Imagining a different world: defining sufficiency using truthmaker semantics

The meanings of certain words are sensitive to a notion of sufficiency. An example of such a word is *because*. Compare the (a) and (b) sentences in (1)–(2).

(1) *Context: Alice is 20 years old. The legal drinking age is 18.*

a. Alice can order alcohol because she is over sixteen years old.

b. Alice can order alcohol because she is over eighteen years old.

(2) *Let x and y be numbers, where $x \neq 0$ and $y = 0$.*

a. xy is 0 because y is less than 10. b. xy is 0 because y is 0.

Intuitively the (a) sentences are unacceptable while the (b) sentences are acceptable. In this talk we propose to capture the contrast in terms of *sufficiency*; proposing, in particular, that *because* entails that the cause is sufficient for the effect with respect to a given set of background conditions (such as the laws governing the legal drinking age and the rules of arithmetic).

A first thought is that the truth of one sentence A is sufficient for the truth of another sentence C just in case the conditional "if A, then C" is true. A problem is that extant semantics of conditionals (such as Stalnaker 1968, Lewis 1973, Kratzer 2012, Cariani and Santorio 2018) validate *strong centering*: where $A > C$ is the conditional construction, strong centering says that if A is true then $A > C$ is true if and only if C is. This is a problem since, for example, in (1), a strongly centered semantics of conditionals predicts *Alice over sixteen > Alice can order alcohol* to be true. As Alice is already over sixteen, a strongly centered semantics will only consider worlds where Alice is her actual age – 20 – in which case she can order alcohol.

So it seems that for A to be sufficient for C, C must hold under the various 'ways' of A being true. We implement this idea using truthmaker semantics (Fine 2017). Our primitive notions are a state space and a notion of exact verification, denoted $\Vdash^e$, between states and sentences. A state space is a partially ordered set $(\mathbb{S}, \sqsubseteq)$, with $\sqsubseteq$ representing parthood, where each state $s \in \mathbb{S}$ is part of a world (a world is a state that is maximal w.r.t. parthood).

Our proposal, informally, is that to check whether one sentence A is sufficient for another, we 'remove' the part of the world responsible for A's truth value and consider the worlds agreeing on what is left over. Definition 1 formalizes this idea, illustrated in Figure 1 overleaf.

Definition 1. *For any state s and sentence A, let $s - A$ be the fusion (least upper bound) of the parts of s disjoint from every exact verifier and falsifier of A:*

$$s - A := \bigsqcup \left\{ s' \sqsubseteq s : \neg\exists u \sqsubseteq s' \; \exists v (u \sqsubseteq v \textit{ and } (v \Vdash^e A \textit{ or } v \Vdash^e \neg A)) \right\}$$

Define that <u>t is an A-variant of s</u> just in case if $s - A$ exists, then $s - A \sqsubseteq t$.

(3) **Proposal.** For any sentences A and C, A is *sufficient for C* in a world w just in case for every A-variant of w where A is true, C is also true.

To see the analysis at work, note that any state exactly verifying that Alice is 20 years old is intuitively identical to – or at least, overlaps – the state of her being over 16. So when we remove the latter, we also remove the former (but do not remove, say, the rules governing the legal drinking age). The remaining states do not specify Alice's age, so there are *Alice is over 16*-variants where she is under 18 and cannot order alcohol. Given the proposal in (3), *Alice is over 16* is predicted to **not** be sufficient for *Alice can order alcohol*, as desired.

More generally, we show that under plausible assumptions the resulting notion of sufficiency is not strongly centered, in the following sense.

Proposition 2. *For any contingent sentence A, every world has an A-variant where A is true and an A-variant where A is false.*

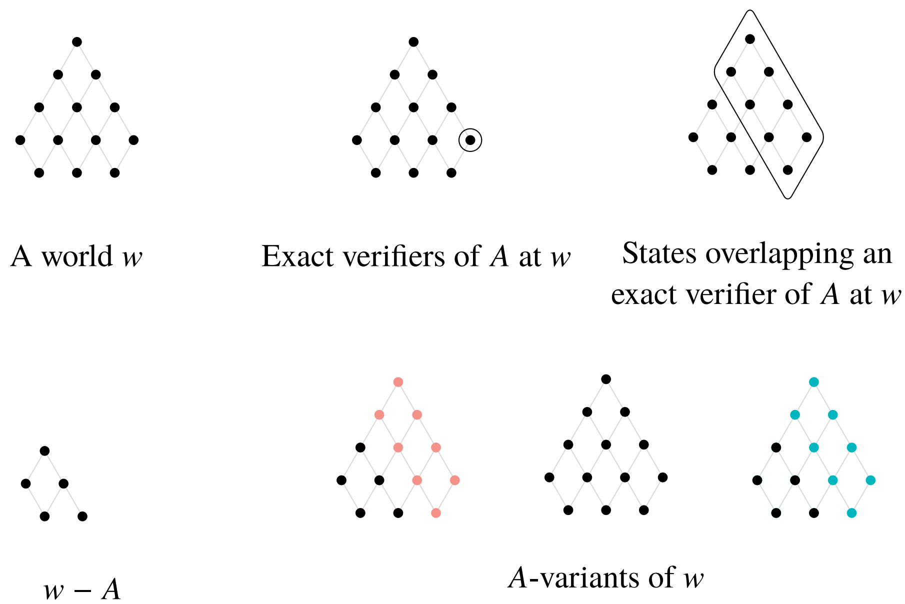

Figure 1: Steps to construct the A-variants of a world.

References

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