Defining sufficiency in truthmaker semantics

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May 1, 2023
The light is off because switch A is down.

The big question

When we interpret causal words, such as *because*, we consider alternatives to reality. How do we decide which ones to consider?
1. **Sufficiency in *because***

2. **Sufficiency via the conditional?**
   - Evidence for strong centering
   - Evidence for conditional excluded middle

3. **Sufficiency in a situation semantics**
(2) The robot took Road B because it took First Street.
(3) *Alice is 20 years old. The legal drinking age is 18.*
   a. Alice can order alcohol because she is over sixteen years old.
   b. Alice can order alcohol because she is over eighteen years old.

(4) *Priya’s mother was born in India. Priya has an Indian passport.*
   a. Priya has an Indian passport because her mother is from Asia.
   b. Priya has an Indian passport because her mother is from India.

(5) *Let* $x$ *and* $y$ *be numbers, where* $x \neq 0$ *and* $y = 0$.
   a. $xy$ *is 0 because* $y$ *is less than 10.
   b. $xy$ *is 0 because* $y$ *is 0.*
If $E$ because $C$ is true, then $C$ was sufficient for $E$ with respect to some set of background conditions.

- A first thought: $C$ is sufficient for $E$ just in case the conditional $if$ $C$, $E$ is true.

- Problem: existing semantics for conditionals validate strong centering

- **Strong centering:** if $A$ is true then $if$ $A$, $C$ is true if and only if $C$ is.
Plan

1. Sufficiency in *because*

2. Sufficiency via the conditional?
   - Evidence for strong centering
   - Evidence for conditional excluded middle

3. Sufficiency in a situation semantics
(6) Alice: “I bet that if you flip the coin, it will land heads.”

Bob flips the coin and it lands heads.
(6) Alice: “I bet that if you flip the coin, it will land heads.”

Bob flips the coin and it lands heads.

Given that Bob flipped the coin, strong centering correctly predicts that Alice’s sentence is true iff the coin actually landed heads.

Without strong centering, Bob could reply:

(7) Bob: What Alice said is false since the coin could have landed tails. So she doesn’t get her money.

Examples featuring will/would conditionals in betting contexts have been previously discussed by Prior (1976, p. 100), Moss (2013), Belnap, Perloff, and Xu (2001, p. 160), Cariani and Santorio (2018) and Cariani (2021, p. 63).
Probability judgements

(8) Alice: “If you flip the coin, it will land heads.”

What is the probability that what Alice said is true?
(8) Alice: “If you flip the coin, it will land heads.”

What is the probability that what Alice said is true?

(9) Bob: Since the coin could have landed tails, what Alice said is false. There is a 0% chance that what Alice said is true.
Strong centering implies conditional excluded middle:

$$\neg (\text{if } p, q) \iff \text{if } p, \neg q$$
Only if

(10) You will succeed only if you work hard.
Only if

(10) You will succeed only if you work hard.

Intuitively (5) implies: if you don’t work hard you won’t succeed.

- \( only(alt)(p) \) is true iff \( \forall q \in alt, \) if \( p \) does not entail \( q \) then \( \neg q \).
- Conditional alternatives: \( \{ if \ p, \ q, if \ \neg p, \ q \} \).
- \( if \ p, \ q \) does not entail \( if \ \neg p, \ q \).

\( \Rightarrow \) (10) is true iff \( \neg(\)you will succeed if you don’t work hard\()\).

- Strong centering implies conditional excluded middle:

\[
\neg(\text{if } p, q) \iff \text{if } p, \neg q
\]

giving us the reading of (10) we observe.

- If will had a universal meaning, (5) would mean in \textit{some cases} where you don’t work hard, you succeed. This is too weak.
(11) a. Everyone will pass if they work hard.
b. No one will fail if they work hard.

Example from Higginbotham 1986

(11a) and (11b) are intuitively equivalent.

(12) a. \( \forall x (\text{if } x \text{ works hard, } x \text{ will pass}) \)
b. \( \neg \exists x (\text{if } x \text{ works hard, } x \text{ will fail}) \)

Assuming fail \( \Leftrightarrow \) not pass, these are equivalent to:

(13) a. \( \forall x (\text{if } x \text{ works hard, } \neg (x \text{ will pass})) \)
b. \( \forall x \neg (\text{if } x \text{ works hard, } x \text{ will fail}) \)

Conditional excluded middle implies the equivalence of (11a) and (11b).

(14) Alice is 20 years old. The legal drinking age is 18.
   a. Alice can order alcohol because she is over sixteen years old.
   b. Alice can order alcohol because she is over eighteen years old.

As Alice is 20 years old, she is already over sixteen years old.

Strong centering predicts (15) to be true.

(15) If Alice is over sixteen years old, she can order alcohol.
1. Sufficiency in *because*

2. Sufficiency via the conditional?
   - Evidence for strong centering
   - Evidence for conditional excluded middle

3. Sufficiency in a situation semantics
If \( x \) and \( y \) are two individuals, then their mereological difference, \( x - y \), is the largest individual contained in \( x \) which has no part in common with \( y \).

Primitive notions (Fine 2017):

1. A state space \((S, \sqsubseteq)\): a partially ordered set with \(\sqsubseteq\) representing parthood, where each state \(s \in S\) is part of a world (a world is a state that is maximal w.r.t. parthood).

2. A notion of exact verification, denoted \(\models^e\), between states and sentences.

Central idea: \(s\) exactly verifies \(A\) (denoted \(s \models^e A\)) just in case \(s\) is wholly relevant to the truth of \(A\).
Definition

For any state \( s \) and sentence \( A \), let \( s \setminus A \) be the fusion (least upper bound) of the parts of \( s \) disjoint from every exact verifier and falsifier of \( A \):

\[
\begin{align*}
  s \setminus A &:= s \setminus \bigsqcup \{ v : v \models^e A \text{ or } v \models^e \neg A \}
\end{align*}
\]

Define that \( t \) is an \( A \)-variant of \( s \) just in case if \( s \setminus A \) exists, then \( s \setminus A \sqsubseteq t \).

(16) **Proposal.** For any sentences \( A \) and \( C \), \( A \) is *sufficient for* \( C \) in a world \( w \) just in case for every \( A \)-variant of \( w \) where \( A \) is true, \( C \) is also true.
Defining remainders

$w - A$

$A$-variants of $w$
A technician is testing whether a printer is calibrated correctly. They want it to print a circle that is a particular shade of blue – baby blue – on a blue piece of paper. In prints this:

Consider:

(17) The machine passed the test because the circle is blue.
(18) Alice is 20 years old. The legal drinking age is 18.
   a. Alice can order alcohol because she is over sixteen years old.
   b. Alice can order alcohol because she is over eighteen years old.

(19) Priya’s mother was born in India. Priya has an Indian passport.
   a. Priya has an Indian passport because her mother is from Asia.
   b. Priya has an Indian passport because her mother is from India.

(20) Let $x$ and $y$ be numbers, where $x \neq 0$ and $y = 0$.
   a. $xy$ is 0 because $y$ is less than 10.
   b. $xy$ is 0 because $y$ is 0.
Sufficiency cannot be defined using conditionals, since they validate strong centering.

Sufficiency can be defined using the notion of $A$-variants, given in terms of truthmaker semantics.


