A First-Order Inquisitive Semantics

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Abstract. This paper discusses the extension of propositional inquisitive semantics (Ciardelli and Roelofsen, 2009b; Groenendijk and Roelofsen, 2009) to the first order setting. We show that such an extension requires essential changes in some of the core notions of inquisitive semantics, and we propose and motivate a semantics which retains the essential features of the propositional system.

1 Introduction

The starting point of this paper is the propositional system of inquisitive semantics (Ciardelli, 2009; Ciardelli and Roelofsen, 2009a,b; Groenendijk and Roelofsen, 2009). Whereas traditionally the meaning of a sentence is identified with its informative content, in inquisitive semantics –originally conceived by Groenendijk (2009b) and Mascarenhas (2009)– meaning is taken to encompass inquisitive content, consisting in the potential to raise issues.

More specifically, the main feature of this system is that a disjunction $p \lor q$ is not only informative, but also inquisitive: it proposes two possibilities, as depicted in figure 1(b), and invites other participants to provide information in order to establish at least one of them.

In inquisitive semantics, questions may also be represented: for instance, the polar question 'whether p' may be expressed by the classical tautology $p \vee \neg p$, whose meaning is depicted in figure 1(c).

The main feature of a first-order extension can be expected to be that existential quantification also has inquisitive effects. A simplified version, assuming finite domains, was used in Balogh (2009) in an analysis of focus phenomena in natural language. However, as was shown in Ciardelli (2009), defining a first order system that can deal with infinite domains is not a trivial affair. While there I proposed to enrich the propositional system in order to make the predicate extension possible, what I outline here is a *conservative* extension of the original framework, which retains most of its essential features, in particular the decomposition of meanings into a purely informative and a purely inquisitive component.

2 Propositional inquisitive semantics

We start by recalling briefly the propositional implementation of inquisitive semantics. We assume a set \mathcal{P} of propositional letters. Our language will consist



Fig. 1. Examples of propositional inquisitive meanings.

of propositional formulas built up from letters in \mathcal{P} and \perp using the connectives \land, \lor and \rightarrow . We write $\neg \varphi$ as an abbreviation for $\varphi \rightarrow \bot$.

Our semantics is based on *information states*, modeled as sets of valuations. Intuitively, a valuation describes a possible state of affairs, and a state s is interpreted as the information that the actual state of affairs is described by one of the valuations in s. In inquisitive semantics, information states are always used to represent the state of the common ground of a conversation, not the information state of any individual participant.

Definition 1 (States). A state is a set of valuations for \mathcal{P} . We denote by ω the state of ignorance, i.e. the state containing all valuations. We use s, t, \ldots as meta-variables ranging over states.

We get to inquisitive meanings passing through the definition of a relation called *support* between states and propositional formulas.

Definition 2 (Support).

 $\begin{array}{ll} s \models p & \Longleftrightarrow \forall w \in s : w(p) = 1 \\ s \models \bot & \Longleftrightarrow s = \emptyset \\ s \models \varphi \land \psi & \Longleftrightarrow s \models \varphi \text{ and } s \models \psi \\ s \models \varphi \lor \psi & \Longleftrightarrow s \models \varphi \text{ or } s \models \psi \\ s \models \varphi \rightarrow \psi & \Longleftrightarrow \forall t \subseteq s : if \ t \models \varphi \ then \ t \models \psi \end{array}$

Support is used to define inquisitive meanings as follows.

Definition 3 (Truth-sets, possibilities, meanings).

- 1. The truth-set $|\varphi|$ of φ is the set of valuations which make φ true.
- 2. A possibility for φ is a maximal state supporting φ .
- 3. The inquisitive meaning $[\varphi]$ of φ is the set of possibilities for φ .

Informativeness The meaning $[\varphi]$ represents the proposal expressed by φ . One effect of the utterance of φ is to *inform* that the actual world lies in one of the specified possibilities, i.e. to propose to eliminate all indices which are not included in any element of $[\varphi]$: thus, the union $\bigcup[\varphi]$ expresses the informative content of φ . A formula which proposes to eliminate indices is called *informative*. It is easy to see that the equality $\bigcup[\varphi] = |\varphi|$ holds, insuring that inquisitive semantics preserves the classical treatment of information.

Inquisitiveness What distinguishes inquisitive semantics from classical update semantics is that now the truth-set $|\varphi|$ of a formula comes subdivided in a certain way, which specifies the possible resolutions of the issue raised by the formula. If resolving a formula φ requires more information than provided by φ itself, which happens iff $|\varphi| \notin [\varphi]$, then φ requests information from the other participants, and thus we say it is *inquisitive*. In the present system (but not in the unrestricted system mentioned below) a formula is inquisitive precisely in case it proposes more than one possibility.

Assertions and questions Notice that formulas which are neither informative nor inquisitive make the trivial proposal $\{\omega\}$ (namely, they propose to stay in the given state). Thus, inquisitive meanings can be seen as consisting of an informative dimension and an inquisitive dimension. Purely informative (i.e., non-inquisitive) formulas are called *assertions*; purely inquisitive (i.e., non-informative) formulas are called *questions*. In other words, assertions are formulas which propose only one possibility (namely their truth-set), while questions are formulas whose possibilities cover the whole logical space ω .





It is easy to see that disjunction is the only source of inquisitiveness in the language, in the sense that any disjunction-free formula is an assertion. Moreover, a negation is always an assertion: in particular, for any formula φ , its double negation $\neg \neg \varphi$, abbreviated by $!\varphi$, is an assertion expressing the informative content of φ .

An example of a question is the formula $p \vee \neg p$ depicted in 1(c), which expresses the polar question 'whether p'. In general, the disjunction $\varphi \vee \neg \varphi$ is a question which we abbreviate by φ .

We say that two formulas φ and ψ are equivalent, in symbols $\varphi \equiv \psi$, in case they have the same meaning. The following proposition, stating that any formula is equivalent with the conjunction of an assertion with a question, simply reflects the fact that inquisitive meanings consist of an informative and an inquisitive component.

Proposition 1 (Pure components decomposition). $\varphi \equiv !\varphi \land ?\varphi$

Obviously, the notions and the results discussed in this section may be relativized to arbitrary common grounds. For more details on the propositional system and its logic, the reader is referred to Groenendijk (2009a) and Ciardelli and Roelofsen (2009b).

3 The maximality problem

In this section I will discuss the main difficulty one encounters when trying to reproduce the above framework in a predicate setting; our analysis will lead to considerations which motivate the solution proposed in the next section.

Fix a first-order language \mathcal{L} . A state will now consist of a set of first-order models for the language \mathcal{L} : not to complicate things beyond necessity, we shall make the simplifying assumption that all models share the same domain and the same interpretation of constants and function symbols. Thus, let \mathbb{D} be a fixed structure consisting of a domain D and an interpretation of all (constants and) function symbols in \mathcal{L} ; a first-order model for \mathcal{L} based on the structure \mathbb{D} is called a \mathbb{D} -model.

Definition 4 (States). A state is a set of \mathbb{D} -models.

If g is an assignment into D, we denote by $|\varphi|_g$ the state consisting of those models M such that $M, g \models \varphi$ in the classical sense. The extension of the definition of support is unproblematic. Just like disjunction, an existential will only be supported in those states where a specific witness for the existential is known.

Definition 5 (First-order support). Let s be a state and let g be an assignment into D.

$s,g\models\varphi$	\iff	$\forall M \in s : M, g \models \varphi \text{ for } \varphi \text{ atomic}$
$Boolean\ connectives$	\iff	as in the propositional case
$s,g\models \exists x\varphi$	\iff	$s, g[x \mapsto d] \models \varphi \text{ for some } d \in D$
$s,g\models \forall x\varphi$	\iff	$s, g[x \mapsto d] \models \varphi \text{ for all } d \in D$

Based on support, we may define the informative content of a formula and prove that the treatment of information is classical. We may also define when a formula is inquisitive. However, there is a crucial thing that we can*not* do: we cannot get a satisfactory notion of meaning by taking maximal supporting states, and indeed in any way which involves support alone. This is what the following examples show.

Example 1. Let our language consist of a binary function symbol + and a unary predicate symbol P; let our domain be the set \mathbb{N} of natural numbers and let + be interpreted as addition. Moreover, let $x \leq y$ abbreviate $\exists z(x + z = y)$.

Let B(x) denote the formula $\forall y(P(y) \to y \leq x)$. It is easy to check that a state s supports B(n) for a certain number n if and only if B(n) is true in all models in s, that is, if and only if n is an upper bound for P^M for any model $M \in s$, where P^M denotes the extension of the predicate P in M.

We claim that the formula $\exists x B(x)$ –which expresses the existence of an upper bound for P– does not have any maximal supporting state. For, consider an arbitrary state s supporting $\exists x B(x)$: this means that there is a number n which is an upper bound for P^M for any $M \in s$.

Now let M^* be the model defined by $P^{M^*} = \{n+1\}$. M^* does not belong to s, since we just said that the extension of P in any model in s is bounded



Fig. 2. The intended possibilities |B(n)| for the boundedness formula and its truth set $|\exists x B(x)|$, which is not itself a possibility.

by n; hence $s \cup \{M^*\}$ is a proper superset of s. It is obvious that for any model $M \in s \cup \{M^*\}$ we have $P^M \subseteq \{0, \ldots, n+1\}$ and thus $M \models B(n+1)$. Hence, $s \cup \{M^*\} \models B(n+1)$ and therefore $s \cup \{M^*\} \models \exists x B(x)$. So, $s \cup \{M^*\}$ is a proper extension of s which still supports $\exists x B(x)$.

This shows that any state that supports $\exists x B(x)$ can be extended to a larger state which still supports the same formula, and therefore no state supporting $\exists x B(x)$ can be maximal.

Let us meditate briefly on this example. What possibilities did we expect to come out of the boundedness example? Now, B(x) is simply supported whenever it is known to be true, so it has a classical behaviour. The existential quantifier in front of it, on the other hand, is designed to be satisfied only by the knowledge of a concrete bound, just like in the propositional case a disjunction (of assertions) is designed to be satisfied only by the knowledge of a disjunct.

Therefore, what we would expect from the boundedness formula is a hybrid behaviour: of course, it should inform that there is an upper bound to P; but it should also raise the issue of *what* number is an upper bound of P. The possible resolutions¹ of this issue are B(0), B(1), B(2), etc., so the possibilities for the formula should be |B(0)|, |B(1)|, |B(2)|, etc.

Now, the definition of possibilities through maximalization has the effect of selecting *alternative* ways to resolve the issue raised by a formula, i.e. ways which are incomparable relative to entailment. The problem is that obviously, if 0 is a bound for P, then so are 1, 2, etc.; if 1 is a bound, then so are 2, 3, etc. So, the ways in which the issue raised by the boundedness formula may be resolved cannot be regarded as *alternatives*. Still, B(0), B(1), etc. are genuine solutions to the meaningful issue raised by the existential, and our semantics should be able to capture this.

This indicates that we need to come up with another way of associating a proposal to a formula; and if we are to be able to deal with the boundedness example, we need our notion to encompass proposals containing non-alternative possibilities. Notice that we cannot hope for a definition of such possibilities in terms of support: this is witnessed by the following example.

¹ For the precise definition of *resolutions* of a formula, the reader is referred to Ciardelli (2009)

Example 2. Consider the following variant of the boundedness formula: $\exists x (x \neq 0 \land B(x))$. Possibilities for this formula should correspond to the possible witnesses for the existential, and since 0 is *not* a witness, we expect |B(0)| *not* to be a possibility.

Thus, a system that represents the inquisitive behaviour of the existential quantifier in a satisfactory way should associate different possibilities to the formulas $\exists x B(x)$ and $\exists x(x \neq 0 \land B(x))$. Capturing this distinction is quite important; for, intuitively, "Yes, zero!" would be a compliant response to "There exists an upper bound for P", but *not* to "There exists a positive upper bound to P", and being able to analyze compliance in dialogue is one of the principal aims of inquisitive semantics. However, the formulas $\exists x B(x)$ and $\exists x(x \neq 0 \land B(x))$ are equivalent in terms of support.

The point here is that, as argued in Ciardelli (2009), support describes the knowledge conditions in which the issue raised by a formula is resolved, but is not sufficiently fine-grained to determine what the resolutions of a formula are.

4 A first-order inquisitive semantics

The discussion in the previous section indicates that we need to devise a non support-based notion of meaning which allows for non-alternative possibilities, i.e. possibilities which may be included in one another. In order to do so, we start from the observation that propositional inquisitive meanings may also be defined recursively, by means of an operator MAX which, given a set Π of states, returns the set MAX(Π) of maximal elements of Π .

Definition 6.

1.
$$[p] = \{|p|\}$$
 if $p \in \mathcal{P}$
2. $[\bot] = \{\emptyset\}$
3. $[\varphi \lor \psi] = MAX([\varphi] \cup [\psi])$
4. $[\varphi \land \psi] = MAX\{s \cap t \mid s \in [\varphi] \text{ and } t \in [\psi]\}$
5. $[\varphi \to \psi] = MAX\{\Pi_f \mid f : [\varphi] \to [\psi]\},$
where $\Pi_f = \{w \in \omega \mid \text{ for all } s \in [\varphi], \text{ if } w \in s \text{ then } w \in f(s)\}$

Restricting the clauses of this definition to indices belonging to a certain state s we obtain the proposal $[\varphi]_s$ made by φ relative to the common ground s.

Now, the most obvious way to allow for non-maximal possibilities is to simply remove the operator MAX from the clauses. This strategy, pursued in my thesis (Ciardelli, 2009), changes the notion of meaning right from the propositional case.

In the resulting system, which we refer to as *unrestricted inquisitive semantics*, informativeness and inquisitiveness no longer exhaust the meaning of a formula. For, formulas such as $p \vee \top$ are neither informative nor inquisitive, but they still make a non-trivial proposal. Ciardelli *et al.* (2009) suggest that such formulas may be understood in terms of *attentive potential* and shows how the enriched notion of inquisitive meaning provides simple tools for an analysis of *might*. In this respect, the unrestricted system is a simple but powerful refinement of the standard system.

However, this solution has also drawbacks. For, in some cases the interpretation of possibilities included in maximal ones in terms of attentive potential does not seem convincing. For instance, consider a common ground s in which a concrete upper bound n for P is known, that is, such that $s \models B(n)$: intuitively, the boundedness formula should be redundant relative to such a common ground, that is, we should have $[\exists x B(x)]_s = \{s\}$. However, in the unrestricted system, the boundedness formula still proposes the range of possibilities $B(0), \ldots, B(n)$, that is, we have $[\exists x B(x)] = \{|B(0)| \cap s, \ldots, |B(n)| \cap s, \emptyset\}$.

The behaviour of the propositional connectives is sometimes also puzzling: for instance, $(p \lor q) \land (p \lor q)$ also proposes the possibility that $p \land q$ (but $p \lor q$ does not), while the implication $p \to ?p$ turns out equivalent with $\neg p \lor \top$.

My aim in the present paper is to outline a different road, to describe a way to extend propositional inquisitive semantics *as it is* to obtain a more "orthodox" predicate inquisitive semantics in which meaning still consists of informative and inquisitive potential.

Definition 7. If Π is a set of states, say that an element $s \in \Pi$ is optimally dominated in case there is a maximal state $t \in \Pi$ with $t \supseteq s$.

In the unrestricted propositional semantics, due to the finitary character of propositional meanings, non-maximal possibilities are always properly included in some maximal one. Therefore, taking the maximal elements or filtering out optimally dominated ones are operations which yield the same result.

On the other hand, the example of the boundedness formula shows that the meanings we want to obtain in the first-order case may consist of an infinite chain of possibilities, none of which is maximal. Here, as we have seen, extracting maximal states in definition 6 leaves us with nothing at all; filtering out optimally dominated states, on the other hand, has no effect in this case and yields the intended meaning of the boundedness formula.

These observations lead to the idea of expanding definition 6 with the natural clauses for quantifiers (where the behaviour of \exists and \forall is analogous to that of \lor and \land respectively), while substituting the operator MAX with a more sensitive filter NOD which, given a set of states Π , returns the set of states in Π which are not optimally dominated. The result is the following definition.

Definition 8 (First-order inquisitive meanings). The inquisitive meaning of a formula φ relative to an assignment g is defined inductively as follows.

1. $[\varphi]_g = \{|\varphi|_g\}$ if φ is atomic 2. $[\bot]_g = \{\emptyset\}$ 3. $[\varphi \lor \psi]_g = \operatorname{NOD}([\varphi]_g \cup [\psi]_g)$ 4. $[\varphi \land \psi]_g = \operatorname{NOD}\{s \cap t \mid s \in [\varphi]_g \text{ and } t \in [\psi]_g\}$ 5. $[\varphi \to \psi]_g = \operatorname{NOD}\{\Pi_f \mid f : [\varphi]_g \to [\psi]_g\}$ $\begin{aligned} & 6. \ [\exists x \varphi]_g = \operatorname{NoD}(\bigcup_{d \in D} [\varphi]_{g[x \mapsto d]}) \\ & 7. \ [\forall x \varphi]_g = \operatorname{NoD}\{\bigcap_{d \in D} s_d \mid s_d \in [\varphi]_{g[x \mapsto d]}\} \end{aligned}$

Again, the proposal $[\varphi]_{s,g}$ made by φ relative to the common ground s and the assignment g is obtained by restricting the clauses to indices in s. Obviously, if φ is a sentence, the assignment g is irrelevant and we may therefore omit reference to it.

There is, however, a subtlety we must take into account. While in the propositional case a formula may propose the empty state only if it is inconsistent, with the given definition the empty state would pop up in totally unexpected circumstances, with unpleasant consequences in terms of entailment and equivalence; for instance, we would have $[\exists x(x = 0 \land B(x))] = \{|B(0)|, \emptyset\} \neq \{|B(0)|\} = [B(0)]$. To fix this problem, we modify slightly our definitions, stipulating that the empty state is optimally dominated in a set of states Π as soon as Π contains a non-empty possibility. For the rest, we can keep the definition of the system unchanged.

Notice that by definition of the operator NOD, we can never end up in an absurd situation like the one discussed in example 1, in which $[\varphi] = \emptyset$ (in which, that is, a formula would propose *nothing*!) Moreover, it is easy to establish inductively the following fact, which shows that we have indeed defined a conservative extension of propositional inquisitive semantics.

Proposition 2. If φ is a quantifier-free formula, then the meaning $[\varphi]$ given by definition 8 coincides with the meaning of φ considered as a propositional formula, as given by definition 3.

The system we defined can cope with the subtleties highlighted by example 2: formulas which are equivalent in terms of support may be assigned different meanings, and may even have *no* common possibility at all, thus differing dramatically in terms of the compliant responses they allow.

Example 3. In the context of example 1, let $E(x) = \exists y(y + y = x)$ and $O(x) = \neg E(x)$; clearly, E(x) and O(x) are assertions stating, respectively, that x is even and that x is odd. We have:

1. $[\exists x B(x)] = \{|B(n)|, n \in \mathbb{N}\}$ 2. $[\exists x(x \neq 0 \land B(x))] = \{|B(n)|, n \neq 0\}$ 3. $[\exists x(E(x) \land B(x))] = \{|B(n)|, n \text{ even}\}$ 4. $[\exists x(O(x) \land B(x))] = \{|B(n)|, n \text{ odd}\}$

On the one hand, one knows an even upper bound for P iff one knows an odd upper bound, so the formulas $\exists x(E(x) \land B(x))$ and $\exists x(O(x) \land B(x))$ are resolved in exactly the same information states, which is what support captures. On the other hand, the sentences "there is an even upper bound to P" and "there is an odd upper bound to P" invite different responses, and the system rightly predicts this by assigning them distinct possibilities. Moreover, unlike the unrestricted system, the proposed semantics correctly predicts that the boundedness formula is redundant in any information state in which an upper bound for P is known: if $s \models B(x)$, then $[\exists x B(x)]_s = \{s\}$.

Many features of the propositional system carry over to this first-order implementation. Crucially, meaning is still articulated in two components, informativeness and inquisitiveness. For, consider a φ which is neither informative nor inquisitive: since φ is not inquisitive, $|\varphi| \in [\varphi]$; and since φ is not informative, $|\varphi| = \omega$; finally, since the presence of the filter NOD explicitly rules out possibilities included in maximal ones, ω must be the *unique* possibility for φ , that is, φ must be an inquisitive tautology.

Assertions and questions may be defined as usual, and it is still the case that for any formula φ , $!\varphi$ is an assertion, $?\varphi$ is a question, and the decomposition $\varphi \equiv !\varphi \land ?\varphi$ holds, where equivalence amounts to having the same meaning.

Obviously, the classical treatment of information is preserved, i.e. we have $\bigcup[\varphi] = |\varphi|$. Finally, the sources of inquisitiveness in the system are disjunction and the existential quantifier, in the sense that any formula not containing disjunction or the existential quantifier is an assertion.

5 Conclusions

In this paper we proposed a conservative extension of propositional inquisitive semantics to the first order setting, focussing on the essential changes that this move required. These were (i) to state the semantics in terms of a recursive specification of the possibilities for a sentence, rather than in terms of support; and (ii) to switch from the requirement of maximality to that of not being optimally dominated. These changes have no effect on the propositional case.

The proposed system was motivated here by the attempt to obtain correct predictions while retaining as much as possible of the propositional system: a very important thing which remains to be done is to provide a more conceptual justification for the given definitions.

Moreover, a task for future work is the investigation of both the logical features of the proposed semantics and its application to natural language, in particular to the semantics of interrogative sentences.

With regard to this latter aspect, notice that our logical semantics as such does not embody a specific theory on the semantic analysis of interrogatives. Instead, it offers a general logical framework in which also opposing empirical analyses may be formulated and studied. This is most obviously so for the Hamblin analysis of questions Hamblin (1973), which is covered by inquisitive existential quantification $(\exists xPx)$, and the partition approach of Groenendijk and Stokhof (1984), which is covered by universal quantification over polar questions $(\forall x?Px)$. The treatment of *which*-questions in Velissaratou (2000), which analyzes such questions in terms of exhaustive answers, but not as partitions, may also be represented (by $\forall x(Px \rightarrow ?Qx)$).

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