***How do you know what she saw? How does he know all that? How do you know what kind of glasses she wore? Maybe they were sunglasses! Maybe she was far-sighted! What do you know about it?***

C: ***I only know the woman's eyesight is in question now.***

E: ***She had to be able to identify a person 60 feet away, at night, without glasses.***

F: ***You can't send someone off to die on evidence like that.***

$$\langle C; gls \Rightarrow ss \rangle \left(\mathrm{Ex}_C(ss) \wedge \langle C; ss! \rangle \, \mathrm{Ex}_{JURY}(ss) \right)$$

"Not guilty"

C: ***Don't you think the woman may have made a mistake?***

D: ***No.***

C: ***It's not possible?***

D: ***No, it's not possible.***

C: ***Is it possible?***

B: ***Not guilty.***

$$\langle B; ss \Rightarrow \neg glt \rangle \left(\mathrm{Ex}_B(\neg glt) \wedge \langle B; \neg glt! \rangle \, \mathrm{Ex}_{JURY}(\neg glt) \right)$$

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So far . . .

- Logical framework for representing ***awareness of***, ***implicit information*** and ***explicit information***.
- Representation of the way these notions evolve via ***consideration***, ***inference*** and ***announcements***.
- Representation of ***interaction*** between agents with diverse information.
- ***Sound*** and ***complete*** logics for the static and dynamic parts.

Further work

- Group-notions of information (e.g., implicit and explicit ***common knowledge***).
- Other notions of information (***belief***, ***safe belief***, ***conditional belief***).
- Analysis of ***deliberation***:
  - Changes of information.
  - Strategies for (not) sharing information.
  - Powers.

Grazie and Gracias