An Analysis of Difference-Making

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Abstract. It is said that causes make a difference to their effects, and that for a belief to count as knowledge, the state of the world must make a difference to the belief. But what does it mean to make a difference? We propose a simple, literal analysis. Something makes a difference if and only if, when we compare its presence with its absence, there is a difference; there is something that holds if the difference-maker is present but not if it is absent. We use this to define the notion of a difference-making relation. A wide variety of relations turn out to be difference-making relations in our sense, such as probability raising, casual dependence, and causation (according to numerous analyses of causation). We show that the notion of difference-making is independent of the semantics of conditionals, in the sense that under minimal assumptions all theories of conditionals agree on which relations count as difference-making relations. It also does not matter whether we require the difference D to not hold if the difference-maker A is absent (i.e. require $\textit{if}\ \neg A, \neg D$), or instead that it is false that the difference holds if the difference-maker is absent (i.e. require $\neg(\textit{if}\ \neg A, D)$). The two resulting notions of difference-making turn out to be equivalent. Finally, we compare our analysis of difference-making with a previous analysis by Carolina Sartorio (2005 'Causes as Difference Makers', *Philosophical Studies*), one that at first glance seems quite different, but is surprisingly equivalent to our own.

Keywords: Difference-making · Dependence · Conditionals · Causation

1 Introduction

Big or small, we all hope to make a difference in the world. The concept of difference-making also plays a central role in philosophy and logic. It is said that causes make a difference to their effects, and that for a belief to count as knowledge, the state of the world must make a difference to the belief.[1] Difference-making has also been central to analyses of scientific explanation (Garfinkel 1981, Strevens 2008), grounding (Krämer and Roski 2017), and relevance (Schurz 2023), concepts that have historically evaded systematic analysis.

[1] For some work on the analysis of causation that appeals to difference-making, see Lewis (1973a), Menzies (2004), Waters (2007), Ney (2009), Sartorio (2005, 2013), Dragulinescu (2017), and Rott (2022). For work that appeals to difference-making in epistemology see Comesaña and Sartorio (2014).

But what does it mean to make a difference? Intuitively, and quite literally, it means that when we compare the presence of the difference-maker with its absence, we find a difference: something is true when the difference-maker is present that is not true when it is absent. A simple thought, and one I propose in this paper as an analysis of the difference-making idea. The goal of this paper is to develop this literal interpretation of difference-making and explore its properties. Formalising this thought requires overcoming some technical challenges (see Section 2.1). For example, intuitively the difference A makes cannot be A itself: when we compare A with its absence, of course we find a difference in A, but this is not what we mean when we say that A makes a difference.

We use our analysis of difference-making to formulate the idea of a difference-making relation (section 2.3), whereby $\mathcal{R}$ is a difference-making relation just in case whenever $\mathcal{R}$ relates A to B, when we compare A with its absence, we find a difference—a sentence $D(A, B)$ that is true if A is true, but this is not the case if A is false. A wide variety of relations turn out to be difference-making relations in our sense: probability raising, Lewis's analysis of causal dependence, various semantics of causal claims, and (according to Comesaña and Sartorio 2014), evidential support (see Section 3).

We formalise the conditional involved in difference-making in Section 4, and use it to prove three surprising equivalences, using minimal assumptions in the semantics of conditionals (with the same assumptions used to show each equivalence).

The first equivalence is that all semantics of equivalences (satisfying our assumptions) give rise to equivalent notions of difference-making. The difference-making idea exhibits remarkable uniformity with respect to the semantics of conditionals: while there are many ways to interpret the conditional in our analysis of difference-making, Section 5 shows that they all agree on which relations count as difference-making relations. The definition of difference-making is independent of the semantics of conditionals one adopts (Theorem 1).

The second equivalence concerns an alternative analysis of difference-making, due to Carolina Sartorio. Taking causation to be a difference-making relation, Sartorio (2005) proposes that a cause makes a difference to its effect just in case, had the cause been absent, its absence would not have also caused the effect. Comesaña and Sartorio (2014) later generalised this notion, proposing that $\mathcal{R}$ is a difference-making relation just in case whenever $\mathcal{R}$ holds between some facts A and B, $\mathcal{R}$ would not have related A's absence to B, had A been absent. This analysis appears at first glance radically unlike the one we propose, and quite restrictive in the kinds of difference it considers; namely, only considering whether $\mathcal{R}$ relates A and $\neg A$ to B. In contrast, for A to make a difference, our analysis merely requires that A make *some* difference, without restricting the kind of difference A may make. Remarkably, despite these apparent differences, under minimal assumptions the two analyses turn out to be equivalent (Theorem 2).

The third equivalence is concerns the interaction of negation and conditionals. In principle, there are two ways to formalise our literal analysis of the difference-making idea: there is some D that is true if A is true, and it is not the case that

D is true if A is false. Call this *wide* difference-making:

$$\textit{if } A, D \quad \textit{and} \quad \neg(\textit{if } \neg A, D).$$

Alternatively, we may require that there is some D that is true if A is true, and D *is false* if A is false. Call this *narrow* difference-making:

$$\textit{if } A, D \quad \textit{and} \quad \textit{if } \neg A, \neg D.$$

Section 8 shows that under our minimal assumptions, the wide and narrow notions of difference-making are equivalent.

2 An analysis of difference-making

Intuitively, and quite literally, something makes a difference just in case, when we compare its presence with its absence, we find that things are not the same. This interpretation appears in Lewis's understanding of causes as difference-makers:

> We think of a cause as something that makes a difference, and the difference it makes must be a difference from what would have happened without it. (Lewis 1973a:557)

Following this idea, we can formulate what is perhaps the most direct interpretation possible of the difference-making idea. A sentence (or fact or proposition) A makes a difference just in case when we compare A with its absence, we find a difference: there is some sentence D—the statement of the difference—such that D is true if A is true, and the same cannot be said for A's negation: it is not the case that D is true if A is false. For something to make a difference is for there to be an asymmetry between the thing's presence and its absence.

We have, then, the following literal interpretation of difference-making.

> Sentence A makes a difference just in case there is a sentence D such that D holds if A is true, but it is not the case that D holds if A is false.

For now we will leave open the interpretation of the conditional in this analysis. We discuss the semantics of conditionals in detail in Section 4.

Let us quickly discuss what kinds of entities we take to be difference-makers. On this analysis it is sentences that makes differences. (One may use facts or propositions or the like in place of sentences as desired.) Now, difference-making also applies to questions and entities, exemplified in (1a) and (1b), respectively.

(1) a. It makes a difference whether it rains/who attends/why they did it.
    b. Your donation makes a difference.

Other constructions exhibit similar flexibility, such as *depends* and *because (of)*.

(2) a. I like the painting because it is blue/because of what colour it is/because of its colour.
b. Who comes depends on what the weather is like/on the weather.

Despite the flexibility of difference-making statements, here we will restrict attention to sentences; analysing, for example,

(3) a. That it is sunny makes a difference to our enjoyment of the hike.
b. The fact that the cause occurred made a difference to the effect occurring.

There have recently been advances in our understanding of the relationship between dependence and questions (Ciardelli 2016, 2018, 2022, Ciardelli, Iemhoff, and Yang 2020). Our analysis of difference-making for sentences can ideally be combined with this work to deliver an analysis of difference-making for questions as well, with entities handled as concealed questions; for example, *its colour* is interpreted as *what colour it is*, and *the weather* as *what the weather is* (Baker 1968, Aloni and Roelofsen 2011, Frana 2017, Kalpak 2020).

2.1 Avoiding triviality

The above analysis of the difference-making idea turns out to be trivially true. When we compare A with its absence, of course we are bound to find *some* difference; namely, A itself.[2] A is true if A is true, and it is not the case that A is true if A is false (assuming A is not a tautology), so on this analysis every (non-tautologous) sentence A whatsoever makes a difference.[3] Everything makes a difference to itself.

One way to resolve the triviality is to restrict what the difference is about. For example, we think of causes as making a difference *to their effects*, and that for a belief to count as knowledge, the state of the world makes a difference *to the belief*. And when we say, for example, "Visit me today or tomorrow, it doesn't make a difference", of course when we visit makes a difference to when we visit; we mean that it doesn't make a difference that is salient in the context, where the salient differences exclude the difference-maker itself. Or when, say, a charity advertises, "Your donation makes a difference!" they mean to say more than that your donation makes a difference to whether or not you give them money.

We thus do not count A itself among the differences that A makes. To formalise this thought, we will take the difference to be modulo comparing A with its negation. In other words, when we compare the presence of A with its absence,

[2] Other trivial differences are $A \vee F$, $A \wedge T$, $(A \wedge T) \vee F$ and $(A \vee F) \wedge T$, where F and T are any contingent sentences that are actually false/true, respectively, and conditionally independent of A in the sense that they remain so whether or not A is true; that is, *if* A*,* $\neg F$, *if* $\neg A$*,* $\neg F$, *if* A*,* T, and *if* $\neg A$*,* T are all true.

[3] Some semantics of conditionals predict *if* A*,* C to be vacuously true when A is contradictory. On such approaches, if A is a tautology, A is true even if A is false. In addition, here we are assuming Identity: *if* A *then* A is always true. See Mandelkern (2019) for some semantics of conditionals that—surprisingly—invalidate Identity.

we take into account that we are swapping A with $\neg A$, and adjust the statement of the difference accordingly. For A to make a non-trivial difference is for it to make a difference modulo the difference between A itself and its negation.

> Sentence A makes a difference just in case there is a sentence D such that D holds if A is true, but it is not the case that D' holds if A is false, where D' is the result of replacing every (if any) occurrence of A in D with $\neg A$.

This updated definition may still permit trivialities. For example, suppose A is equivalent to a distinct sentence B. Then A makes a difference to B: if A is true, B is true, and (again assuming A is not a tautology) it is not the case that if A is false, B is true. We may wish to rule out such trivial differences by taking a coarse-grained view of sentences, so that equivalent sentences count as one sentence for the purposes of the analysis, though we will not pursue the matter further here.

2.2 Making a difference to

The above analysis tells us what it means for something to make a difference. We often go further, describing the particular difference A makes; saying, for example, that A makes a difference *to* B. Intuitively this means that D, the statement of the difference, is about B. It is a rich and challenging task to say what it means in general for a sentence to be about something (for discussion see Yablo 2014, Hawke 2018, Berto 2022, McHugh 2023a:110–118), and not something we can reasonably expect an analysis of difference-making to answer. We may assume that we have some grasp of what it means for D to be about something (such as another object, or a sentence). This furnishes the following analysis of what it means to make a difference to something

> A sentence A makes a difference to B just in case for some sentence D <u>that is about B</u>, D holds if A is true, but it is not the case that D' holds if A is false, where D' is the result of replacing every (if any) occurrence of A in D with $\neg A$.

In what follows we will omit the aboutness requirement, since it raises larger questions beyond the scope of the analysis of difference-making.

2.3 Difference-making relations

In addition to the concept of difference-making, we also have the concept of a difference-making relation. Causation is often called a difference-making relation, and Comesaña and Sartorio (2014) propose that evidential support is a difference-making relation. It is a short step from the idea of difference-making to the idea of a difference-making relation: call $\mathcal{R}$ a difference-making relation just in case whenever $\mathcal{R}$ relates A to B, A makes a difference to B.

There are two ways to formalise this thought, what we call *uniform* and *variable* difference-making. When we say that the A's makes a difference to the B's, we may require that the A's make a uniform difference—in the sense that they all make the same kind of difference—or allow the A's to make various kinds of differences. The two kinds correspond to two readings of the statement *The A's make a difference to the B's*, depending where the indefinite *a difference* is interpreted. For example:

(4) Each cause makes a difference to its effect.
  a. There is a difference such that each cause makes that difference to its effect. (Uniform reading)
  b. For each cause, there is a difference that the cause makes to its effect. (Variable reading)

On the uniform reading the difference varies with the A's and B's, so to formulate this reading precisely we will need some variables. Where $D(X, Y)$ is a sentence, perhaps containing X and Y as subsentences, let $D(A, B)$ be the result of replacing every occurrence of X in $D(X, Y)$ with A and every occurrence of Y in $D(X, Y)$ with B.[4] For example, if $D(X, Y)$ is $X \vee (Y \wedge \neg Y)$ then $D(A, B)$ is $A \vee (B \wedge \neg B)$. And if $D(X, Y)$ is Y then $D(A, B)$ is B.

We can now formalise the two kinds of difference-making relations. Let $\mathcal{R}$ to be a relation between sentences. We define:

Uniform difference-making
$\mathcal{R}$ is a uniform difference-making relation just in case there is a sentence $D(X, Y)$ such that for any sentences A and B, $\mathcal{R}(A, B)$ implies that $D(A, B)$ holds if A is true, but it is not the case that $D(A, B)$ holds if A is false.

Variable difference-making
$\mathcal{R}$ is a variable difference-making relation just in case for any sentences A and B, $\mathcal{R}(A, B)$ implies that there is a sentence D such that D holds if A is true, but it is not the case that D holds if A is false.

When we survey the literature of difference-making, we find it is often the uniform rather than the variable notion that one has in mind. Consider again Lewis on difference-making:

> We think of a cause as something that makes a difference, and the difference it makes must be a difference from what would have happened without it. Had it been absent, its effects—some of them, at least, and usually all—would have been absent as well. (Lewis 1973a:557)

For Lewis, causes make a uniform difference to their effects: for every cause, the difference it makes to the effect is whether or not the effect occurs. The

[4] For this operation to be well-defined we will assume that X and Y do not share any atomics. For example, if we let X be $P \wedge Q$, Y be $Q \wedge R$, and $D(X, Y)$ be $P \wedge Q \wedge R$, then this substitution operation is not well-defined. This assumption will not pose a problem in what follows.

relationship between the cause and effect is uniform across the various causes. Keeping with this idea, we will adopt the uniform notion in what follows.

Summing up this section, we have developed a literal interpretation of the difference-making idea: something makes a difference just in case, when we compare its presence with its absence, we find a difference. This seems to be the simplest and most natural analysis available. We then used this definition as the basis for formulating the notion of a difference-making relation.

3 Examples of difference-making relations

A wide variety of interesting relations turn out to be difference-making relations on our analysis. Here we show this for five:

1. Probability raising
2. Lewis's definition of causal dependence (Lewis 1973a)
3. Becker's semantics of *is an actual cause of* (Beckers 2016)
4. McHugh's semantics of *cause* and *because* (McHugh 2023a)
5. Knowledge on the basis of evidence (following Comesaña and Sartorio 2014).

The fact that so many independently-discussed relations count as difference-making relations on this analysis speaks, we believe, to the its naturality.

3.1 Probability raising

A raises the probability of C just in case $P(C \mid A) > P(A)$. Let $\mathcal{PR}(A, C)$ state that A raises the probability of C, and let us take the conditionals in our analysis of difference-making to express probabilistic conditionalisation. That is, *if* A restricts the probability function to those cases where A holds. This assumes that the probability of a conditional is the conditional probability: $P(\textit{if } A,\ C) = P(C \mid A)$. This principle is as plausible as it is controversial, in light of Lewis's triviality result (Lewis 1976) and the rich subsequent discussion. If one finds this assumption controversial, we may take our analysis to be restricted to those conditionals for which the probability of the conditional is the conditional probability.

Our analysis classifies probability raising as a difference-making relation. To see this, let d be the probability of C. We find the following difference between the A-cases and the $\neg A$-cases; *if* A holds, C has probability greater than d, while not if A does not hold: $P(C \mid A) > d$ but it is not the case that $P(C \mid \neg A) > d$, i.e. $P(C \mid \neg A) \leq d$. That is, we take $D(A, C)$ to be the sentence $P(C) > d$.

$\mathcal{PR}(A, C)$ entails *if* $A,\ P(C) > d$ and $\neg(\textit{if } \neg A,\ P(C) > d)$.

This follows from the following fact A raises the probability of C just in case $\neg A$ lowers the probability of C: $P(C \mid A) > P(C)$ if and only if $P(C \mid \neg A) < P(C)$.[5]

[5] This fact has been previously observed in the context of difference-making by Comesaña and Sartorio (2014:371).

Indeed, this is a special case of the following fact, in which we quantify the difference A makes to C: A raises the probability of C by d if and only if $\neg A$ lowers the probability of C by d times the ratio of the probability of A to $\neg A$, assuming $P(A)$ and $P(\neg A)$ are non-zero.

$$P(C \mid A) = P(C) + d \qquad \text{if and only if} \qquad P(C \mid \neg A) = P(C) - d\frac{P(A)}{P(\neg A)}$$

(For a proof see the appendix.)

3.2 Lewis's analysis of causal dependence

Lewis (1973a:563) proposed that C causally depends on E just in case the following conditionals are true: $C \mathrel{\Box\!\!\rightarrow} E$ and $\neg(C \mathrel{\Box\!\!\rightarrow} \neg E)$, where $\Box\!\!\rightarrow$ denotes the *would*-conditional on Lewis's analysis. If *would*-conditionals express universal quantification over worlds, as Lewis (1973a) thought, and there are some accessible worlds where the cause does not occur, $\neg C \mathrel{\Box\!\!\rightarrow} \neg E$ implies $\neg(\neg C \mathrel{\Box\!\!\rightarrow} E)$.

Lewis's definition of causal dependence is a difference-making relation in our sense. The difference is the effect's occurrence: we take $D(C, E)$ to be E.

$\mathcal{CD}_{\mathrm{L}}(C, E)$ entails $C \mathrel{\Box\!\!\rightarrow} E$ and $\neg(\neg C \mathrel{\Box\!\!\rightarrow} E)$.

3.3 Beckers and Vennekens's semantics of *is an actual cause of*

Following Sartorio (2005), let us take $\mathcal{R}$ to be *cause*, so $\mathcal{C}(C, E)$ is the sentence C *cause* E. Let us introduce a relation of production (Hall 2004, Illari 2011), and following McHugh (2023a:41), take the difference that causes make to their effects to consist in the cause producing the effect: $D(C, E)$ is the sentence C *produce* E.

Beckers and Vennekens (2018) offer a semantics of *is an actual cause of* using structural causal models (Pearl 2000), where C is an actual cause of E just in case C and E are actually true, C produced E, and after intervening to make C false, $\neg C$ does not produce E (in symbols, $\neg([C = 0](C = 0 \textit{ produce } E = 1))$, where for any variables X and Y, $[X = x](Y = y)$ denotes that Y takes value y after intervening to set X to take value x.

This definition does not appear to fit our analysis of difference-making, though it is equivalent to one that does:

$\mathcal{C}_{\mathrm{B\&V}}(C, E)$ entails $[C = 1](C = 1 \textit{ produce } E = 1)$
and $\neg([C = 0](C = 0 \textit{ produce } E = 1))$.

This is equivalent to Beckers and Vennekens's semantics since interventions in structural causal models satisfy two rules. The first is *modus ponens*: $X = x \wedge [X = x]Y = y$ implies $Y = y$. The second is *conjunctive sufficiency*: if X has value x, then when we intervene to set X to x, nothing changes. $X = x \wedge Y = y$ implies $[X = x]Y = y$. Then given that C is true ($C = 1$), $C = 1$ *produce* $E = 1$ is equivalent to $[C = 1](C = 1 \textit{ produce } E = 1)$.

3.4 McHugh's semantics of *cause* and *because*

McHugh (2023a,b) proposes a semantics of *cause* and *because* with a similar shape to Beckers and Vennekens's semantics of *is an actual cause of*. A key difference is that in place of interventions, McHugh uses the more general notion of sufficiency: C *cause* E and E *because* C are true just in case C is true, and C is sufficient to produce E but $\neg C$ is not (for a proof that interventions are a special case of sufficiency claims see McHugh 2023a:254–260). Where $\gg$ denotes sufficiency, McHugh proposes:

$\mathcal{C}_{\mathrm{M}}(C, E)$ entails $C \gg (C\ \textit{produce}\ E)$ and $\neg(\neg C \gg (\neg C\ \textit{produce}\ E))$,

which fits our analysis of difference-making.

3.5 Evidential support

Moving from causation to epistemology—and adapting observations by Comesaña and Sartorio (2014)—let us take $\mathcal{K}(E, P)$ to express that the agent in question knows proposition P on the basis of evidence E. For example, the agent may know that it is raining by seeing drops slide down their window. Let us take the difference that E mthe evidence akes to P to consi propositionst in E evidentially supporting P: $D(E, P)$ says that E evidentially supports P. And let, us, say, take the modal force to be selectional. Then Comesaña and Sartorio (2014) propose that if an agent knows P on the basis of evidence E, E evidentially supports P, and if E is false, the absence of E does not evidentially support P.

$\mathcal{K}(E, P)$ entails *if* E, E *evidentially supports* P and $\neg($*if* $\neg E, \neg E$ *evidentially supports* $P)$.

4 The interpretation of the conditional

Our definition of difference-making appeals to conditionals: for A to make a difference is for there for something—besides A itself—that is true if A is true, but this does not hold if A is false. So far we have left the interpretation of the conditional open. This was a deliberate choice. As we saw in the previous section, difference-making relations use a variety of conditionals notions, such as probabilistic conditionalisation, Lewisian counterfactuals, interventions, and sufficiency. In addition to these, there is Stalnaker's selection function approach, and existential conditionals (arguably expressed by *could*-conditionals), stating that if the antecedent holds, there is some possibility where the consequent holds.

Intuitively, we interpret a conditional by considering cases where the antecedent holds, and checking whether the consequent holds there. Formalising this thought abstractly, let Q be a quantifier over worlds—a function taking two sets of worlds and returning a truth-value. In the terminology of Kratzer (1981b:45), Q represents the modal force. For each sentence A of the language, let R_A be a binary accessibility relation, where wR_Aw' holds just in case w' is among the worlds we consider when we interpret a conditional with antecedent A at world w.[6] We assume that A is true at every A-accessible world. Let $R_A[w] = \{w' : wRw'\}$ be the set of R_A-accessible worlds from w, and for any sentence C let $|C|$ be the set of worlds where C is true.

For each quantifier Q, then, we have a corresponding conditional telling us that Q of the accessible antecedent worlds are consequent worlds.

$$A \rightsquigarrow C \text{ is true at } w \qquad \text{if and only if} \qquad Q(R_A[w], |C|)$$

We restrict to the accessible worlds since, as is well-known, conditionals in general do not express logical entailment, quantifying over all logically possible worlds whatsoever, but over a specific subset of them in which certain facts are held fixed. For example, on a similarity approach (Todd 1964, Stalnaker 1968, Lewis 1973b) the accessible worlds are the minimally different/most similar worlds to the actual world where the antecedent is true. On an aboutness approach (McHugh 2022, 2023a), the accessible worlds are those where the antecedent is true that result from allowing the part of the world the antecedent is about to vary and playing the laws forward. The question of which worlds we consider when we interpret a conditional, while rich and complex, does not affect our analysis of difference-making, so we will not discuss it further here.

We have, then, a family of difference-making relations, one for each conditional the we plug into our analysis of difference-making.

Definition 1 (Difference-making relation). *Let $\mathcal{R}$ be a relation between sentences, Q a quantifier over worlds (a function taking two sets of worlds and returning a truth-value), R_A a binary accessibility relation over worlds, and for any world w let $A \rightsquigarrow C$ be true at w just in case $Q(R_A[w], |C|)$ holds.*

$\mathcal{R}$ is a difference-making relation with respect to Q just in case there is a sentence $D(X, Y)$ such that for any sentences A and B,

$$\mathcal{R}(A, B) \qquad \textit{entails} \qquad A \rightsquigarrow D(A, B) \;\; \textit{and} \;\; \neg(\neg A \rightsquigarrow D(\neg A, B)).$$

The notion of entailment is standard; namely, that for every model M and world w, if $\mathcal{R}(A, B)$ is true at M at w then $A \rightsquigarrow D(A, B)$ and $\neg(\neg A \rightsquigarrow D(\neg A, B))$ are also true at M at w, though one may adopt alternative notions of entailment here if desired.

5 Difference-making unified

A natural question to ask is whether the difference-making idea is really many ideas, one for each kind of conditional, or uniform, with a single principle capturing the entire family at once. It turns out that under minimal assumptions

[6] A techinality: on the semantics of conditionals by Lewis (1973b) and Kratzer (1981a), this formulation requires the limit assumption, whereby the accessible worlds from w are the closest worlds to w where A is true. For discussion of the limit assumption see Kaufmann (2017).

about the conditional, the difference-making idea is uniform, in the sense that all theories of conditionals agree on which relations count as difference-making relations.

While this uniformity may seem surprising, it becomes more natural once we reflect on the nature of difference-making. For A to make a difference is for there to be some difference between what holds if A is true and if A is false. The difference D may be a difference in some concrete fact—say, that a party happens—but it may also be a modal difference; for example, that the party *necessarily* happens, i.e. happens in *all* A-accessible worlds. The difference could be that D *necessarily* holds if A is true but does not necessarily hold if A is false. This results from plugging a universal interpretation of the conditional into our analysis of difference-making. Or the difference could be that D is *possible* if A is true, but impossible if A is false—an existential interpretation of the conditional. Or, adopting Stalnaker's semantics (Stalnaker 1968), when we interpret a conditional we could select a unique world where the antecedent holds. Then the difference would be that D holds at the selected world where A is true but not at the selected world where A is false. Or the difference could be that D is *likely* when A is true, but unlikely when A is false.

These are all variations on the same theme: something is true when A is true but not when A is false. The difference may be a difference in some concrete fact or a modal difference—necessary versus unnecessary, possible versus impossible, likely versus unlikely, and so on.

Let us now show the equivalence of the family of difference-making relations, one for each conditional. We need the following minimal assumptions.

5.1 Assumption 1: Between universal and existential quantification

The first assumption we need is that the conditional is *between universal and existential* in the following sense: if all A-accessible worlds are C-worlds, Q-many A-accessible worlds are C-worlds, and if Q-many A-accessible worlds are C-worlds, some A-accessible world is a C-world. In symbols, $\forall \Rightarrow Q \Rightarrow \exists$.

Definition 2 (Between universal and existential). *Let $Q : X \times Y \to \{0,1\}$ be a function taking two sets and returning a truth-value. We say Q is between universal and existential just in case for all $x \in X$ and $y \in Y$, $x \subseteq y$ implies $Q(x,y)$, and $Q(x,y)$ implies $x \cap y \neq \emptyset$.*

A wide range of natural quantifiers are between universal and existential. Examples of quantifiers where $\forall$ implies Q are *every* itself, *at least half*, *likely*, and (assuming the domain is nonempty), *some* and *the selected world*, where the quantifier picks a particular unique world and talks about that. These quantifiers also all imply $\exists$, assuming seriality of R_A, i.e. that every world has at least some A-accessible world.

This assumption is met by semantics of conditionals that select a particular world, such as Stalnaker's semantics (Stalnaker 1968) and interventions in recursive structural causal models (see Halpern and Pearl 2005:849)

This assumption is also met when we take Q to be probability-raising, that is, $Q(X, Y)$ just in case $P(Y \mid X) > P(X)$, assuming that the probabilities of A, $\neg A$, C, $\neg C$ are each nonzero.

Of course, many other quantifiers are not between universal and existential, such as *no*, *less than half*, and *exactly three*. It is nonetheless true that this assumption is met by practically all semantics of conditionals one would reasonably propose.

5.2 Assumption 2: Idempotent restriction

The second assumption we need we call *idempotent restriction*: if w' is an A-accessible world at w, then the A-accessible worlds from w' are just the A-accessible worlds from w. In other words, restricting twice to the A-accessible worlds is the same as restricting once.

Definition 3 (Idempotent restriction). *Let $R_A \subseteq W \times W$ be a binary relation and $R_A[w] = \{w' : wR_Aw'\}$. R_A has idempotent restriction just in case*

$$wR_Aw' \text{ implies } R_A[w] = R_A[w']$$

for all $w, w' \in W$.

This is Zimmerman's (2000) self-reflection principle, albeit with different interpretation of the relation. Here the relation represents the set of worlds we consider when we interpret a conditional with antecedent A, whereas Zimmerman interprets the relation as epistemic compatibility: world w is related to world w' just in case w' is compatible with the relevant agent's knowledge in w. In this epistemic setting, the principle states that if w' is compatible with an agent's knowledge in w, then the worlds compatible with their knowledge in w are the same as those compatible with their knowledge in w', something which, Zimmerman notes, "seems reasonable" (2000:284). A similar principle in the epistemic domain is endorsed by Aloni (2023:14).

With idempotent restriction we derive the following implications.

Lemma 1. *Let Q and Q' be quantifiers over worlds (that is, functions taking two sets of worlds and returning a truth value), and for any world w, define*

$$\begin{array}{lll} A \rightsquigarrow C \text{ is true at } w & \text{if and only if} & Q(R_A[w], |C|), \\ A \rightsquigarrow' C \text{ is true at } w & \text{if and only if} & Q'(R_A[w], |C|). \end{array}$$

Assume R_A has idempotent restriction.

1. *If $\forall$ implies Q', $A \rightsquigarrow C$ implies $A \rightsquigarrow' (A \rightsquigarrow C)$.*
2. *If Q' implies $\exists$, $A \rightsquigarrow' (A \rightsquigarrow C)$ implies $A \rightsquigarrow C$.*

Proof. 1. Pick any world w where $A \rightsquigarrow C$ is true. To show that $A \rightsquigarrow' (A \rightsquigarrow C)$ is true at w, we have to show that $A \rightsquigarrow C$ is true at Q'-worlds $w' \in R_A[w]$.

We show that $A \rightsquigarrow C$ is true at all worlds $w' \in R_A[w]$. Pick any $w' \in R_A[w]$. We show that C is true at Q-worlds $w'' \in R_A[w']$. Since $w' \in R_A[w]$, by idempotent restriction, $R_A[w] = R_A[w']$. Then as $A \rightsquigarrow C$ is true at w, C is true at Q-worlds $w'' \in R_A[w]$, and so C is true at Q-worlds $w'' \in R_A[w']$. Hence $A \rightsquigarrow C$ is true at all worlds $w' \in R_A[w]$. Since $\forall$ implies Q', $A \rightsquigarrow C$ is true at Q'-worlds $w' \in R_A[w]$, i.e. $A \rightsquigarrow' (A \rightsquigarrow C)$ is true at w.

2. Pick any world w where $A \rightsquigarrow' (A \rightsquigarrow C)$ is true, i.e. $A \rightsquigarrow C$ is true at Q'-worlds $w' \in R_A[w]$. Since Q' implies $\exists$, $A \rightsquigarrow C$ is true at some $w' \in R_A[w]$. Then C is true at Q-worlds $w'' \in R_A[w']$. Since $w' \in R_A[w]$, by idempotent restriction, $R_A[w'] = R_A[w]$, so C is true at Q-worlds $w'' \in R_A[w]$, i.e. $A \rightsquigarrow C$ is true at w.

From this Lemma we can show that all semantics of conditionals satisfying our assumptions—idempotent restriction and between universal and existential quantification—give rise to equivalent difference-making principles.

Theorem 1. *Let Q, Q', $\rightsquigarrow$ and $\rightsquigarrow'$ be given as in Lemma 1. Assume that Q and Q' are between universal and existential and that R_A has idempotent restriction. Then the following principles are equivalent.*

(DM) *There is a sentence $D(X, Y)$ such that for all sentences A and B, $\mathcal{R}(A, B)$ entails $A \rightsquigarrow D(A, B)$ and $\neg(\neg A \rightsquigarrow D(\neg A, B))$.*

(DM′) *There is a sentence $D'(X, Y)$ such that for all sentences A and B, $\mathcal{R}(A, B)$ entails $A \rightsquigarrow' D'(A, B)$ and $\neg(\neg A \rightsquigarrow' D'(\neg A, B))$.*

Proof. Take $D'(X, Y)$ to be $X \rightsquigarrow D(X, Y)$. Then DM′ states that for all sentences A and B, $\mathcal{R}(A, B)$ entails $A \rightsquigarrow' (A \rightsquigarrow D(A, B))$ and $\neg(\neg A \rightsquigarrow' (\neg A \rightsquigarrow D(\neg A, B))$. Given idempotent restriction and that $\forall$ implies Q' and Q' implies $\exists$, by Lemma 1, $A \rightsquigarrow' (A \rightsquigarrow D(A, B)$ is equivalent to $A \rightsquigarrow D(A, B)$ and $\neg A \rightsquigarrow' (\neg A \rightsquigarrow D(\neg A, B))$ is equivalent to $\neg A \rightsquigarrow D(\neg A, B)$. Hence also $\neg(\neg A \rightsquigarrow' (\neg A \rightsquigarrow D(\neg A, B)))$ is equivalent to $\neg(\neg A \rightsquigarrow D(\neg A, B))$. Thus DM implies DM′. By symmetry, DM′ implies DM.

This theorem tells us that the difference-making idea is independent of what particular theory of conditionals we adopt, provided it has idempotent restriction and that its quantification is between universal and existential. The various choices for the conditional give rise to equivalent difference-making principles, and therefore all agree on which relations count as difference-making relations.

6 Sartorio's analysis of difference-making

We are not the first to offer an analysis of difference-making. Sartorio (2005) offers an analysis of the idea that causes are difference-makers, proposing what she calls the *Causes as Difference-Makers* principle:

> **Sartorio's Causes as Difference-Makers Principle.** If C caused E, then, had C not occurred, the absence of C wouldn't have caused E.

Comesaña and Sartorio (2014) later generalised this principle, offering the following definition of a difference-making relation.

> $\mathcal{R}$ is a difference-making relation if and only if, whenever $\mathcal{R}$ holds between some facts F and G, $\mathcal{R}$ wouldn't have related F's absence to G, if F had been absent.

Comesaña and Sartorio argue that, in addition to causation, evidential support is also a difference-making relation. If E evidentially supports proposition P, then had E been absent, the absence of E wouldn't have also evidentially supported P. For example, if drops on a windowpane evidentially support the proposition that it is raining, then had there been no drops on the window, the absence of drops on the window could not have also evidentially supported the proposition that it is raining.

Comesaña and Sartorio point out that this is weaker than requiring that if E evidentially supports P, then had E been absent, the agent would not have believed P, similar to the sensitivity constraint on knowledge from Dretske (1970) and Nozick (1983): if S knows that P, then had P been false, S would not have believed P). They point to Frankfurt (1969) style examples to show why this would be too demanding. Imagine a neuroscientist implants a chip in Alice's brain that detects whether she believes it is raining. If not, the chip activates at a given time and forces Alice to believe that it is. As it happens, Alice sees the drops on the window and comes to believe that it is raining, without the chip needing to activate. In this scenario, intuitively, the drops on the window evidentially support the proposition that it is raining even though sensitivity fails: if the drops hadn't been there, Alice would still have believed that it is raining (in that case due to the brain implant).

6.1 Motivating Sartorio's analysis

Let us briefly discuss Sartorio's motivation for introducing her principle. The principle promises to solve a longstanding problem in the analysis of causation: distinguishing switching and preemption cases. Here is a classic example of a switching case, due to Hall (2000:205) and depicted in Figure 1.

> An engineer is standing by a switch in the railroad tracks. A train approaches in the distance. She flips the switch, so that the train travels down the right-hand track, instead of the left. Since the tracks reconverge up ahead, the train arrives at its destination all the same.

Intuitively, the engineer flipping the switch did not cause the train to reach the station. Asked why, a natural response is that flipping the switch did not make a difference to whether the train reached the station. We might follow up by saying that had the engineer not flipped the switch, the train would have reached the station anyway.

This brings us to preemption cases: something causes an effect, but had it been absent, a backup would have caused the effect instead. A classic example is

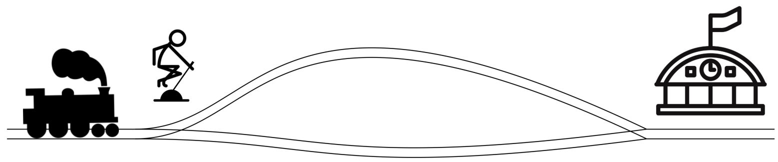

Fig. 1. Hall's switching scenario.

the Billy and Suzy case from Hall and Paul (2003:110). The following formulation is from Hall (2004:235).

> Suzy and Billy, expert rock-throwers, are engaged in a competition to see who can shatter a target bottle first. They both pick up rocks and throw them at the bottle, but Suzy throws hers before Billy. Consequently Suzy's rock gets there first, shattering the bottle. Since both throws are perfectly accurate, Billy's would have shattered the bottle if Suzy's had not occurred, so the shattering is overdetermined.

Intuitively, Suzy's throw caused the bottle to break. This is despite the fact that, had she not thrown, the bottle would have broken anyway.

Analyses of causal claims have historically struggled to distinguish switching and preemption cases. Take for example the simple *but-for* test that C causes E just in case if C had not occurred, E would not have occurred. Such an account fails to predict that Suzy's throw caused the bottle to break: it would have broken anyway without her throw. Alternatively, we might say—with Paul (1998) and Lewis (2000)—that causation requires the cause to make a difference to how the effect occurs, such as when it occurs. Suzy caused the bottle to break earlier than it would have without her throw, but equally, we may suppose that the engineer caused the train to arrive earlier than it would have without flipping the switch (it took a shortcut rather than the scenic route, say). Still we judge that the engineer did not cause the train to reach the station.

Sartorio's analysis of difference-making offers a principled way to distinguish switching and preemption cases. Sartorio (2005:74–75) observes:

> One thing that catches the eye about Switch is that, just as the flip doesn't make a difference to the [train reaching the station], the failure to flip wouldn't have made a difference to the [train reaching the station] either. ... what might be missing in Switch is some kind of asymmetry between my flipping the switch and my failing to flip the switch.

Given the symmetry of the switching scenario, whatever relation holds between flipping the switch and the train reaching the station would also have held between not flipping the switch and the train reaching the station, had the switch not been flipped. So the switching scenario violates Sartorio's principle.

Turning to our preemption case, if Suzy had not thrown, Billy would have caused the bottle to break. But what about Suzy *not* throwing? Would that have caused the bottle to break? Intuitively not. Sartorio's principle is met.

The problem of articulating the difference between switching and preemption cases has haunted the literature on causation for years. The fact that Sartorio's analysis of difference-making offers a principled way to distinguish them represents a major breakthrough in the analysis of causation. The principle therefore appears to be something that we would like any analysis of causation to satisfy. Now suppose we had a semantics of *cause* – call it *proto-cause* – that does not satisfy Sartorio's Principle. We would like to be able to modify our theory minimally to satisfy the principle.

There is, however, a problem. Sartorio's principle does not have the right form to do so. The problem is its non-recursive nature: the principle contains *cause* in both the antecedent and consequent. In essence Sartorio's Principle is an inequality of the form $c \geq f(c)$, where c denotes C *cause* E, $\geq$ represents logical strength and f takes a sentence of the form C *cause* E and returns *if* $\neg C$*,* ($\neg C$ *cause* E). To turn this into our desired necessary condition, we would like to express this inequality with all occurrences of *cause* on one side. Given an inequality in arithmetic, say, $x \geq 3x - x^2$, in school we learn how to put the x's all on one side. Unfortunately the same tricks will not work here. While the operations of arithmetic have inverses (addition/subtraction, multiplication/division), logical operations in general do not.[7]

In contrast, our analysis of difference-making is far easier to work with. For A to make a difference to C, we only require that something is true if A is true, but that this is not the case if A is false. Besides A itself, the difference could be any proposition whatsoever. Given this, it is straightforward to take any analysis of causation and turn it into a difference-making relation in our sense. For it is plausible to assume that any analysis of causation whatsoever will say something about what is the case if the cause is present, or what is the case if it is absent. If the former, the analysis predicts that, D holds if the cause C holds, then we add that this no longer holds when we replace the cause with its negation. And if the analysis predicts that D holds if $\neg C$ holds, we add that this no longer holds when we replace $\neg C$ with C. By design, the resulting analysis of causation will be a difference-making relation according to our analysis.

What we would really like is to have the best of both worlds: a notion of difference-making as simple and easy to work with as ours, while also having the benefits of Sartorio's analysis; in particular, a principled distinction between switching and preemption cases.

As it turns out, we do have the best of both worlds. Under minimal assumptions on the semantics of conditionals, both analyses of difference-making turn out to be equivalent. Let us prove this now.

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[7] Though there is a rich literature on the topic of logical subtraction, such as Peirce (1867), Jaeger (1973, 1976), Hudson (1975), Fuhrmann (1996, 1999), Humberstone (1981, 2011), Yablo (2014), and Hoek (2018).

7 The best of both worlds

Under plausible assumptions on the conditional, Sartorio's definition of difference-making relations is equivalent to the literal analysis we propose (Definition 1). The assumptions are given in (5), where $\rightsquigarrow$ is our general conditional construction familiar from Section 4.

(5) a. **Consistency.** $A \rightsquigarrow \neg C$ entails $\neg(A \rightsquigarrow C)$.
b. **Stability.** $\mathcal{R}(A, B)$ entails $A \rightsquigarrow \mathcal{R}(A, B)$.
c. **Negative idempotence.** $\neg(A \rightsquigarrow C)$ entails $A \rightsquigarrow \neg(A \rightsquigarrow C)$.
d. **Right weakening.** If C entails D then $A \rightsquigarrow C$ entails $A \rightsquigarrow D$.

We discuss each in turn.

Consistency. (5a) says that conditionals are consistent, in the sense that $A \rightsquigarrow C$ and $A \rightsquigarrow \neg C$ are never true together. If the conditional expresses universal quantification over worlds, then this is an effect a assumption that there is some accessible world where the antecedent is true; in other words, R_A is serial, $R_A[w]$ is non-empty. This is a standard assumption to make—common to quantificational elements in general (Cooper 1983, von Fintel 1994, Beaver 1995, Ippolito 2006). For other semantics of the conditional consistency follows automatically. Suppose, following Stalnaker (1968), that conditionals talk about a particular world where the antecedent is true. Consistency follows since the consequent and its negation cannot both be true at the selected world. Similarly, if the conditional quantifies existentially, consistency states that if the the consequent is false at some A-accessible world, it cannot also be true at every such world—a direct consequence of bivalence.

Stability. (5b) is a stability principle. It says that if $\mathcal{R}(A, B)$ holds, then if A holds, $\mathcal{R}(A, B)$ still holds. This is automatically satisfied by many semantics of conditionals when $\mathcal{R}$ is factive, in the sense that $\mathcal{R}(A, B)$ implies A. Causation and knowledge on the basis of evidence both turn out to be factive in this sense. For a cause to have an effect, the cause must occur: $\mathcal{C}(C, E)$ implies C. And for an agent to know a proposition P on the basis of evidence E, the evidence E must hold: $\mathcal{K}(E, P)$ implies E.[8]

When $\mathcal{R}$ is factive, $\mathcal{R}(A, B)$ clearly also implies $A \wedge \mathcal{R}(A, B)$. Now, a number of conditional semantics satisfy *conjunctive sufficiency*, that $A \wedge C$ implies $A \rightsquigarrow C$ (this is also known as *conjunction conditionalisation*). For example, on Lewis's semantics this rule follows from strong centering: the idea that every world is more similar to itself than any other world is to it. On selectional semantics such as that from Cariani and Santorio (2018), conjunctive sufficiency follows here from the selection function's centering requirement: if A is already true at the world of evaluation w then the selection function must pick it.[9] Though note

[8] For some work in epistemic logic that makes use of the factivity of evidence, see Özgün (2017) and Baltag, Özgün, and Vargas Sandoval (2017).

[9] For some discussion of why selection functions would be subject to the centering requirement, see McHugh (2023a:64–65).

that other semantics of *would*-conditionals also validate conjunctive sufficiency, such as Stalnaker's (1968) and Lewis's (1973) semantics of conditionals, and interventions in structural causal models (Pearl 2000).

Altogether, factivity and conjunctive sufficiency together imply stability:

$$\begin{aligned} \mathcal{R}(A,B) &\Rightarrow A \wedge \mathcal{R}(A,B) && \text{(Factivity)} \\ &\Rightarrow A \rightsquigarrow \mathcal{R}(A,B) && \text{(Conjunctive sufficiency)} \end{aligned}$$

Of course, facticity and conjunctive sufficiency are not necessary for stability. We require neither factivity nor conjunctive sufficiency in our proof that Sartorio's Principle is equivalent to the Perfection Principle, merely the weaker requirement of stability.

Idempotence. (5c) says that conditional restriction is idempotent in the following sense: if, restricting to the A-accessible worlds, we do not find C-worlds, then restricting to the A-accessible worlds, and then restricting to the A-accessible worlds again, we still do not find C-worlds. This follows from assumptions we already encountered in Section 5; namely, $\forall$ implies Q and idempotent restriction.

Fact 1 *Assume that $\forall$ implies Q and idempotent restriction. Then $\neg(A \rightsquigarrow C)$ entails $A \rightsquigarrow \neg(A \rightsquigarrow C)$.*

Proof. Suppose $\neg(A \rightsquigarrow C)$ is true at w. Pick any world w' with wR_Aw'. Then by idempotent restriction, $R_A[w] = R_A[w']$. Since $\neg(A \rightsquigarrow C)$ is true at w, we have $\neg Q(R_A[w], |C|)$, and so $\neg Q(R_A[w'], |C|)$. Thus $\neg(A \rightsquigarrow C)$ is true at w'. Since w' was arbitrary, $\neg(A \rightsquigarrow C)$ is true at all worlds in $R_A[w]$. As $\forall$ implies Q, $\neg(A \rightsquigarrow C)$ is true at Q-worlds in $R_A[w]$, so $A \rightsquigarrow \neg(A \rightsquigarrow C)$ is true at w.

Right weakening. Lastly, (5d) says that the conditional satisfies right-weakening. This is a highly plausible constraint, one which—to my knowledge—is satisfied by almost every theory of conditionals available.[10]

Let us remind ourselves of the two analyses of difference-making in question. The first is our literal analysis of difference-making, the second Sartorio's.

The Literal Analysis
There is a sentence $D(X,Y)$ such that for any sentences A and B, $\mathcal{R}(A,B)$ entails $A \rightsquigarrow D(A,B)$ and $\neg(\neg A \rightsquigarrow D(\neg A,B))$.

Sartorio's Analysis
For any sentences A and B, $\mathcal{R}(A,B)$ entails $\neg A \rightsquigarrow \neg\mathcal{R}(\neg A,B)$.

Theorem 2. *Sartorio's Analysis is equivalent to the Literal Analysis, given the assumptions in (5).*

[10] Exceptions to right weakening include Rott's 'contraposing conditional' Rott (2020) and the family of conditionals studied by Casini, Meyer, and Varzinczak (2019).

Proof. (⇒) Suppose Sartorio's Analysis. Take $D(X,Y) = \mathcal{R}(X,Y)$. Pick any sentences A and B where $\mathcal{R}(A,B)$. By Stability, $\mathcal{R}(A,B)$ implies $A \rightsquigarrow \mathcal{R}(A,B)$, which is $A \rightsquigarrow D(A,B)$. We also have the following chain of implications.

$$\begin{aligned}
&\mathcal{R}(A,B) && \\
&\neg A \rightsquigarrow \neg\mathcal{R}(\neg A,B) && \text{(Sartorio's Analysis)} \\
&\neg(\neg A \rightsquigarrow \mathcal{R}(\neg A,B)) && \text{(Consistency)} \\
&\neg(\neg A \rightsquigarrow D(\neg A,B)) && (D(X,Y) = \mathcal{R}(X,Y))
\end{aligned}$$

Hence C *cause* E entails $A \rightsquigarrow D(A,B)$ and $\neg(\neg A \rightsquigarrow D(\neg A,B))$.

(⇐) Suppose the Literal Analysis. So $\mathcal{R}(\neg A,B)$ entails $\neg A \rightsquigarrow D(\neg A,B)$. Then by contraposition we have ($*$): $\neg(\neg A \rightsquigarrow D(\neg A,B))$ entails $\neg\mathcal{R}(\neg A,B)$. Observe the following chain of implications.

$$\begin{aligned}
&\mathcal{R}(A,B) && \\
&\neg(\neg A \rightsquigarrow D(\neg A,B)). && \text{(The Literal Analysis)} \\
&\neg A \rightsquigarrow \neg(\neg A \rightsquigarrow D(\neg A,B)). && \text{(Negative idempotence)} \\
&\neg A \rightsquigarrow \neg\mathcal{R}(\neg A,B). && \text{(Right weakening and } *)
\end{aligned}$$

Hence $\mathcal{R}(A,B)$ entails $\neg C \rightsquigarrow \neg\mathcal{R}(\neg A,B)$, which is Sartorio's Analysis.

8 Negation in difference-making

Our literal analysis of difference-making, and Sartorio's analysis, both involve an interaction between negation and conditionals. The literal analysis requires that for $\mathcal{R}$ to count as a difference-making relation, $\mathcal{R}(A,B)$ must entail that it is not the case that the difference D is true if A is false: $\neg(\neg A \rightsquigarrow D(A,B))$. One could instead insist that $\mathcal{R}(A,B)$ entail that D is *false* if A is false: $\neg A \rightsquigarrow \neg D(A,B)$. Similarly, Sartorio's analysis requires that $\mathcal{R}(A,B)$ entail $\neg A \rightsquigarrow \neg\mathcal{R}(A,B)$, though one could alternatively require that $\mathcal{R}(A,B)$ entail $\neg(\neg A \rightsquigarrow \mathcal{R}(A,B))$. What exactly the difference comes down to depends on one's theory of conditionals. For example, if conditionals quantify universally, $\neg(\neg A \rightsquigarrow D(A,B))$ says that in $D(A,B)$ is false in *some* $\neg A$-accessible case, while $\neg A \rightsquigarrow \neg D(A,B)$ says that $D(A,B)$ is false in *every* $\neg A$-accessible case. Assuming there is some $\neg A$-accessible world, the latter implies the former. In contrast, if the conditional quantifies existentially, this entailment relation is reversed. And for theories such as Stalnaker's, whereby conditionals select a unique world, the two variants are equivalent (again assuming there is some $\neg A$-accessible world). A natural question to ask is how these variants are related, and whether one offers a more satisfying analysis of the difference-making idea.

It turns out that the notion of difference-making exhibits remarkable uniformity on this front, just as it exhibited remarkable uniformity regarding the semantics of conditionals. Under minimal assumptions, for each analysis (the Literal Analysis and Sartorio's), its wide- and narrow-scope versions are equivalent. Those assumptions are exactly the same assumptions in (5) we used to prove the equivalence of the Literal Analysis and Sartorio's Analysis; namely, consistency, stability, negative idempotence, and right weakening.

8.1 Negation in the Literal Analysis

We begin with the literal analysis, which now comes in wide and narrow forms.

The Narrow Literal Analysis.
There is a sentence $D(X,Y)$ such that for any sentences A and B, $\mathcal{R}(A,B)$ entails $A \rightsquigarrow D(A,B)$ and $\neg A \rightsquigarrow \neg D(\neg A,B))$.

The Wide Literal Analysis.
There is a sentence $D(X,Y)$ such that for any sentences A and B, $\mathcal{R}(A,B)$ entails $A \rightsquigarrow D(A,B)$ and $\neg(\neg A \rightsquigarrow D(\neg A,B))$.

We give two proofs of their equivalence, using slightly different assumptions. The first proof uses the same assumptions as our proof that Sartorio's Analysis is equivalent to the Literal Analysis; the second uses only positive and negative idempotence, with positive idempotence defined in analogy with negative idempotence as follows.

(6) a. **Positive idempotence.** $A \rightsquigarrow C$ entails $A \rightsquigarrow (A \rightsquigarrow C)$.
b. **Negative idempotence.** $\neg(A \rightsquigarrow C)$ entails $A \rightsquigarrow \neg(A \rightsquigarrow C)$.

Proposition 1. *Given consistency, the narrow Literal Analysis implies the wide Literal Analysis. Given stability, negative idempotence and right weakening, or positive and negative idempotence, the wide Literal Analysis implies the narrow Literal Analysis.*

Proof. The first is immediate. For the second, suppose the wide Literal Analysis.

1. Suppose further stability, negative idempotence, and right weakening. Take $D'(X,Y)$ to be $\mathcal{R}(X,Y)$. Suppose $\mathcal{R}(A,B)$. Then by stability, $A \rightsquigarrow \mathcal{R}(A,B)$, which is $A \rightsquigarrow D'(A,B)$. Now, by the (wide) Literal Analysis, $\mathcal{R}(\neg A,B)$ entails $\neg A \rightsquigarrow D(\neg A,B)$. Then by contraposition, (†): $\neg(\neg A \rightsquigarrow D(\neg A,B))$ entails $\neg\mathcal{R}(\neg A,B)$. Observe the following chain of implications.

$\mathcal{R}(A,B)$	
$\neg(\neg A \rightsquigarrow D(\neg A,B))$	(Wide Literal Analysis)
$\neg A \rightsquigarrow \neg(\neg A \rightsquigarrow D(\neg A,B))$	(Negative idempotence)
$\neg A \rightsquigarrow \neg\mathcal{R}(\neg A,B)$	(Right weakening and †)
$\neg A \rightsquigarrow \neg D'(\neg A,B)$	(Definition of D')

Hence $\mathcal{R}(X,Y)$ entails $A \rightsquigarrow D'(A,B)$ and $\neg A \rightsquigarrow \neg D'(\neg A,B)$, so the narrow Literal Analysis is satsifed.

2. Suppose instead both positive and negative idempotence. Take $D'(X,Y)$ to be $X \rightsquigarrow D(X,Y)$. Suppose $\mathcal{R}(A,B)$. Then by the wide Literal Analysis, $A \rightsquigarrow D(A,B)$ and $\neg(\neg A \rightsquigarrow D(\neg A,B))$. By positive idempotence, $A \rightsquigarrow (A \rightsquigarrow$

$D(A, B))$, which is $A \rightsquigarrow D'(A, B)$, and by negative idempotence, $\neg A \rightsquigarrow \neg(\neg A \rightsquigarrow D(\neg A, B))$, which is $\neg A \rightsquigarrow \neg D'(\neg A, B)$. Hence $\mathcal{R}(A, B)$ entails $A \rightsquigarrow D'(A, B)$ and $\neg A \rightsquigarrow \neg D'(\neg A, B)$, so the narrow Literal Analysis is satisfied.

8.2 Negation in Sartorio's Analysis

Turning to Sartorio's Analysis, we have two versions, differing again in the scope of negation.

Sartorio's Narrow Analysis.
For any sentences A and B, $\mathcal{R}(A, B)$ entails $\neg A \rightsquigarrow \neg\mathcal{R}(\neg A, B)$.

Sartorio's Wide Analysis.
For any sentences A and B, $\mathcal{R}(A, B)$ entails $\neg(\neg A \rightsquigarrow \mathcal{R}(\neg A, B))$.

Proposition 2. *Given consistency, Sartorio's Narrow Analysis implies Sartorio's Wide Analysis. Given stability, idempotence, and right weakening, Sartorio's Wide Analysis implies Sartorio's Narrow Analysis.*

Proof. The first is immediate. We show the second. By stability, $\mathcal{R}(\neg A, B)$ entails $\neg A \rightsquigarrow \mathcal{R}(\neg A, B)$, so by contraposition (‡), $\neg(\neg A \rightsquigarrow \mathcal{R}(\neg A, B))$ entails $\neg\mathcal{R}(\neg A, B)$. We have the following chain of implications.

$\mathcal{R}(A, B)$	
$\neg(\neg A \rightsquigarrow \mathcal{R}(\neg A, B))$	(Sartorio's Wide Analysis)
$\neg A \rightsquigarrow \neg(\neg A \rightsquigarrow \mathcal{R}(\neg A, B))$	(Negative idempotence)
$\neg A \rightsquigarrow \neg\mathcal{R}(\neg A, B)$	(Right weakening and ‡)

Hence $\mathcal{R}(A, B)$ entails $\neg A \rightsquigarrow \neg\mathcal{R}(\neg A, B)$, which is Sartorio's Narrow Analysis.

Appendix

Fact 2 *$P(C \mid A) = P(C) + d$ if and only if $P(C \mid \neg A) = P(C) - d\frac{P(A)}{P(\neg A)}$, where $P(A)$ and $P(\neg A)$ are non-zero.*

Proof. We have the following chain of equivalences.

$$
\begin{aligned}
P(C \mid A) &= P(C) + d \\
\frac{P(A \mid C)P(C)}{P(A)} &= P(C) + d && \text{(Bayes' Rule)} \\
P(A \mid C) &= P(A) + d\frac{P(A)}{P(C)} \\
1 - P(\neg A \mid C) &= 1 - P(\neg A) + d\frac{P(A)}{P(C)} \\
P(\neg A \mid C) &= P(\neg A) - d\frac{P(A)}{P(C)} \\
\frac{P(C \mid \neg A)P(\neg A)}{P(C)} &= P(\neg A) - d\frac{P(A)}{P(C)} && \text{(Bayes' Rule)} \\
P(C \mid \neg A) &= P(C) - d\frac{P(A)}{P(\neg A)}
\end{aligned}
$$

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