The analysis of sufficiency

Philosophical Foundations of Explanation

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Plan

1. Analysing sufficiency
2. Reciprocity
3. Sufficiency via aboutness
4. Independent motivation for sufficiency from conditionals
   - Strengthening with a possibility

The need for sufficiency

(1) a. Ali has an Irish passport because he was born in Ireland.
    b. Ali has an Irish passport because he was born in Europe.

The need for sufficiency

(1) a. Ali has an Irish passport because he was born in Ireland.
b. Ali has an Irish passport because he was born in Europe.

(2) a. Being born in Ireland caused Ali to get an Irish passport.
b. Being born in Europe caused Ali to get an Irish passport.

The need for sufficiency

(3) a. Sue was allowed into the bar because she's over 21.
    b. Sue was allowed into the bar because she's over 16.

(4) a. The fact that Sue is over 21 caused the bouncer to let her in.
    b. The fact that Sue is over 16 caused the bouncer to let her in.

The need for sufficiency

(5) *The radio spontaneously starts playing music.*

A: Why did the radio turn on?
B: I have no idea. I didn't touch it.
A: I see it's plugged in, and it needs to be plugged in to turn on.
B: Right, but I still have no idea why it started playing.

The need for sufficiency with reasons

Sami and Jan are fun on their own, but always fight when together. A heard that they are both attending a party and therefore decids to skip it.

(6) a. I'm skipping the party for two reasons: because Sami is going and because Jan is going.
b. I'm skipping the party for one reason: because Sami and Jan are going.

(7) a. The reasons why I'm skipping the party are that Sami is going and that Jan is going.
b. The reason why I'm skipping the party is that Sami and Jan are going.

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My intuitive judgement: the (a)-sentences are odd, the (b)-sentences are fine.

The need for sufficiency with reasons

Sami and Jan are each miserable people. Even one of them going to a party is enough to make it a dull event.

(8) a. I'm skipping the party for two reasons: because Sami is going and because Jan is going.
  b. I'm skipping the party for one reason: because Sami and Jan are going.

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  b. The reason I'm skipping the party is that Sami and Jan are going.

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My intuitive judgement: the (a)- and (b)-sentences are both fine.

The sufficiency requirement

- *E because C* $\Rightarrow$ *C* is sufficient for *E*.
- *C cause E* $\Rightarrow$ *C* is sufficient for *E*.

What does it mean for *C* to be sufficient for *E*?

Sufficiency is not logical entailment

(10) a. My laptop turned on because I pushed the power button.
b. Pushing the power button caused the laptop to turn on.

$\not\Rightarrow$ In every logically possible world where I push the power button, the laptop turns on.

These are assertable even though there is a logically possible world where the laptop's battery is empty.

Is C sufficient for E just in case *if C would E* is true?

Problem

Many existing semantics of conditionals validate conjunctive sufficiency, predicting that C and E together entail *if C would E*.

The selection function approach (Stalnaker 1968)

> *Consider a possible world in which A is true, and which otherwise differs minimally from the actual world. 'If A, then B' is true (false) just in case B is true (false) in that possible world.*

Let W be the set of possible worlds, and $f : \wp(W) \times W \to W$ a function from propositions to worlds.

Proposal: $A > B$ is true at world w just in case B is true at $f(A, w)$.

Constraints on the selection function:

1. A is true at $f(A, w)$.
2. $f(A, w)$ is the absurd world λ (the world where every proposition is true) only if there is no possible world with respect to w in which A is true.
3. If A is true in w then $f(A, w) = w$.
4. If A is true in $f(B, w)$ and B is true in $f(A, w)$, then $f(A, w) = f(B, w)$.

Condition 3 ensures that $A \wedge C$ entails $A > C$.

The ordering approach (Lewis 1973)

Let W be a set. For any $w \in W$ let $\leq_w$ be a reflexive and transitive binary relation over W. For any sentences A and C and $w \in W$ define that a conditional $A > C$ is true at w (denoted $w \models A > C$) as follows:

$$w \models A > C \quad \text{iff} \quad \forall x \models A \; \exists y \models A \; \big(y \leq_w x \wedge \forall z \leq_w y \; (z \models A \to C)\big),$$

where $A \to C$ is the material conditional (that is, equivalent to $\neg A \vee C$).

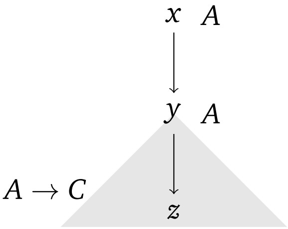


Figure: Illustrating the truth conditions of $A > C$.

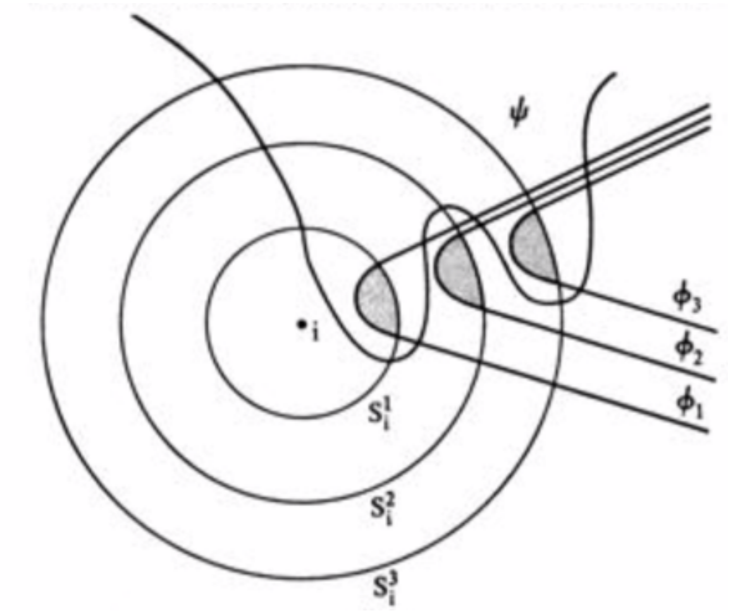


Figure: Lewis (1973) assumes strong centering: every world is more similar to itself than any other world is to it: $w \leq_w v$ and $\not(v \leq_w w)$ for all w, v where

Kratzer's semantics of *would*-conditionals

> *There is an intuitive and appealing way of thinking about the truthconditions for counterfactuals. It is an analysis that, in my heart of hearts, I have always believed to be correct...*
>
> *A "would"-counterfactual is true in a world w iff every way of adding propositions that are true in w to the antecedent while preserving consistency reaches a point where the resulting set of propositions logically implies the consequent.*
>
> — Kratzer (2012, p. 127)

Kratzer's semantics of *would*-conditionals

Definition (Admissible base set)

Let S be the set of situations (i.e. sets of parts of possible worlds). For any world w, an *admissible Base Set* is a subset F_w of $P(S)$ satisfying:

1. Truth: Every proposition in F_w is true at w: $w \in \bigcap F_w$.
2. Persistence: All proposition in F_w is persistent (for all $p \in F_w$ and situations s, s', if $s \in p$ and s is part of s', then $s' \in p$).
3. Cognitive Viability: All $p \in F_w$ are cognitively viable.
4. Non-Redundancy: F_w is not redundant (it does not contain propositions p and q such that $p \neq q$ and $p \cap W \subseteq q \cap W$).
5. Completeness: $\bigcap F_w$ contains all and only worlds that are indistinguishable from w, given the grain set by Cognitive Viability.

Define that proposition p *lumps* proposition q at world w just in case for any situation that is part of w in which p is true, q is true.

Kratzer's semantics of *would*-conditionals

Given an admissible base set and a proposition p, Kratzer defines the *crucial set* $F_{w,p}$ as follows.

Definition (The crucial set)

For any world w, admissible base set F_w, and proposition p, $F_{w,p}$ is the set of all subsets A of $F_w \cup \{p\}$ satisfying the following conditions:

1. A is consistent
2. $p \in A$
3. A is closed under lumping: for all $q \in A$ and $r \in F_w$: if q lumps r in w, then $r \in A$.

Definition (Truth conditions of "would"-counterfactuals)

Given a world w and an admissible Base Set F_w, a "would"-counterfactual with antecedent p and consequent q is true in w iff for every set in $F_{w,p}$ there is a superset in $F_{w,p}$ that logically implies q.

Reciprocity implies conjunctive sufficiency

$$\frac{A > B \qquad B > A \qquad B > C}{A > C}\text{ Reciprocity}$$

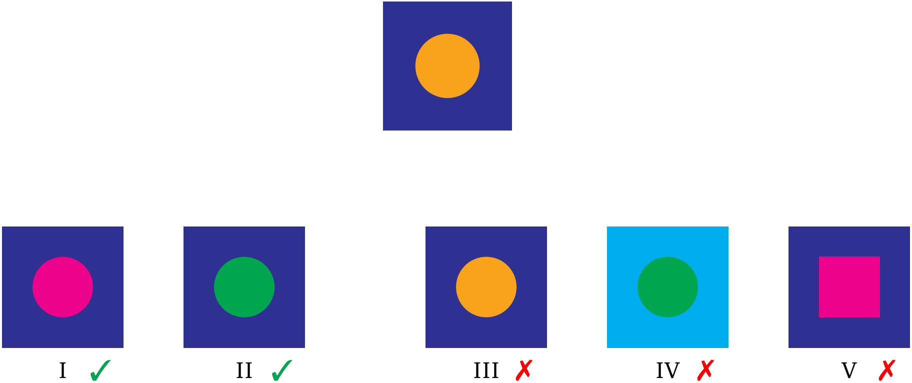
$$\frac{A \wedge B}{A > C}\text{ Conjunctive Sufficiency}$$

Walters and Williams (2013) show that, under mild assumptions, reciprocity also ensures that $A \wedge C$ implies $A > C$.

Consider any true A, C, and any B that is irrelevant to A and C, in the sense that $(B \vee \neg B) > A$ and $(B \vee \neg B) > C$ hold.

$$\frac{A > (B \vee \neg B) \qquad (B \vee \neg B) > A \qquad (B \vee \neg B) > C}{A > C}\text{ Reciprocity}$$

Given the existence of such a B, Reciprocity tells us that $A \wedge C$ implies $A > C$.


I ✓
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III ✗
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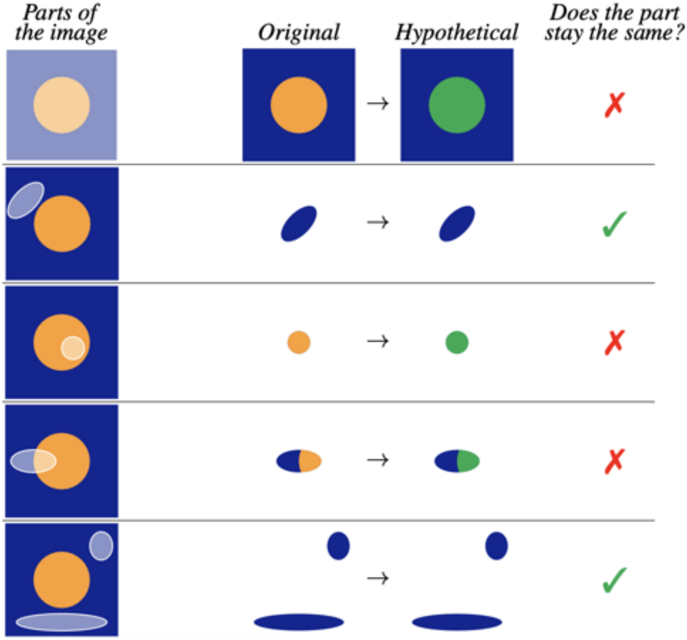
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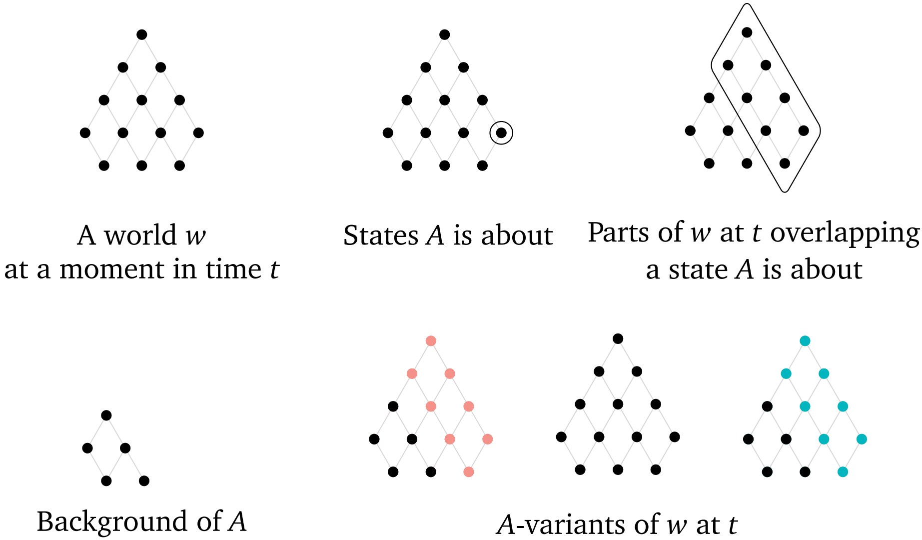
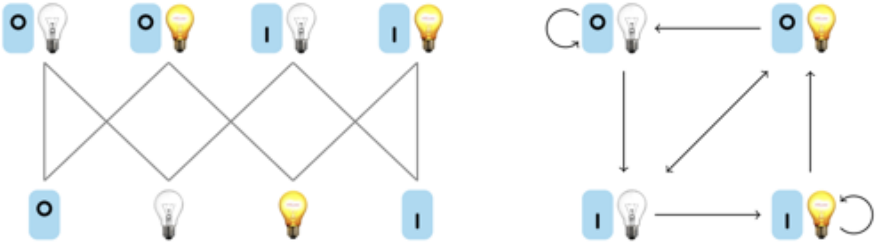


Figure: Steps to construct the A-variants of a world at a moment in time.



(a) Mereological structure. (b) Nomic possibilities.

Figure: Light switch example. Nomically possible worlds correspond to directed paths in (b).

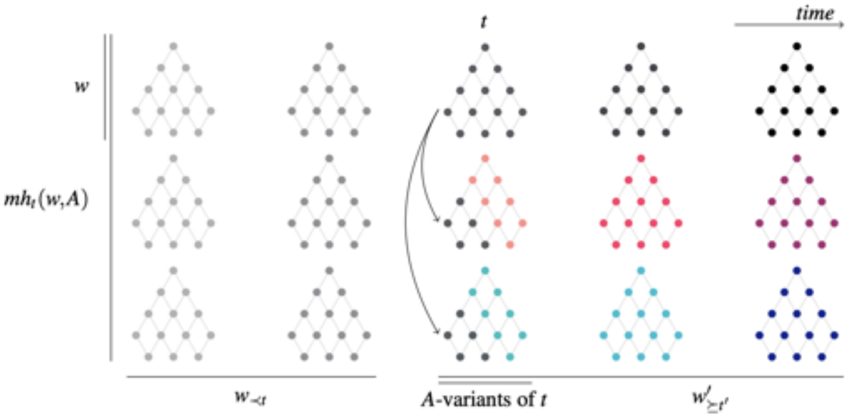


Figure: Constructing the modal horizon.

Definition (Nomic aboutness model)

Where S is a set and $\leq$ a binary relation on S, define

$$\begin{aligned} Sit &:= S \times I, \text{ where } I \text{ is an arbitrary label set,} \\ M &:= \{t_i \in Sit : t \leq u \textit{ implies } t = u \textit{ for all } u \in S\}, \\ W &:= \{(M', \preceq) : M' \subseteq M, \preceq \text{ is a linear order}\}. \end{aligned}$$

Definition (The modal horizon)

For any sentence A, moment $t \in M$ and world $w \in W$, define

$$mh_t(w, A) := \{w_{\prec t} \frown w'_{\succeq t'} : t' \textit{ is an A-variant of } t, t' \in w' \text{ and } w' \in P\}.$$

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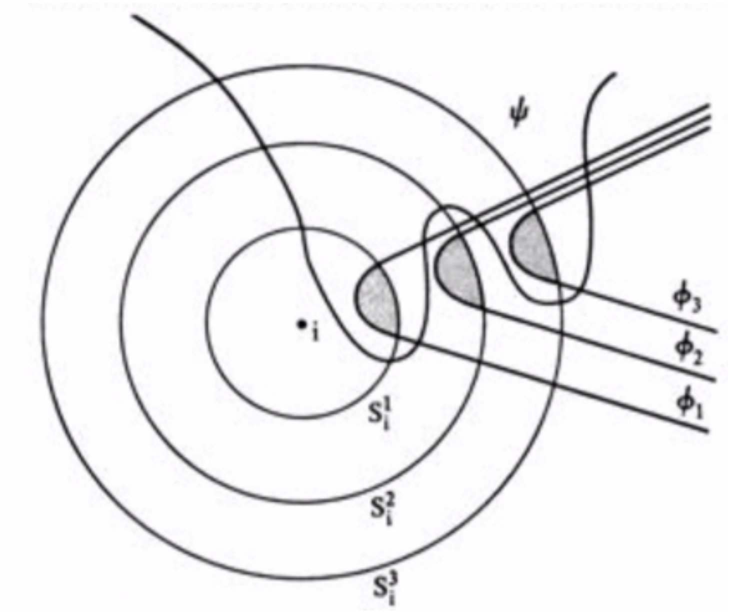


Figure: Lewis (1973)

Strengthening with a Possibility (aka rational monotonicity):

$$\frac{A > C \qquad A \mathbin{\Diamond\!\!\rightarrow} B}{(A \wedge B) > C}$$

This is valid in Lewis' (1973) sphere semantics for counterfactuals.

Counterexample

Boylan and Schultheis (2017, 2021)
Alice, Billy, and Carol are playing a simple game of dice. Anyone who gets an odd number wins $10; anyone who gets even loses $10. The die rolls are, of course, independent. What Alice rolls has no effect on what Billy rolls and vice versa. Likewise for Alice and Carol as well as for Billy and Carol.
Each player throws their dice. Alice gets odd; Billy gets even; Carol gets odd.

(11) a. If Alice and Billy had thrown the same type of number, then at least one person would still have won $10.
b. If Alice and Billy had thrown the same type of number, then Alice, Billy, and Carol could have all thrown the same type of number.
c. If Alice, Billy, and Carol had all thrown the same type of number, then at least one person would still have won $10.

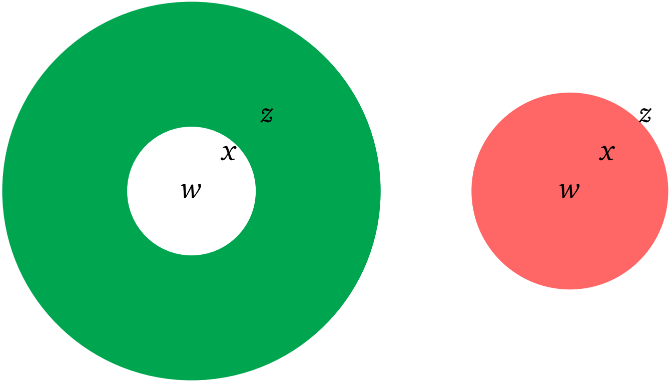
Ordering semantics

For every world w, let $<_w$ be a strict partial order over worlds.

$<_w$ is *almost connected* iff for all worlds w, x, y, z,
if $x <_w z$ then $x <_w y$ or $y <_w z$.

Strengthening with a Possibility is valid iff $<_w$ is almost connected (Veltman 1985, p. 103).

Our intuitive concept of closeness is total, and hence almost connected.



$x <_w y$ $y <_w z$

Figure: y must be in either the green or red region.

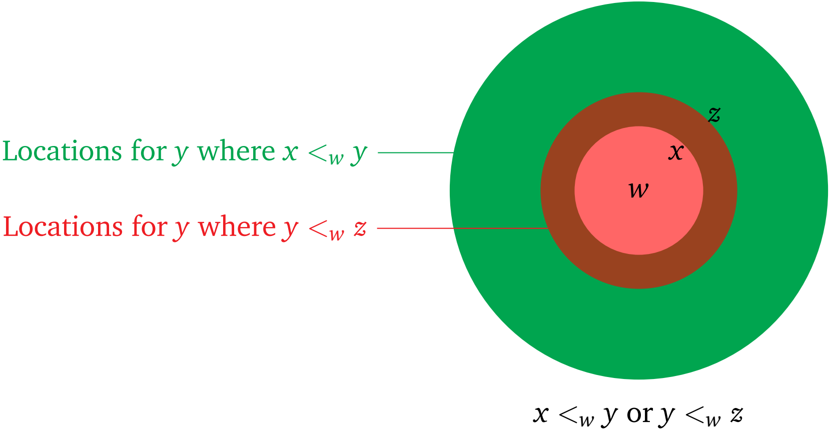


Figure: The green and red regions exhaust the domain: y must be in either the green or red region.

Our concept of closeness is almost connected.

If the semantics of counterfactuals is given by an order over worlds, it is an order that is not almost connected.

And therefore one that cannot be described in terms of ‘closer’ or ‘closest’ worlds.

Upshot: when we speak of the semantics of counterfactuals in terms of ‘closer’ or ‘closest’ worlds, we are strictly speaking making a mistake.

“Alice and Billy threw the same type of number” is about the state of Alice’s throw and Billy’s throw.
“Alice and Billy and Carol” threw the same type of number” is about the state of all three throws.

Bacon's counterexample to reciprocity

$$\frac{A > B \qquad B > A \qquad B > C}{A > C}\text{ Reciprocity (a.k.a. CSO)}$$

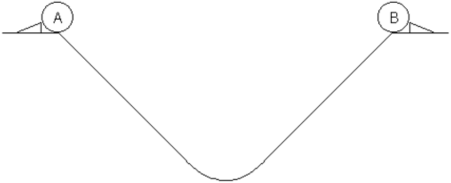
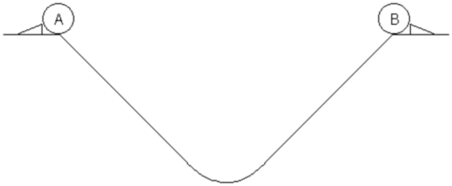


Figure: Bacon's counterexample to reciprocity.

(12) a. If A fell, B would fall.
b. If B fell, A would fall.
c. If A fell, the light would turn green.
d. If B fell, the light would turn red.



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(13) a. If A had fallen, B would have fallen.
b. If B had fallen, A would have fallen.
c. If A had fallen, the light would have turned green.
d. If B had fallen, the light would have turned red.

The selection function approach validates reciprocity

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Let W be a set. For any $w \in W$ let $\leq_w$ be a reflexive and transitive binary relation over W. For any sentences A and C and $w \in W$ define that a conditional $A > C$ is true at w (denoted $w \models A > C$) as follows:

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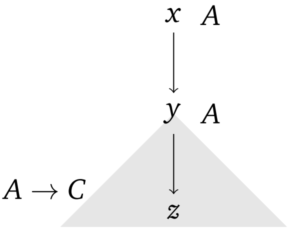

Figure: Illustrating the truth conditions of $A > C$.

The ordering approach validates reciprocity

Proof.

Pick any $w \in W$ and suppose that $A > B$, $B > A$ and $B > C$ are true at w. To show that $A > C$ is true at w, pick any $x \models A$. We have to show that

there is a $y \models A$ such that $y \leq_w x$ and for all $z \leq_w y$, $z \models A \rightarrow C$.

Since $w \models A > B$ and $x \models A$, there is a $v \models A$ such that $v \leq_w x$ and (i) for all $v' \leq_w v$, $v' \models A \rightarrow B$. Since $\leq_w$ is reflexive, $v \leq_w v$, so $v \models A \rightarrow B$. Thus $v \models B$.
Since $w \models B > A$ and $v \models B$, there is a $u \models B$ such that $u \leq_w v$ and (ii) for all $u' \leq_w u$, $u' \models B \rightarrow A$.
Since $w \models B > C$ and $u \models B$, there is a $y \models B$ such that $y \leq_w u$ and (iii) for all $z \leq_w y$, $z \models B \rightarrow C$.
Since $y \leq_w u$, by (ii), $y \models B \rightarrow A$. Then as $y \models B$, $y \models A$.
And as $y \leq_w u \leq_w v \leq_w x$, by transitivity of $\leq_w$, $y \leq_w x$.
We show that for all $z \leq_w y$, $z \models A \rightarrow C$. Pick any $z \leq_w y$. Then $z \leq_w y \leq_w u \leq_w v$, so by transitivity of $\leq_w$, $z \leq_w v$. Then by (i), $z \models A \rightarrow B$.
And since $z \leq_w y$, by (iii), $z \models B \rightarrow C$. Hence $z \models A \rightarrow C$. □

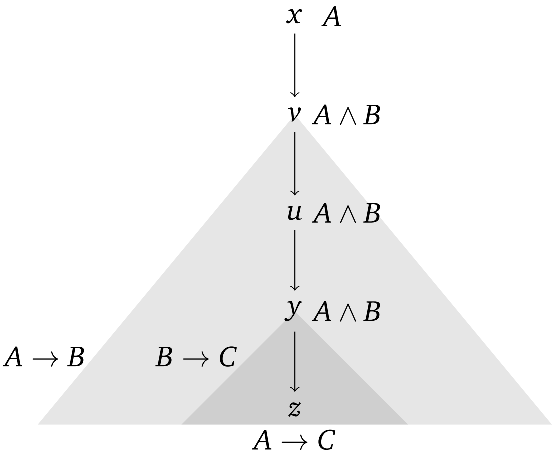

Figure: Illustrating the proof that reciprocity is valid on the ordering semantics.