

# Inquisitive semantics: a new notion of meaning

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## Abstract

This paper presents a notion of meaning that captures both informative and inquisitive content, and forms the cornerstone of inquisitive semantics. The new notion of meaning is explained and motivated in detail, and compared to previous inquisitive notions of meaning.

## 1 Introduction

The central aim of the inquisitive semantics research program is to develop a new notion of semantic meaning that captures both *informative* and *inquisitive* content. This enriched notion of meaning is intended to provide new foundations for the analysis of linguistic discourse that is aimed at *exchanging* information.

The way in which inquisitive semantics enriches the notion of meaning changes our perspective on *logic* as well. Besides the classical notion of entailment, the semantics also gives rise to a new notion of *inquisitive entailment*, and to new logical notions of *relatedness*, which determine, for instance, whether one sentence compliantly addresses or resolves the issue raised by another.

The enriched notion of semantic meaning also changes our perspective on *pragmatics*. The general objective of pragmatics is to explain aspects of interpretation that are not directly dictated by semantic content, in terms of general features of rational human behavior. Gricean pragmatics has fruitfully pursued this general objective, but is limited in scope. Namely, it is only concerned with what it means for *speakers* to behave rationally in *providing* information. Inquisitive pragmatics is broader in scope: it is both *speaker-* and *hearer-*oriented, and is concerned more generally with what it means to behave rationally in cooperatively *exchanging* information rather than just in providing information. This makes it possible to derive a wider range of implicatures, in particular ones that arise from inquisitiveness.

This paper focuses on the new notion of meaning that forms the cornerstone of the most basic implementation of inquisitive semantics. Our aim here is to explain and motivate this new notion of meaning in detail, and to place it in a wider context of previous ideas about inquisitive notions of meaning. In other work (e.g. [Ciardelli, 2009](#); [Ciardelli and Roelofsen, 2011](#); [Groenendijk and Roelofsen, 2009](#); [Roelofsen, 2011](#)) we discussed how these new type of meanings

could be assigned to expressions in simple logical languages, like the language of propositional logic and the language of first-order predicate logic, and we explored some of the logical and pragmatic repercussions of this shift in perspective. The current paper can be seen as a retrospective preface to this body of work, aiming to provide a detailed description and principled motivation for its most basic notion.

The paper is organized as follows. We will start in section 2 by formulating and motivating the new notion of meaning in general terms. In sections 3 and 4, these terms will be spelled out in more detail. Finally, in section 5, we will relate our proposal to previous work on inquisitive aspects of meaning.

## 2 Meaning as information exchange potential

The meaning of a sentence can be thought of as something that determines the intended effect of an utterance of that sentence on the discourse context in which the sentence is uttered. That is, when a speaker utters a sentence in a certain discourse context, he intends his utterance to change the discourse context in a particular way, and the meaning of the sentence determines this intended effect. Thus, the meaning of a sentence can be conceived of as its *context change potential*, which can be modeled formally as a function that maps every discourse context to a new discourse context.

This general conception of meaning can be made more precise in several ways, depending on what exactly we take a discourse context to be. The simplest and most common option is to think of a discourse context as the body of information that has been established in the discourse so far. This body of information is usually referred to as the *common ground* of the conversation, and it is formally modeled as a set of possible worlds—those worlds that are compatible with the established information.

If the discourse context is identified with the information established so far, then the context change potential of a sentence boils down to its *information change potential*. Formally, the meaning of a sentence can then be modeled as a function that maps information states—sets of possible worlds—to other information states.

Classically, the meaning of a sentence is identified with its *truth-conditions*, rather than a function over information states. However, a specific connection is assumed in this setting between the truth-conditions of a sentence and the intended effect of uttering that sentence in a certain discourse context. Namely, it is assumed that the intended effect of uttering a sentence in a certain discourse context is to restrict that discourse context to precisely those worlds that satisfy the truth-conditions of the uttered sentence, i.e., to those worlds in which the sentence is true. Thus, the truth-conditions of a sentence completely determine the sentence's information change potential. In light of this connection, the classical truth-conditional framework can also be seen as one in which the meaning of a sentence is something that determines the sentence's context change potential.

Now, as noted above, identifying a discourse context with the body of information that has been established in the discourse so far, is only one particular way of spelling out the notion of a discourse context. Of course, the general conception of meaning as context change potential is in principle compatible with richer notions of discourse contexts. And to analyze many types of discourse, such richer notions are indeed required. We will focus here on a very basic type of discourse, namely one in which a number of participants *exchange information* by raising and resolving issues. In order to analyze this type of discourse, discourse contexts should not only embody the information that has been established so far, but also the issues that have been raised so far. And similarly, the meaning of a sentence should not only embody its informative content, i.e., its potential to provide information, but also its inquisitive content, i.e., its potential to raise issues. In short, the meaning of a sentence should embody its *information exchange potential*. Below we will spell out in detail how to model such richer types of discourse contexts and meanings.

### 3 Discourse contexts: information and issues

The first step is to formulate a notion of discourse contexts that embodies both the information that has been established so far and the issues that have been raised so far. We already know how to model the information established so far, namely as a set of possible worlds. Throughout the paper we will assume a set of possible worlds  $\omega$  as our logical space.

**Definition 1** (Information states).

An information state is a set of possible worlds  $s \subseteq \omega$ .

We will often refer to information states simply as *states*, and for any discourse context  $c$  we will use  $\text{info}(c)$  to denote the information state that represents the information available in  $c$ . The crucial question that remains to be addressed is how to model *issues*.

**Issues.** Suppose we are in a discourse context  $c$ . Then, according to the information established so far, the actual world is located somewhere in  $\text{info}(c)$ . We can think of every subset  $s \subseteq \text{info}(c)$  as an information state that is more informed than  $\text{info}(c)$ , i.e., one that locates the actual world more precisely than  $\text{info}(c)$  itself. Thus, we may refer to every  $s \subseteq \text{info}(c)$  as a possible *enhancement* of  $\text{info}(c)$ .

Now, an issue in  $c$  should represent a certain request for information, a request to locate the actual world more precisely inside  $\text{info}(c)$ . Thus, an issue in  $c$  can be modeled as a non-empty set  $\mathcal{I}$  of enhancements of  $\text{info}(c)$ , namely those enhancements that would satisfy the given request, i.e., that would locate the actual world with sufficient precision.

Importantly, not just any non-empty set  $\mathcal{I}$  of enhancements of  $\text{info}(c)$  can properly be thought of as an issue in  $c$ . First, if  $\mathcal{I}$  contains a certain enhancement  $s$  of  $\text{info}(c)$ , and  $t \subset s$  is a further enhancement of  $s$ , then  $t$  must also be in  $\mathcal{I}$ .

After all, if  $s$  locates the actual world with sufficient precision, then  $t$  cannot fail to do so as well. So  $\mathcal{I}$  must be *downward closed*.

Second, the elements of  $\mathcal{I}$  must together form a *cover* of  $\text{info}(c)$ . That is, every world in  $\text{info}(c)$  must be included in at least one element of  $\mathcal{I}$ . After all, any world in  $\text{info}(c)$  may be the actual world according to the information available in  $c$ . Now suppose that  $w$  is a world in  $\text{info}(c)$  that is not included in any element of  $\mathcal{I}$ . Then, according to the information available in  $c$ ,  $w$  may very well be the actual world. But if it *is* indeed the actual world, then it would be impossible to satisfy the request represented by  $\mathcal{I}$  without discarding the actual world. Thus, in order to ensure that it is possible to satisfy the request represented by  $\mathcal{I}$  without discarding the actual world,  $\mathcal{I}$  should form a cover of  $\text{info}(c)$ . This leads us to the following notion of an issue.

**Definition 2** (Issues).

Let  $s$  be an information state, and  $\mathcal{I}$  a non-empty set of enhancements of  $s$ . Then we say that  $\mathcal{I}$  is an issue over  $s$  if and only if:

1.  $\mathcal{I}$  is *downward closed*: if  $t \in \mathcal{I}$  and  $t' \subseteq t$  then also  $t' \in \mathcal{I}$
2.  $\mathcal{I}$  forms a *cover* of  $s$ :  $\bigcup \mathcal{I} = s$

**Definition 3** (Settling an issue).

Let  $s$  be an information state,  $t$  an enhancement of  $s$ , and  $\mathcal{I}$  an issue over  $s$ . Then we say that  $t$  settles  $\mathcal{I}$  if and only if  $t \in \mathcal{I}$ .

Now we are ready to return to the notion of a discourse context, our main concern in this section.

**Discourse contexts.** Given what we have said so far, the most straightforward way to proceed would be to define a discourse context  $c$  as a pair  $\langle \text{info}(c), \text{issues}(c) \rangle$ , where  $\text{info}(c)$  is an information state, and  $\text{issues}(c)$  a non-empty set of issues over  $\text{info}(c)$ . The initial discourse contexts would then be  $\langle \omega, \{\wp(\omega)\} \rangle$ , consisting of the trivial information state, which does not rule out any world, and the trivial issue, which is settled by all states.

This would indeed be a suitable notion of discourse contexts. However, for our current purposes, it will be convenient to simplify this notion somewhat. We will do this in two steps. First, rather than thinking of a discourse context  $c$  as a pair  $\langle \text{info}(c), \text{issues}(c) \rangle$  where  $\text{issues}(c)$  is a *set* of issues over  $\text{info}(c)$ , we could think of a discourse context as a pair  $\langle \text{info}(c), \text{issue}(c) \rangle$  where  $\text{issue}(c)$  is a *single* issue over  $\text{info}(c)$ . This simplification is justified by the observation that any non-empty set of issues  $I$  over a state  $s$  can be reduced to a single issue  $\mathcal{I} := \bigcap I$  over  $s$  such that any enhancement  $t \subseteq s$  settles  $\mathcal{I}$  just in case it settles every issue in  $I$ . After all, a state settles all the issues in  $I$  just in case it is contained in all those issues, which is true just in case it is contained in  $\bigcap I$ .<sup>1</sup>

<sup>1</sup>Notice that for the informative component of a discourse context we have implicitly assumed a similar reduction: we do not keep track of all the separate pieces of information that have been established in the discourse so far, but rather of the set of worlds that are

So we can think of a discourse context  $c$  as a pair  $\langle \text{info}(c), \text{issue}(c) \rangle$ , where  $\text{info}(c)$  is an information state, and  $\text{issue}(c)$  a single issue over  $\text{info}(c)$ . But this representation can be simplified even further. After all, since  $\text{issue}(c)$  is an issue over  $\text{info}(c)$ , it must form a *cover* of  $\text{info}(c)$ . So we always have that  $\text{info}(c) = \bigcup \text{issue}(c)$ . That is,  $\text{info}(c)$  can always be retrieved from  $\text{issue}(c)$ . But then  $\text{info}(c)$  can just as well be left out of the representation of  $c$ . Thus, a discourse context  $c$  can simply be represented as an issue over some information state  $s$ , i.e., a non-empty, downward closed set of enhancements of  $s$  that together form a cover of  $s$ . This information state  $s$  is then understood to embody the information available in  $c$ .

**Definition 4** (Discourse contexts).

- A discourse context  $c$  is a non-empty, downward closed set of states.
- The set of all discourse contexts will be denoted by  $\mathcal{C}$ .

**Definition 5** (The information available in a discourse context).

- For any discourse context  $c$ :  $\text{info}(c) := \bigcup c$

So we have moved from the classical notion of a discourse context as a set of possible worlds—representing the information established so far—to a richer notion of discourse contexts as non-empty, downward closed sets of states—representing both the information established so far and the issues raised so far. With this enriched notion of discourse contexts, we are in principle ready to return to our main concern, which is to specify a notion of *meaning* that embodies both informative and inquisitive content. However, before turning to meanings, it will be useful to briefly identify some special properties that discourse contexts may have.

First of all, we can make a distinction between *informed* and *ignorant* discourse contexts, ones in which some information has been established and ones in which no information has been established, respectively.

**Definition 6** (Informed and ignorant discourse contexts).

- A discourse context  $c$  is informed iff  $\text{info}(c) \neq \omega$ .
- A discourse context  $c$  is ignorant iff  $\text{info}(c) = \omega$ .

Similarly, we can make a distinction between *inquisitive* and *indifferent* discourse contexts. A discourse context  $c$  is indifferent iff the information that has been established so far settles all the issues that have been raised, i.e.,  $\text{info}(c) \in c$ . Otherwise, i.e., if there are unresolved issues, then  $c$  is called inquisitive.

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compatible with all these pieces of information (formally, this is again the *intersection* of all the separate established pieces of information). For certain purposes it may be convenient, or even necessary, to keep track of all the separate pieces of information and/or issues that have been established/raised in a discourse, cf., Stalnaker's (1978) distinction between the *common ground* and the *context set*. However, for our current purposes, this would only add unnecessary complexity.

**Definition 7** (Inquisitive and indifferent discourse contexts).

- A discourse context  $c$  is indifferent iff  $\text{info}(c) \in c$ .
- A discourse context  $c$  is inquisitive iff  $\text{info}(c) \notin c$ .

There are two special discourse contexts: the *initial* and the *absurd* discourse context. The initial context,  $c_{\top}$ , is the only context that is both ignorant and indifferent. The absurd context,  $c_{\perp}$ , is one in which the established information is inconsistent and therefore rules out all possible worlds.

**Definition 8** (The initial and the absurd discourse context).

- $c_{\top} := \wp(\omega)$
- $c_{\perp} := \{\emptyset\}$

Two discourse contexts can be compared in terms of the information that has been established or in terms of the issues that have been raised. One context  $c'$  is at least as informed as another context  $c$  if and only if  $\text{info}(c') \subseteq \text{info}(c)$ .

**Definition 9** (Informative order on discourse contexts).

Let  $c, c' \in \mathcal{C}$ . Then:

- $c' \geq_{\text{info}} c$  iff  $\text{info}(c') \subseteq \text{info}(c)$

Similarly, for any two discourse contexts  $c$  and  $c'$  that are equally informed, i.e.,  $\text{info}(c) = \text{info}(c')$ , we can say that  $c'$  is at least as inquisitive as  $c$  if and only if every state that settles all the issues that have been raised in  $c'$  also settles all the issues that have been raised in  $c$ , i.e., if and only if  $c' \subseteq c$ .

**Definition 10** (Inquisitive order on discourse contexts).

Let  $c, c' \in \mathcal{C}$  and  $\text{info}(c) = \text{info}(c')$ . Then:

- $c' \geq_{\text{inq}} c$  iff  $c' \subseteq c$

We will say that one context  $c'$  is an *extension* of another context  $c$  just in case (i)  $c'$  is at least as informed as  $c$ , and (ii)  $c'$  is at least as inquisitive as the context that is obtained by *restricting*  $c$  to  $\text{info}(c')$ , i.e.,  $c' \geq_{\text{inq}} c \upharpoonright \text{info}(c')$ , where  $c \upharpoonright \text{info}(c') := \{s \cap \text{info}(c') \mid s \in c\}$ . It can be shown that these two conditions are fulfilled just in case  $c' \subseteq c$ . So the extension relation between discourse contexts can simply be defined in terms of inclusion.

**Definition 11** (Extending discourse contexts). Let  $c, c' \in \mathcal{C}$ . Then:

- $c'$  is an extension of  $c$ ,  $c' \geq c$ , iff  $c' \subseteq c$

The extension relation forms a *partial order* on  $\mathcal{C}$ , and  $c_{\top}$  and  $c_{\perp}$  constitute the extremal elements of this partial order:  $c_{\perp}$  is an extension of every discourse context, and every discourse context is in turn an extension of  $c_{\top}$ .

**Fact 1** (Partial order and extrema).

- $\geq$  forms a partial order on  $\mathcal{C}$
- For every  $c \in \mathcal{C}$ :  $c_{\perp} \geq c \geq c_{\top}$

Finally, for any two discourse contexts  $c$  and  $c'$ , we will refer to  $c \cap c'$  as the *merge* of  $c$  and  $c'$ . It can be shown that  $c \cap c'$  is always a proper discourse context, i.e., a non-empty downward closed set of states, and moreover, that the information available/requested in  $c \cap c'$  is exactly the information available/requested in  $c$  plus the information available/requested in  $c'$ .

**Definition 12** (Merging two discourse contexts).

For any  $c, c' \in \mathcal{C}$ ,  $c \cap c'$  is called the *merge* of  $c$  and  $c'$ .

**Fact 2** (Merging yields a new discourse context).

For any  $c, c' \in \mathcal{C}$ ,  $c \cap c'$  is also in  $\mathcal{C}$ .

**Fact 3** (Merging information and issues). Let  $c, c' \in \mathcal{C}$ . Then:

1. A possible world is discarded by the information available in  $c \cap c'$  just in case it is discarded by the information available in  $c$  or by the information available in  $c'$ .
2. A state settles all the issues raised in  $c \cap c'$  just in case it settles all the issues raised in  $c$  and all the issues raised in  $c'$ .

**Proof.** The second claim follows immediately from the fact that a state  $t$  settles all the issues raised in a context  $c$  just in case  $t \in c$ . To establish the first claim, we have to show that  $\text{info}(c \cap c') = \text{info}(c) \cap \text{info}(c')$ . Clearly, if  $w \in \text{info}(c \cap c')$ , then  $w$  must be contained in at least one state that is included in both  $c$  and  $c'$ . It follows, then, that  $w \in \text{info}(c)$  and  $w \in \text{info}(c')$ . This establishes that  $\text{info}(c \cap c') \subseteq \text{info}(c) \cap \text{info}(c')$ . For the other inclusion, suppose that  $w \in \text{info}(c)$  and  $w \in \text{info}(c')$ . Then  $w$  is contained in at least one state in  $c$  and at least one state in  $c'$ . But since  $c$  and  $c'$  are both downward closed, this means that the singleton state  $\{w\}$  is included in both  $c$  and  $c'$ . Thus,  $\{w\}$  is also included in  $c \cap c'$ , and this means that  $w \in \text{info}(c \cap c')$ .  $\square$

This completes our brief exploration of discourse contexts. We are now ready to turn to meanings.

## 4 Meanings and propositions

In section 2 we characterized the meaning of a sentence in general terms as its context change potential, which can be modeled formally as a function that maps every discourse context to a new discourse context. We have now specified what we take discourse contexts to be, so it has also become clearer what we take meanings to be.

However, not just any function  $f$  over discourse contexts can properly be seen as a meaning. First of all, we want meanings to be functions that map any discourse context  $c$  to a new discourse context that is an *extension* of  $c$ . Second, if we have two discourse contexts  $c$  and  $c'$ , one an extension of the other, then  $f(c)$  and  $f(c')$  must be related in a certain way. The idea is that if  $f(c)$  and  $f(c')$  differ, this difference should be traceable to the initial difference in information and issues between  $c$  and  $c'$ . Once the initial gap between  $c$  and  $c'$  is filled, the difference between  $f(c)$  and  $f(c')$  should also vanish.

Let us make this intuition precise. Consider a discourse context  $c$ , and an extension of it,  $c'$ . Now consider  $f(c')$  and  $f(c)$ . First of all, it should be the case that  $f(c')$  is still an extension of  $f(c)$ . That is,  $f$  must be a *monotonic* function w.r.t. the extension order on discourse contexts. However, we will require something stronger than this: namely, if we add the information and the issues present in  $c'$  to  $f(c)$ , i.e., if we take the *merge* of  $c'$  and  $f(c)$ , we should end up exactly in  $f(c')$ . We will refer to this condition as the *compatibility condition*.

**Definition 13** (Compatibility condition).

A function  $f$  over discourse contexts satisfies the compatibility condition if and only if for every  $c, c' \in \mathcal{C}$  such that  $c' \geq c$ , we have that  $f(c') = f(c) \cap c'$ .

**Definition 14** (Meanings).

A meaning is a function  $f$  which maps every discourse context  $c$  to a new discourse context  $f(c) \geq c$ , in compliance with the compatibility condition.

Now recall that in the classical setting, the meaning of a sentence  $\varphi$  is identified with its truth-conditions—a function from worlds to truth-values—or equivalently, with the set of worlds  $|\varphi|$  that satisfy these truth-conditions. This set of worlds is usually referred to as the *proposition* expressed by the sentence. Furthermore, as pointed out above, in the classical setting the proposition expressed by a sentence  $\varphi$  is taken to determine the sentence’s context change potential: it is assumed that the intended effect of an utterance of  $\varphi$  is to restrict the discourse context—classically modeled as a set of possible worlds—to  $|\varphi|$ . In other words, the new discourse context is obtained by intersecting the old discourse context with the proposition expressed by  $\varphi$ .

In the present setting, we may also introduce a notion of propositions that fulfills precisely the same role. To get at the right notion, recall from fact 1 that every discourse context  $c$  is an extension of  $c_\top$ . Thus, the compatibility condition ensures that for every meaning  $f$  and every discourse context  $c$ :

$$f(c) = f(c_\top) \cap c$$

This means that  $f$  is completely determined by  $f(c_\top)$ . If we have a certain discourse context  $c$  and we want to know what the new discourse context is that results from applying  $f$  to  $c$ , we can simply take the intersection of  $c$  with  $f(c_\top)$ . Thus, just like in the classical case, a meaning  $f$  can be identified with a unique static object,  $f(c_\top)$ , which we will call the *proposition* associated with  $f$ . So our new notion of meanings also gives rise to a new notion of propositions. Namely, propositions are non-empty, downward closed sets of states.

**Definition 15** (Propositions).

- A proposition is a non-empty, downward closed set of states.
- The set of all propositions is denoted by  $\Pi$ .

We will proceed to characterize some special properties that propositions may have. Notice that there is a one-to-one correspondence between propositions and meanings: for every meaning  $f$ , the associated proposition is  $f(c_{\top})$ , and for any proposition  $A$ , the associated meaning is the function  $f_A$  that maps every discourse context  $c$  to  $c \cap A$ . This means that everything we will say below about propositions directly pertains to the associated meanings as well.

Also notice that, as in the classical setting, propositions and discourse contexts are the same type of objects. This means that many properties of discourse contexts that we discussed in section 3 can also be predicated, in much the same way, of propositions.

For instance, we will say that the *informative content* of a proposition  $A$ ,  $\text{info}(A)$ , is the information that is available in the discourse context  $c_{\top} \cap A$ . This information is embodied by  $\bigcup A$ .

**Definition 16** (Informative content of a proposition).

- For any proposition  $A$ :  $\text{info}(A) = \bigcup A$

We will say that a state  $t$  *settles* a proposition  $A$  just in case it settles the issue in  $c_{\top} \cap A$ , which is the case if and only if  $t \in A$ .

**Definition 17** (Settling a proposition).

- An information state  $t$  settles a proposition  $A$  if and only if  $t \in A$ .

We will say that a proposition  $A$  is *informative* just in case its informative content is non-trivial, i.e.,  $\text{info}(A) \neq \omega$ . We will say that  $A$  is *inquisitive* just in case it is not settled by its own informative content, i.e.,  $\text{info}(A) \notin A$ .

**Definition 18** (Informative and inquisitive propositions).

- A proposition  $A$  is informative iff  $\text{info}(A) \neq \omega$ .
- A proposition  $A$  is inquisitive iff  $\text{info}(A) \notin A$ .

Just like we identified two special discourse contexts,  $c_{\top}$  and  $c_{\perp}$ , we can also identify two special propositions, namely the tautological proposition  $A_{\top} := \wp(\omega)$ , whose associated meaning maps every discourse context to itself, and the contradictory proposition  $A_{\perp} := \{\emptyset\}$ , whose associated meaning maps every discourse context to the absurd context.

**Definition 19** (Tautology and contradiction).

- $A_{\top} := \wp(\omega)$
- $A_{\perp} := \{\emptyset\}$

Just like discourse contexts, propositions can be ordered either in terms of their informative component or in terms of their inquisitive component. We say that one proposition  $A$  is at least as informative as another proposition  $B$  just in case for every discourse context  $c$ ,  $c \cap A$  is at least as informed as  $c \cap B$ , which is true if and only if  $\text{info}(A) \subseteq \text{info}(B)$ .

**Definition 20** (Informative order on propositions).

Let  $A, B \in \Pi$ . Then:

- $A \models_{\text{info}} B$  iff  $\text{info}(A) \subseteq \text{info}(B)$

Similarly, if  $A$  and  $B$  are equally informative, then we say that  $A$  is at least as inquisitive as  $B$  just in case any state that settles  $A$  also settles  $B$ , which is true if and only if  $A \subseteq B$ .

**Definition 21** (Inquisitive order on propositions).

Let  $A, B \in \Pi$  and  $\text{info}(A) = \text{info}(B)$ . Then:

- $A \models_{\text{inq}} B$  iff  $A \subseteq B$

We will say that one proposition  $A$  *entails* another proposition  $B$  just in case (i)  $A$  is at least as informative as  $B$ , and (ii)  $A$  is at least as inquisitive as the proposition that results from *restricting*  $B$  to the informative content of  $A$ , i.e.,  $A \models_{\text{inq}} B \upharpoonright \text{info}(A)$ , where  $B \upharpoonright \text{info}(A) := \{s \cap \text{info}(A) \mid s \in B\}$ . It can be shown that these two conditions are satisfied exactly if  $A \subseteq B$ . So entailment can simply be defined in terms of inclusion.

**Definition 22** (Entailment).

Let  $A, B \in \Pi$ . Then:

- $A \models B$  iff  $A \subseteq B$

Entailment forms a partial order on  $\Pi$ , and  $A_{\top}$  and  $A_{\perp}$  constitute the extremal elements of this partial order:  $A_{\perp}$  entails every proposition, and every proposition in turn entails  $A_{\top}$ .

**Fact 4** (Partial order and extrema).

- $\models$  forms a partial order on  $\Pi$
- For every  $A \in \Pi$ :  $A_{\perp} \models A \models A_{\top}$

In fact, it can be shown that  $\langle \Pi, \models \rangle$  forms a complete Heyting algebra, with infinitary meet and join operators, and a (relative) pseudo-complement operator. Incidentally, the meet of a set of propositions amounts to their intersection, and the join of a set of propositions amounts to their union, just as in the complete Boolean powerset algebra that underlies classical logic. This algebraic result gives rise to an inquisitive semantics for the language of propositional logic, in which conjunction is taken to behave semantically as a meet operator, disjunction as a join operator, negation as a pseudo-complement operator, and

implication as a relative pseudo-complement operator (Roelofsen, 2011). Essentially the same system has been presented from a different, non-algebraic perspective in (Ciardelli, 2009; Groenendijk and Roelofsen, 2009; Ciardelli and Roelofsen, 2011). We regard this system as the most basic implementation of inquisitive semantics, and refer to it as  $\text{Inq}_B$ .

To end this section, we would like to emphasize that we take the meaning of a sentence to determine the *intended* effect of an utterance of that sentence on the discourse context. Whether this effect is achieved depends on how other discourse participants react to the utterance. Thus, an utterance can be thought of as a *proposal* to accept a certain piece of information and to settle a certain issue. Other participants may comply with such a proposal, but they may also reject it, or come up with a counter-proposal. A detailed analysis of this interactive process is beyond the scope of this paper (see, e.g. Groenendijk, 2008; Balogh, 2009), but it should be clear that we do not take the meaning of a sentence to determine the *actual* effect of an utterance of that sentence on the discourse context, but rather the intended effect.

This completes our exploration of inquisitive meanings and propositions. We will end the paper by relating the ideas presented here to previous work on inquisitive notions of meaning.

## 5 Previous inquisitive notions of meaning

There is a large body of work on the semantics of questions, which has given rise to several inquisitive notions of meaning. We will restrict our attention here to those proposals that are most closely related to our own. That is, we will consider the classical work on the semantics of questions by Hamblin (1973), Karttunen (1977), and Groenendijk and Stokhof (1984), and a number of more recent theories that are couched in a dynamic semantic framework (Jäger, 1996; Hulstijn, 1997; Groenendijk, 1999, 2009; Mascarenhas, 2009). Our work most naturally fits within the latter tradition, which in turn builds on the former.

**Classical semantic theories of questions.** According to Hamblin (1973), questions denote sets of classical propositions. The central idea is that “questions set up a choice-situation between a set of propositions, namely those propositions that count as answers to it” (Hamblin, 1973, p.48). In the closely related theory of Karttunen (1977), questions also denote sets of classical propositions, but only those propositions that correspond to *true* answers. Thus, in both systems, the *meaning* of a question is a function from worlds to sets of classical propositions. In Hamblin’s system, this function maps every possible world to the same set of propositions, corresponding to the set of all possible answers; in Karttunen’s system, every world is mapped to a subset of all possible answers, namely those that are true in the given world. Notice that, as acknowledged by Karttunen (1977, p.10), the difference is inessential. In both cases, the meaning of a question is fully determined by—and could be identified with—the set of all classical propositions that correspond to a possible answer.

A fundamental problem with these accounts is that they do not specify in more detail what “possible answers” are supposed to be. Of course, Hamblin and Karttunen do provide a compositional semantics for a fragment of English, and thereby specify what they take to be the possible answers to the questions in that fragment. But in order for these theories to be evaluated, we first need to know what the notion of a “possible answer” is supposed to capture. To illustrate this point, consider the following example:

- (1) Who is coming for dinner tonight?
  - a. Paul is coming.
  - b. Only Paul and Nina are coming.
  - c. Some girls from my class are coming.
  - d. I don’t know.

In principle, all the responses in (1a-d) could be seen as possible answers to (1). For Hamblin and Karttunen, only (1a) counts as such. However, it is not made clear what the precise criteria are for being considered a possible answer, and on which grounds (1a) is to be distinguished from (1b-d).

One natural criterion would be the following. We could say that a response to a question counts as a proper answer just in case it *resolves* the issue that the question raises. However, if we adopt this criterion then we also have to impose a certain condition on question-meanings. That is, in this case question-meanings cannot just be arbitrary sets of classical propositions. Rather, they should be *downward closed* sets of classical propositions. After all, suppose that  $\alpha$  is an element of the meaning of a question  $Q$ . Given our criterion, this means that  $\alpha$  corresponds to an issue-resolving response to  $Q$ . But then every  $\beta \subseteq \alpha$  corresponds to an even more informative, and therefore also issue-resolving response. So, given our criterion,  $\beta$  must also be an element of the meaning of  $Q$ .

The property of downward closedness is exactly the property that is characteristic for the notion of propositions that we proposed here. This is because propositions are characterized as sets of states that *settle* a certain issue, and such states correspond exactly to pieces of information that *resolve* the given issue. So the elements of a propositions can be thought of as states that settle the issue in question, or alternatively as issue-resolving responses. Thus, our proposal is compatible with the general philosophy of Hamblin and Karttunen, but adopts a more specific notion of “possible answers” and constrains question-meanings accordingly.

According to [Groenendijk and Stokhof \(1984\)](#), a question denotes, in each world, a single classical proposition embodying the *true exhaustive* answer to the question in that world. For instance, if  $w$  is a world in which Paul and Nina are coming for dinner, and nobody else is coming, then the denotation of (1) in  $w$  is the classical proposition expressed by (1b).

The *meaning* of a question, then, is a function from worlds to classical propositions. These classical propositions have two special properties: they are *mutually exclusive* (since two different exhaustive answers are always incompatible), and together they form a *cover* of the entire logical space (since every world is

compatible with at least one exhaustive answer). So the meaning of a question can be identified with a set of classical propositions which form a *partition* of logical space.

Now, partitions can be seen as a specific kind of propositions (in our sense). That is, for every partition  $\mathcal{P}$ , there is a corresponding proposition  $A_{\mathcal{P}}$ , consisting of all states that are contained in one of the blocks in  $\mathcal{P}$ :

$$A_{\mathcal{P}} := \{s \subseteq b \mid b \in \mathcal{P}\}$$

However, not every proposition corresponds to a partition. In fact, a proposition  $A$  corresponds to a partition if and only if (i)  $A$  is closed under union of consistent sets of states, i.e., for every set of states  $A' \subseteq A$  such that  $\bigcap A' \neq \emptyset$ , the state  $\bigcup A'$  is also in  $A$ , and (ii) together, the states in  $A$  cover the entire logical space. There are many propositions that do not have these two special properties.

Thus, the notion of meaning developed here is more general than the notion of question meanings as partitions. And this generalization is useful when considering certain linguistic constructions. In particular, if we restrict ourselves to partitions it is difficult, if not impossible, to deal satisfactorily with conditional questions, disjunctive questions, and mention-some constituent questions, exemplified below.

- (2) If Ann is coming, will Ben come as well?
- (3) Is Ann $\uparrow$  coming, or Bill $\uparrow$ ? ( $\uparrow$  indicates rising intonation)
- (4) Where can I buy an Italian newspaper?

With a more general notion of meaning like the one developed here, such constructions are much easier to deal with (see, e.g., Groenendijk and Roelofsen, 2009; Roelofsen and van Gool, 2010; Pruitt and Roelofsen, 2011).

**Questions in dynamic semantics.** Our proposal is most closely related to theories that aim to capture the semantics of both question and assertions in a *dynamic* framework. The first such theories were developed by Jäger (1996), Hulstijn (1997), and Groenendijk (1999).<sup>2</sup> All these theories essentially reformulate the partition theory of questions in the format of an update semantics (Veltman, 1996). This means that they explicitly identify meanings with *context change potentials*, i.e., functions over discourse contexts, just as we did in the present paper. Moreover, rather than modeling a discourse context simply as a set of worlds—embodying the information established so far—these theories provide a more refined model of the discourse context, one that also embodies the issues that have been raised so far. More specifically, a discourse context is modeled as an *equivalence relation*  $R$  over a set of worlds  $D \subseteq \omega$ , which is called the *domain* of  $R$ . Such an equivalence relation can be taken to encode both information and issues. On the one hand, the domain  $D$  can be taken to consist precisely of those worlds that are compatible with the information established

<sup>2</sup>See the book *Questions in dynamic semantics* (Aloni et al., 2007) for several papers elaborating on these early proposals.

so far. And on the other hand,  $R$  can be taken to relate two distinct worlds  $w$  and  $v$  just in case the difference between  $w$  and  $v$  is not (yet) at-issue, i.e., the discourse participants have not yet expressed an interest in information that would distinguish between  $w$  and  $v$ . In other words,  $R$  can be conceived of as a relation encoding *indifference* (Hulstijn, 1997).

Both assertions and questions can then be taken to have the potential to change the context in which they are uttered. Assertions restrict the domain  $D$  to those worlds in which the asserted sentence is true (strictly speaking, they remove all pairs of worlds  $\langle w, v \rangle$  from  $R$  such that the asserted sentence is false in at least one of the two worlds). Questions *disconnect* words, i.e., they remove a pair  $\langle w, v \rangle$  from  $R$  just in case the true exhaustive answer to the question in  $w$  differs from the true exhaustive answer to the question in  $v$ .

Thus, the dynamic framework of Jäger (1996), Hulstijn (1997), and Groenendijk (1999) provides a notion of context and meaning that embodies both informative and inquisitive content in a uniform way. However, as discussed in detail by Mascarenhas (2009), several issues, both empirical and conceptual, remain open. Empirically, it is difficult in this framework, if not impossible, to deal with conditional questions, disjunctive questions, and mention-some constituent questions. Clearly, these problems are inherited from the classical partition theory of questions (see the discussion of examples (2)-(4) above).

Conceptually, if  $R$  is primarily thought of as a relation encoding *indifference*, then it is not clear why it should always be an *equivalence relation*. In particular, it is not clear why  $R$  should always be *transitive*. The discourse participants could very well be interested in information that distinguishes  $w$  from  $v$ , while they are not interested in information that distinguishes either  $w$  or  $v$  from a third world  $u$ . To model such a situation, we would need an indifference relation  $R$  such that  $\langle w, u \rangle \in R$  and  $\langle u, v \rangle \in R$  but  $\langle w, v \rangle \notin R$ . This is impossible if we require  $R$  to be transitive.

These concerns led Groenendijk (2009) and Mascarenhas (2009) to develop a system in which indifference relations are defined as reflexive and symmetric, but not necessarily transitive relations. Otherwise, the architecture of the system is the same as that of Jäger (1996), Hulstijn (1997), and Groenendijk (1999). This early version of inquisitive semantics is referred to as the *pair-semantics*, because it still defines contexts as sets of world-pairs, and propositions, although formally defined differently, can also be characterized as sets of world-pairs.

Groenendijk (2009) and Mascarenhas (2009) argue that the pair-semantics, besides addressing the conceptual issue concerning indifference relations, also overcomes the empirical issues concerning conditional questions, disjunctive questions, and mention-some constituent questions. However, whereas disjunctive questions with two disjuncts, like (3) above, can be dealt with satisfactorily in the pair-semantics, disjunctive questions with three or more disjuncts are still problematic, and the same holds for mention-some constituent questions. Unfortunately, we do not have the space here to show exactly why these problems arise, but a detailed explanation can be found in Ciardelli and Roelofsen (2011).

This observation has led to the development of  $\text{Inq}_B$ , whose notion of discourse contexts and meaning has been explained and motivated in detail in

the present paper. Every discourse situation that can be modeled in the pair-semantics can also be modeled in  $\text{Inq}_B$ , but not the other way around. In particular, discourse situations that arise from uttering a disjunctive question with more than two disjuncts, or a mention-some constituent question, can straightforwardly be modeled in  $\text{Inq}_B$ . Thus,  $\text{Inq}_B$  naturally fits within the tradition of dynamic semantic theories of informative and inquisitive discourse, but it is more general and empirically more adequate than its predecessors.

Finally, we would like to remark that the notion of meaning proposed here can be further refined in order to capture more aspects of issues than just their resolving answerhood conditions. Such refinements are explored in much ongoing work, see for instance Ciardelli *et al.* (2009, 2010); Roelofsen and van Gool (2010); Farkas and Roelofsen (2011).

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