LINT
Logic for Interaction

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ILLC, University of Amsterdam
Can mathematics, and thereby many other sciences which are based on mathematics, be treated effectively, that is, based on simple logical rules that even a computer can follow?
What has been the main source of ideas for mathematics and thereby for logic in the past?
What will probably be the main source of ideas for mathematics and thereby for logic in the future?
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Customers who bought this item also bought

- Theory of Moves by Steven J. Brams
- Biblical Games: Game Theory and the Hebrew Bible by Steven J. Brams
- Negotiation Games by Steven J. Brams
- The Strategy of Conflict by Thomas C. Schelling
- Fair Division: From Cake-Cutting to Dispute Resolution by Steven J. Brams
Logic and games - background

• Dialogical logic (Lorenzen)
• Game theoretic semantics (Henkin)
• Semantic tableaux (Beth)
• EF game (Ehrenfeucht, Fraïssé)
### Change we can

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
</tr>
</thead>
<tbody>
<tr>
<td>individual</td>
<td>group</td>
</tr>
<tr>
<td>perfect information</td>
<td>imperfect information</td>
</tr>
<tr>
<td>classical logic</td>
<td>classical+constructive+linear</td>
</tr>
<tr>
<td>agent makes choices</td>
<td>choices make the agent</td>
</tr>
<tr>
<td>features are fixed predicates</td>
<td>features are fixed variables</td>
</tr>
</tbody>
</table>
Is the shirt color dependent on the pants color?

meaningless question
Is the shirt color dependent on the pants color?

meaningless question
Is the shirt color dependent on the pants color?

Yes! White pants $\rightarrow$ yellow shirt, blue pants $\rightarrow$ red shirt
Is the shirt color dependent on the socks and shoes color?

Yes!
<table>
<thead>
<tr>
<th>record</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
<th>A6</th>
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</table>
Whatever decisions the governments make in the next 10 years, it seems likely that by the year 2050 the sea levels will rise, but whether the rise is over 25 cm depends on whether the industrialized nations start reducing their greenhouse gas emissions now. (Modal dependence logic)
Can you write this in a decidable fragment of first order:

A passport official at an airport only wants to know whether you have a valid visa or not. If you do, she lets you in; if not, she sends you back on the next flight. *(Social software, Rohit Parikh)*
If the social welfare function respects unanimity and independence of irrelevant alternatives, it is a dictatorship. (Social choice theory, Arrow’s Theorem)
In a game a play is built up from the choices of the players.

By looking at many plays we can learn about the players.
First I picks a natural number $x_0$.
Then II picks a natural number $x_1$.
Then II picks a natural number $x_2$, without now using knowledge of what $x_0$ is.
II wins if $x_2 > x_0$. 
Is II obeying the rules?

<table>
<thead>
<tr>
<th>I: $x_0$</th>
<th>II: $x_1$</th>
<th>II: $x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
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<tr>
<td>3</td>
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<td>100</td>
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<tr>
<td>53111468</td>
<td>53111468</td>
<td>53111469</td>
</tr>
</tbody>
</table>

<table>
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<tr>
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<tbody>
<tr>
<td>1</td>
<td>0</td>
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<tr>
<td>53111468</td>
<td>0</td>
<td>53111469</td>
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Summing up: dependence arises in

- Causality
- Data
  - Biological
  - Financial
  - Sociological
  - Cosmological
- Social software
- Social choice
- Natural language
- Games
Let us investigate Dependence Logic
• Dependence does not manifest itself in a **single** event or observation.

• We need semantics where the basic concept is a **set** of observations

• Such sets are called **teams**.
The Intuition of Teams

• Teams accomplish tasks by
  – Every member doing the same
  – Dividing into subteams (skills)
  – Supplementing a new feature, (a skill)
  – Duplicating along a feature, (gender)

• Teams manifest dependence by e.g.
  – Letting rank, not gender, determine salary
Plays where rook or queen was sacrificed:
Example

• If $\neg \varphi \lor \psi$ is valid then $\varphi$ *logically implies* $\psi$. 

This part must be empty!
<table>
<thead>
<tr>
<th>Team X</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Duplicated team</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Finnish driver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swedish author</td>
</tr>
<tr>
<td>Norwegian skier</td>
</tr>
</tbody>
</table>

| Finnish **male** driver |
| Finnish **female** driver |
| Swedish **male** author |
| Swedish **female** author |
| Norwegian **male** skier |
| Norwegian **female** skier |
• Dependence logic has features related to
  – Game theory: games of imperfect information
  – Intuitionism: dependence on evidence
  – Linear logic: dependence on resources

Unpublished joint work with Abramsky
Conclusion

There is a mathematical theory of dependence with applications to games, logic, computer science, linguistics, economics, etc.
The price

- An exponential complexity jump.
- Logarithmic compression.
True in all models

False in all models

True in some models

False in some models

Non-determined
• There is a natural logic of dependence based on the lift from individuals to teams.
• Concepts can be clarified: compositionality, imperfect information, non-determinacy.
• A basic Logic for Interaction.
LIN T subprojects

- **DepLog** *(Amsterdam)* To engage in a thorough investigation of the new dependence logic, to find its axiomatization, its fine-structure, and its relation to modal dependence theories.

- **ImpInf** *(Aachen)* To develop a uniform logical and operational framework for handling imperfect information in logical games and other interactive systems.

- **Dyn** *(Oxford)* To relate and merge the two major existing approaches to the logic of interaction: ‘local’ and ‘global’ dynamics.

- **LogCon** *(Gothenburg)* To apply existing methods from logic and mathematics for characterizing the standard logical constants for proof and truth to logical frameworks specifically designed to deal with interaction.
Principal Investigators

- **Grädel**, Aachen
  - Algorithmic model theory, fixed-point logics, algorithmic theory of games
- **Väänänen**, Amsterdam
  - Game-theoretic methods in logic, dependence logic
- **Westerståhl**, Gothenburg
  - Model-theoretic semantics, generalized quantifiers in natural language
- **Hella**, Tampere
  - Finite model theory, generalized quantifiers

Associated Partners

- **Abramsky**, Oxford
  - Semantics of computation, game semantics
- **Sandu**, Paris
  - Evaluation games, game theoretic semantics
Modelling intelligent interaction - Logic in the Humanities, Social and Computational sciences (LogICCC)

What does LINT have to do with the other projects?
- Dialogical Foundations of Semantics
  - Incorporating interaction into logical semantics
- The Logic of Causal and Probabilistic Reasoning in Uncertain Environments
  - Probability logic, causality, conditional independence
- Logical Models of Reasoning with Vague Information
  - Imperfect information, data extraction
- Games for Analysis and Synthesis of Interactive Computational Systems
  - Game-theoretic formalizations of interactive computational systems
- Vagueness, Approximation, and Granularity
  - “Is he tall?” “Depends on what you mean by tall!”
- Computational Foundations of Social Choice
  - Logic-based languages for modeling and reasoning about choice problems and preference structures
- SOCIAL SOFTWARE for elections, the allocation of tenders and coalition/alliance formation
  - Social software