Noname manuscript No.
(will be inserted by the editor)

Inquisitive dynamic epistemic logic

Ivano Ciardelli · Floris Roelofsen

December 23, 2012

Abstract Information exchange can be seen as a dynamic process of raising and resolving issues. The goal of this paper is to provide a logical framework to model and reason about this process. We develop an inquisitive dynamic epistemic logic, IDEL, which enriches the standard framework of dynamic epistemic logic, incorporating insights from recent work on inquisitive semantics. At a static level, IDEL does not only allow us to model the *information* available to a set of agents, like standard epistemic logic, but also the *issues* that the agents entertain. At a dynamic level, IDEL does not only allow us to model the effects of public announcements that provide new information, like standard DEL, but also the effects of public announcements that *raise new issues*. Thus, IDEL provides the fundamental tools needed to analyze information exchange as a dynamic process of raising and resolving issues.

Keywords Logics of questions · dynamic epistemic logic · inquisitive semantics

Contents

1 Introduction . . . 2
2 Dynamic epistemic logic . . . 2
  2.1 Epistemic logic . . . 3
  2.2 Dynamics . . . 5
3 Inquisitive dynamic epistemic logic . . . 6
  3.1 Inquisitive epistemic logic . . . 7
  3.2 Dynamics . . . 17
4 Related work . . . 19
  4.1 Epistemic logic with issues . . . 19
  4.2 Dynamics . . . 20
  4.3 Comparison . . . 21
5 Conclusion . . . 27

This paper has been submitted for publication. Comments of any kind are more than welcome. Acknowledgments will be added after the reviewing process.

1 Introduction

Much attention has been paid in recent years to the development of logical tools to describe and reason about information exchange through linguistic interaction. One prominent framework that has emerged from this work is the framework of *dynamic epistemic logic* (DEL) (van Ditmarsch et al., 2007; van Benthem, 2011, among many others). In its basic incarnation, this framework allows us to formally describe and reason about the information available to a number of agents, and how this information changes when one of the agents makes an assertion and thereby provides new information.

However, in modeling information exchange it is not only important to keep track of the information that is available to the agents involved, but also of the information that they would like to obtain, i.e., the *issues* that they entertain. Moreover, while assertions evidently play an important role in the process of exchanging information, an equally important role is played by *questions*. Information exchange is a collaborative process of raising and resolving issues. Agents raise issues by asking questions, and resolve these issues by making assertions.

This paper develops an *inquisitive dynamic epistemic logic*, IDEL, incorporating insights from recent work on inquisitive semantics (Groenendijk and Roelofsen, 2009; Ciardelli, 2009; Ciardelli et al., 2012b, among others). At a static level, IDEL does not only allow us to model the information available to a set of agents, like standard epistemic logic, but also the issues that the agents entertain. And similarly, at a dynamic level, IDEL does not only allow us to capture the effects of assertions, which provide new information, but also the effects of questions, which raise new issues.

We will compare our approach in some detail with the *dynamic epistemic logic with questions* (DELQ) recently developed by van Benthem and Ştefan Minică (2012). Although the two proposals are very much in the same spirit, we will argue that IDEL has some crucial advantages.

The paper is organized as follows. Section 2 provides an overview of standard dynamic epistemic logic. In section 3 we present our inquisitive dynamic epistemic logic, and in section 4 we compare our framework with DELQ. Finally, section 5 concludes with some directions for future work.

2 Dynamic epistemic logic

We start with an overview of the standard, most basic incarnation of dynamic epistemic logic. Our exposition will be quite detailed, which will allow us later on to draw very explicit parallels between the treatment of issues in IDEL and the treatment of information in DEL. As is usually done, we will present DEL in two stages. First, in section 2.1, we will discuss the essential elements of epistemic logic, which forms the static component of DEL. Then, in section 2.2, we will discuss the dynamic component of the system.

2.1 Epistemic logic

Epistemic logic is designed to formally describe and reason about a certain set of facts together with what certain agents know about these facts and about one another's knowledge. Epistemic situations are represented by models that are based on the notion of a possible world. Intuitively, a possible world represents a possible state of affairs. When a certain set of facts is all that is at stake, a state of affairs may well be identified with a valuation that tells us which facts are true and which are false. In this way we get worlds characterized uniquely by a propositional valuation, which are suitable models for classical propositional logic. However, if we do not just want to take the facts themselves into consideration, but also what certain agents know about the facts, and how such information is exchanged, then we need a richer characterization of states of affairs, reflecting all the additional features that are relevant for the purpose at hand. In the case of epistemic logic, what is relevant is the information available to each agent. Thus, in an epistemic model, a possible world w is laid out by specifying two things: (i) a valuation for the facts under discussion and (ii) an *information state* for each agent. Formally, an information state is modeled as a set of possible worlds, to be conceived of as those worlds that are compatible with the available information. We thus arrive at the following definition.

Definition 1 (Epistemic models) An epistemic model for a set $\mathcal{P}$ of atomic sentences and a set $\mathcal{A}$ of agents is a tuple $M = \langle \mathcal{W}, V, \sigma_{\mathcal{A}} \rangle$ where:

- $\mathcal{W}$ is a set, whose elements are called *possible worlds*.
- $V : \mathcal{W} \to \wp(\mathcal{P})$ is a *valuation map* that specifies for every world w which atomic sentences are true at w.
- $\sigma_{\mathcal{A}} = \{\sigma_a \mid a \in \mathcal{A}\}$ is a set of *epistemic maps* from $\mathcal{W}$ to $\wp(\mathcal{W})$, each of which assigns to any world w an information state $\sigma_a(w)$ in accordance with:

  Factivity : for any $w \in \mathcal{W}$, $w \in \sigma_a(w)$

  Introspection : for any $w, v \in \mathcal{W}$, if $v \in \sigma_a(w)$, then $\sigma_a(v) = \sigma_a(w)$

Together, the valuation map V and the epistemic maps in $\sigma_{\mathcal{A}}$ equip every world w in the model with a complete specification of the state of affairs that it represents, given that we take a state of affairs to be completely determined in this setting by what the facts are like and what the agents know.

The epistemic maps need to satisfy two conditions. The factivity condition requires that the information available to agents be truthful, so that the information state of an agent is indeed a knowledge state, and not merely a belief state. The introspection condition requires that agents know their knowledge state, so that if the information state of a in w differs from her state in v, then a can tell the worlds w and v apart.[1]

The epistemic maps $\sigma_a : \mathcal{W} \to \wp(\mathcal{W})$ can be equivalently regarded as binary relations $\sim_a \subseteq \mathcal{W} \times \mathcal{W}$, where for any w and v: $w \sim_a v$ iff $v \in \sigma_a(w)$. The factivity and introspection conditions on σ_a then translate to the requirement that $\sim_a$

[1] Either of these conditions may be dropped or weakened to model scenarios of false information or not fully introspective agents (see, for instance, Fagin et al., 1995). The system considered here is usually taken to be the most basic variant of epistemic logic. For this reason we take it as a point of departure here, but we do not expect to encounter particular difficulties in adapting our proposal to weaker variants.

be an equivalence relation. While the presentation of epistemic models that uses equivalence relations rather than functions is more common in the literature, the functional notation has an important advantage for our current purposes: it brings out more clearly how the maps σ_a are one of the ingredients that, together with the valuation V, characterize the state of affairs associated with each possible world. This suggests that, if we wanted to take into account more aspects of a state of affairs than just the information available to all the agents involved, we could add more maps to our models to describe these additional aspects. This is indeed the approach we will take in section 3.

The logical language used to talk about epistemic models is a propositional language enriched with modal operators K_a for each $a \in \mathcal{A}$. The interpretation of the modality K_a relies on the epistemic map σ_a. In a model M and at a world w, $K_a\varphi$ is true iff any world compatible with a's information at w is one where φ is true:

$$\langle M, w\rangle \models K_a\varphi \iff \text{for any } v \in \sigma_a(w),\ \langle M, v\rangle \models \varphi$$

In other words, $K_a\varphi$ is true at w if the truth of φ follows from the information available to a at w, that is, if a knows that φ at w.

The *proposition* expressed by a sentence φ in an epistemic model M, which we will denote as $|\varphi|_M$, is the set of worlds in M where φ is true. Notice that the modality K_a can be regarded as making a claim about the relation between two sets of worlds, the information state of a at the evaluation world and the proposition expressed by its argument, as the following reformulation of the clause for K_a shows:

$$\langle M, w\rangle \models K_a\varphi \iff \sigma_a(w) \subseteq |\varphi|_M$$

This perspective will help us understand how modalities generalize beyond usual modal logic in the richer semantic picture that we will introduce.

Besides the agents' individual knowledge, notions of group knowledge also play an important role in the analysis of information exchange. One notion that is of particular importance is that of *common knowledge*, i.e., the information that is publicly available to all the agents. One might think that treating this notion would require enriching our models with a map σ_* that specifies, for each world w, an information state $\sigma_*(w)$ embodying the information that is publicly available to all the agents in w. We could then expand our language with a corresponding modality K_*, interpreted as follows:

$$\langle M, w\rangle \models K_*\varphi \iff \text{for any } v \in \sigma_*(w),\ \langle M, v\rangle \models \varphi$$

However, common knowledge is very closely tied to the agents' individual knowledge: in fact, it is *determined* by it. A sentence φ is common knowledge if and only if every agent a knows that φ, and every agent a knows that every agent b knows that φ, and every agents a knows that every agent b knows that every agent c knows that φ, and so on ad infinitum. Thus, whether the sentence $K_*\varphi$ is true or false at a world should be completely determined by the following condition:

$$\langle M, w\rangle \models K_*\varphi \iff \langle M, w\rangle \models K_{a_1}K_{a_2}\dots K_{a_n}\varphi \text{ for any } a_1, \dots, a_n \in \mathcal{A},\ n \geq 0$$

One can show that, in order to guarantee this equivalence for any particular valuation V, the common knowledge map σ_* must be defined precisely as follows:

$$\sigma_*(w) = \{v \mid \text{there exist } u_0, \dots, u_{n+1} \in \mathcal{W} \text{ and } a_0, \dots, a_n \in \mathcal{A} \text{ such that } u_0 = w,\ u_{n+1} = v, \text{ and for } i \leq n,\ u_{i+1} \in \sigma_{a_i}(u_i)\}$$

This means that the common knowledge map σ_* is uniquely determined by the set of individual epistemic maps $\sigma_\mathcal{A}$, and need not be added to our models as an additional component.

2.2 Dynamics

Epistemic logic allows us to describe the information available to a group of agents in a particular state of affairs. Dynamic epistemic logic allows us to describe how such a state of affairs may *change* as a result of certain actions that the agents may perform. In the case of information exchange through linguistic interaction, the relevant actions are the *speech acts* that the agents may perform.

In the most basic version of DEL, only one kind of action is considered, namely, the act of *publicly announcing* a sentence φ.[2] Such an announcement is taken to have the effect of making φ common knowledge. That is, as a result of a public announcement of φ, all agents learn that φ, and they learn that everyone now knows that φ, and that everyone knows that everyone knows, and so on ad infinitum. Technically, this is achieved by letting a public announcement of φ have the effect of eliminating all worlds where φ is false from the model, and restricting the epistemic maps of the agents accordingly.[3] That is, a public announcement of φ transforms an epistemic model $M = \langle \mathcal{W}, V, \sigma_a \rangle$ into the model $M^\varphi = \langle \mathcal{W}^\varphi, V^\varphi, {\sigma_\mathcal{A}}^\varphi \rangle$, where:

– $\mathcal{W}^\varphi = \mathcal{W} \cap |\varphi|_M$
– $V^\varphi = V\restriction_{\mathcal{W}^\varphi}$
– ${\sigma_\mathcal{A}}^\varphi = \{{\sigma_a}^\varphi \mid a \in \mathcal{A}\}$, where for every $w \in \mathcal{W}^\varphi$: ${\sigma_a}^\varphi(w) = \sigma_a(w) \cap |\varphi|_M$

For any type of action A that one may want to consider, the language of epistemic logic could be enriched with a corresponding dynamic modality $[A]$ that talks about what will be the case in the model after A is performed. In the case of basic public announcement logic, a dynamic modality $[\varphi]$ is introduced that talks about what is the case after a public announcement of φ. Now suppose we want to evaluate the formula $[\varphi]\psi$ in a model M at a world w. If $\langle M, w\rangle \models \neg\varphi$, then the public announcement removes w from the model, and there is no fact of the matter as to what holds at w after the announcement. In this basic framework, such non-truthful announcements are treated as a case of inconsistency: if $\langle M, w\rangle \models \neg\varphi$, then one lets $\langle M, w\rangle \models [\varphi]\psi$ for all ψ. On the other hand, if $\langle M, w\rangle \models \varphi$, then w survives the public announcement of φ, and one lets $\langle M, w\rangle \models [\varphi]\psi$ hold if and only if $\langle M^\varphi, w\rangle \models \psi$ holds. Summing up:

$$\langle M, w\rangle \models [\varphi]\psi \iff w \notin |\varphi|_M \text{ or } \langle M^\varphi, w\rangle \models \psi$$

[2] Public announcement logic was first proposed by Plaza (1989) and was further developed by Gerbrandy and Groeneveld (1997), Baltag et al. (1998), and van Ditmarsch (2000), among others. Recent overviews of the system and its role in the general dynamic epistemic logic landscape are provided by van Ditmarsch et al. (2007) and van Benthem (2011).

[3] Actually, restricting the epistemic maps to φ-worlds would be all we need to model the intended change, which is a merely epistemic one. The only reason why the $\neg\varphi$-worlds also have to be eliminated from the model is that, if we did not eliminate them, the resulting model would no longer be an epistemic model in the sense of our definition, since the new epistemic maps would not validate the factivity requirement in the $\neg\varphi$-worlds.

The fact that a public announcement establishes common knowledge is witnessed by the logical validity of the formula $[\varphi]K_*\varphi$. At any world in any model, a public announcement of φ makes φ common knowledge among all the agents.

Of course, it is possible to consider many more actions than just public announcements. To mention just one important case, private announcements, directed only to a subset of the agents involved, have received much attention in the literature (see, for instance, Baltag et al., 1998). However, we will restrict our attention here to the most basic system, with public announcements only, in order to explicate our proposal in a more perspicuous way.

One may worry that the given treatment of public announcements is too strong, in that it does not give the addressees the option to reject the proposed informational update. This is clearly unrealistic if our goal is to model an actual conversation, where disagreement may occur. However, recall that we are working here under the assumption that an agent's information is always truthful. In such a setting, disagreement cannot occur, since any two agents always have compatible information states. Assuming a Gricean pragmatic rule that requires agents to only announce what they know, a situation in which one of the addressees has a reason to reject a public announcement can never arise. Of course, a more realistic picture in which knowledge is replaced by belief, and disagreement may occur, will be more interesting from a conversational perspective. We leave the investigation of the details of such a picture for future work.

3 Inquisitive dynamic epistemic logic

In a nutshell, the picture of information exchange assumed in DEL is that of a group of agents, each equipped with a certain body of information, sharing some of their individual knowledge with the other participants by making informative announcements. Something crucial is missing from this picture. When agents enter an information exchange, they are not just equipped with a certain body of information, but they also entertain certain issues that they would like to see resolved. In many cases, the desire to resolve these issues actually constitutes the motivation for the agents to engage in the exchange in the first place. Furthermore, the exchange itself does not merely consist in a sequence of informative announcements. Rather, it is an interactive process of raising and resolving issues. Agents ask questions to raise new issues, and they make assertions to resolve these issues.

In order to do justice to this more comprehensive picture of information exchange, we need to make room for issues in our logical framework. Just like agents are modeled as having certain information and are given the ability to share this information in the exchange by making informative announcements, they should also be modeled as entertaining certain issues, and they should be given the ability to raise these issues in the exchange by making inquisitive announcements, i.e., by asking questions.

3.1 Inquisitive epistemic logic

Our first task is to add an inquisitive dimension to epistemic logic. That is, we will develop a framework in which it is not only possible to model the information available to a set of agents, but also the issues that they entertain.

Semantic structures. While in epistemic logic a possible world w was laid out by specifying (i) a valuation for the atomic sentences in the language, and (ii) an information state for each agent, we now also need to specify (iii) an *inquisitive state* for each agent, encoding the issues that the agent entertains in w. But what kind of formal object should these inquisitive states be? In other words, what is a good mathematical representation of an issue? We will adopt the formal notion of issues that has been developed in recent work on *inquisitive semantics*.[4] The fundamental idea is to lay out an issue by specifying what it takes for the issue to be resolved. That is, an issue is identified with a set of information states: those information states that contain enough information to resolve the issue.

We assume that every issue can be resolved in at least one way, which means that issues should be identified with *non-empty* sets of information states. Moreover, a set of information states can only suitably embody an issue if it is *downward closed*. That is, if t is a state in an issue I, then any $u \subseteq t$ should be in I as well. After all, if $t \in I$, then t contains enough information to resolve I; but then any $u \subseteq t$ clearly also contains enough information to resolve I, and should therefore be included in I as well. Thus, issues are defined as non-empty, downward closed sets of information states.

Definition 2 (Issues)
An *issue* is a non-empty, downward closed set of information states.

It is only possible to truthfully resolve an issue I if the actual world is contained in at least one state $t \in I$, i.e., if the actual world is contained in $\bigcup I$. Therefore, we say that an issue I *assumes* the information that the actual world is located in $\bigcup I$. Moreover, if s is a state, then we say that I is an issue *over* s just in case $\bigcup I = s$. The set of all issues over a state s is denoted by Π_s. Finally, we say that an information state t *settles* an issue I just in case $t \in I$.

Figure 1 depicts some issues over the state $s = \{w_1, w_2, w_3, w_4\}$. In order to keep the figures neat, we have depicted only the maximal elements of these issues. The issue in (a) can only be settled by specifying precisely which world in s is the actual one. The issue in (b) can be settled either by locating the actual world in $\{w_1, w_2\}$, or by locating it in $\{w_3, w_4\}$. The issue in (c) can be settled either by locating the actual world in $\{w_1, w_3, w_4\}$, or by locating it in $\{w_2, w_3, w_4\}$. Finally, the issue in (d) is the *trivial* issue over s, which is already settled by s itself.

This notion of issues is precisely what we need to give epistemic logic an inquisitive dimension. Recall that in epistemic logic, every agent a is assigned an

[4] A detailed exposition of inquisitive semantics, in particular the notion of issues that we will adopt here, can be found in Ciardelli et al. (2012b). Earlier expositions of the framework can be found in Groenendijk (2009); Mascarenhas (2009); Ciardelli (2009); Groenendijk and Roelofsen (2009); Ciardelli and Roelofsen (2011), and Roelofsen (2011a). However, these do not explicitly define and motivate the notion of issues that will play a crucial role here.

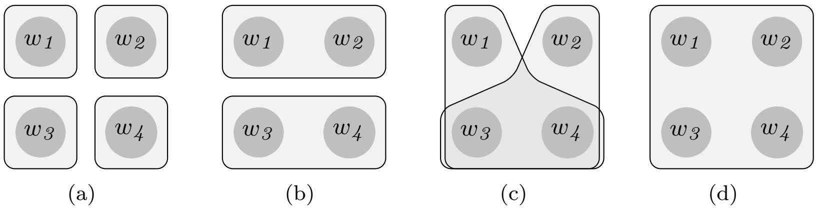

Fig. 1 Issues over the state $\{w1, w2, w3, w4\}$.

information state $\sigma_a(w)$ in every world w, determining the range of worlds that she considers possible candidates for the actual one. Now, every agent will also be assigned an inquisitive state $\Sigma_a(w)$, which will be modeled as an issue over $\sigma_a(w)$, reflecting the agent's desire to locate the actual world more precisely inside her information state.

Since $\Sigma_a(w)$ will be modeled as an issue over $\sigma_a(w)$, we will always have that $\sigma_a(w) = \bigcup \Sigma_a(w)$. This means that from the inquisitive state $\Sigma_a(w)$ of an agent a in a world w, we can always derive the information state $\sigma_a(w)$ of that agent in that world, simply by taking the union of $\Sigma_a(w)$. Thus, in effect, $\Sigma_a(w)$ encodes both the information available to a and the issues entertained by a at w. This means that the map Σ_a suffices as a specification of the state of the agent a at each world, encompassing both information and issues. We do not have to list σ_a explicitly as an independent component in the definition of an inquisitive epistemic model: we can simply use $\sigma_a(w)$ as an abbreviation for $\bigcup \Sigma_a(w)$, keeping in mind that this set of worlds represents the information state of agent a in w. We thus arrive at the following definition.

Definition 3 (Inquisitive epistemic models)
An inquisitive epistemic model is a tuple $M = \langle \mathcal{W}, V, \Sigma_{\mathcal{A}} \rangle$ where:

- $\mathcal{W}$ is a set, whose elements will be called possible worlds.
- $V : \mathcal{W} \to \wp(\mathcal{P})$ is a *valuation map* that specifies for every world w which atomic sentences are true at w.
- $\Sigma_{\mathcal{A}} = \{\Sigma_a \mid a \in \mathcal{A}\}$ is a set of *state maps* from $\mathcal{W}$ to $\Pi_{\mathcal{W}}$, each of which assigns to any world w an issue $\Sigma_a(w)$, in accordance with:

  Factivity : for any $w \in \mathcal{W}$, $w \in \sigma_a(w)$
  Introspection : for any $w, v \in \mathcal{W}$, if $v \in \sigma_a(w)$, then $\Sigma_a(v) = \Sigma_a(w)$

The factivity condition is just as before, ensuring that the agents' information states are truthful. The introspection condition now concerns both information and issues: agents must be introspective in that they must know not only what information they have, but also what issues they entertain. That is, if the state of a in world w differs from the state of a in v, either in information or in issues, then a must be able to tell w and v apart.

As the reader will have noticed, there is a striking similarity between inquisitive epistemic models and standard epistemic models. In both frameworks, a model consists simply of a set of worlds, each equipped with (i) a valuation for atomic sentences and (ii) a state for each agent. The only difference is that while in standard epistemic logic the agents' states describe just their information, in the present setting they encompass both their information and their issues.

Logical language. So far we have introduced and motivated a notion of inquisitive epistemic models, to serve as semantic structures for our framework. The next step is to define a logical language that will enable us to talk about such models. As issues play a prominent role in our semantic picture, we want to endow our language with sentences whose meaning is *inquisitive*, i.e., can be identified not with a piece of information, but with an issue. We will do this by extending the usual declarative language of epistemic logic with sentences of a new syntactic category, the category of *interrogatives*. As we will see, the semantic labor will be rigidly divided between these two categories: declaratives will be *informative*, while interrogatives will be *inquisitive* (if not tautological).[5]

As will be immediately clear from the syntax of our language, interrogatives will not only play a role in their own right, but they will also play a role as components of larger sentences, as they may be embedded under various modal operators. Therefore, due to the presence of interrogatives, the declarative fragment of our language will also be richer than usual. The set $\mathcal{L}_!$ of declaratives and the set $\mathcal{L}_?$ of interrogatives are laid out by simultaneous recursion via the following definition.

Definition 4 (Syntax)
Let $\mathcal{P}$ be a set of atomic sentences and let $\mathcal{A}$ be a set of agents.

1. For any $p \in \mathcal{P}$, $p \in \mathcal{L}_!$
2. $\bot \in \mathcal{L}_!$
3. If $\alpha \in \mathcal{L}_!$, then $?\alpha \in \mathcal{L}_?$
4. If $\varphi \in \mathcal{L}_\circ$ and $\psi \in \mathcal{L}_\circ$, then $\varphi \wedge \psi \in \mathcal{L}_\circ$, where $\circ \in \{!, ?\}$
5. If $\alpha \in \mathcal{L}_!$ and $\varphi \in \mathcal{L}_\circ$, then $\alpha \to \varphi \in \mathcal{L}_\circ$, where $\circ \in \{!, ?\}$
6. If $\varphi \in \mathcal{L}_\circ$ for $\circ \in \{!, ?\}$ and $a \in \mathcal{A}$, then $K_a\varphi \in \mathcal{L}_!$
7. If $\varphi \in \mathcal{L}_\circ$ for $\circ \in \{!, ?\}$ and $a \in \mathcal{A}$, then $E_a\varphi \in \mathcal{L}_!$
8. Nothing else belongs to either $\mathcal{L}_!$ or $\mathcal{L}_?$

We start out by classifying atomic sentences and the falsum as declarative sentences. The third clause makes it possible to construct an interrogative $?\alpha$ from a declarative α, using the polar interrogative operator ?. Notice that the declarative α does not have to be atomic; in particular, it may in turn contain an interrogative as a sub-constituent. The fourth clause allows us to conjoin two sentences of the same category to obtain a sentence of the same category.[6] The fifth clause states that a sentence of either category may be conditionalized by a declarative antecedent, resulting in a conditional sentence of the same category. By means of implication and falsum we define negation for declaratives, letting $\neg\alpha := \alpha \to \bot$.

[5] In inquisitive semantics it is actually common practice to assume a language that does not make a categorical distinction between declaratives and interrogatives (see, e.g., Groenendijk and Roelofsen, 2009; Ciardelli and Roelofsen, 2011; Ciardelli et al., 2012b). However, the semantics can equally naturally be applied to a bi-categorical language (Groenendijk, 2011). The reason that we choose to adopt a bi-categorial language here is twofold. First, it seems that the intuitions are somewhat easier to get across this way. And second, assuming a distinction between declaratives and interrogatives makes it easier to compare our proposal to others, in particular that of van Benthem and Ştefan Minică (2012), which will be done in section 4.

[6] Since our system assumes a strict partition of sentences into declaratives and interrogatives, hybrid conjunctions like $p \wedge ?q$ are not included in our logical language. Such conjunctions do in fact occur quite widely in natural language, both as standalone sentences and embedded under modal operators (e.g., *Ann is coming, but is Bill coming as well?*, *I know that Ann is coming and whether Bill is coming as well*), and can be handled straightforwardly in the standard hybrid system of inquisitive semantics (see the references in footnote 5).

Other propositional connectives joining declaratives, such as disjunction, may be defined by means of the usual abbreviations.

The last two clauses introduce the two modalities that we will consider. Since agents are equipped with both information and issues, we will consider a *knowledge* modality K_a, which will allow us to talk about the agents' information, and an *entertain* modality E_a, which will allow us to talk about the issues that the agents entertain. Notice that both modalities are allowed to embed sentences of either category. Our knowledge modality will coincide with its standard counterpart when its complement is a declarative. However, it is more flexible, since its complement may also be interrogative. This enables us to construct sentences like $K_a?p$, expressing the fact that *a knows whether p*. The entertain modality E_a on the other hand, is specifically designed to talk about issues, and as such does not have a counterpart in standard epistemic logic.

Finally, let us point out once more how the definitions of declaratives and interrogatives are intertwined. The polar interrogative operator forms basic interrogatives out of declaratives, from which more complex interrogatives may then be constructed. On the other hand, the modalities form declaratives out of sentences of either kind, including interrogatives. This allows us to construct sentences such as $K_a?K_b\,?p$, expressing complex facts like *a knows whether b knows whether p.*

Semantics. Our next task is to provide an interpretation for the sentences of our logical language. This is a crucial step in our enterprise, since this is where we need to go beyond the usual techniques of epistemic logic, relying fundamentally on insights from inquisitive semantics.

In epistemic logic, like in any other modal logic, sentences are interpreted relative to a world in a model. The semantics recursively specifies the conditions under which a sentence is true at a given world. This is a suitable approach as long as we consider only declarative sentences. But our language also contains interrogatives, and it is not clear at an intuitive level what it would mean to ask whether a given interrogative, or the issue that it expresses, is true in a certain world. Rather, what we naturally ask of an issue is whether it is resolved in a certain information state. Thus, the natural evaluation points for interrogatives are information states, rather than worlds, and the meaning of an interrogative should be taken to consist in its *resolution conditions* rather than its truth conditions.

At first sight, it may seem that this forces us to develop a double-face semantics in which declaratives are evaluated at worlds and interrogatives at states. However, there is a solution which is both conceptually more elegant and formally much more efficient. Namely, as is commonly done in inquisitive semantics, we will lift the interpretation of declaratives from the level of worlds to the level of information states as well, in a way that will allow us to promptly recover truth conditions if needed. This will enable us to provide a uniform semantic treatment of all sentences in our language. The following definition specifies recursively when a sentence is *supported* by a state s. Intuitively, for declaratives being supported amounts to being *known*, or *true everywhere* in s, while for interrogatives it amounts to being *resolved* in s.

Definition 5 (Semantics)
Let M be an inquisitive epistemic model and let s be an information state in M.

In the following, α denotes an arbitrary declarative, μ an arbitrary interrogative, and φ and ψ arbitrary sentences of either category.

1. $\langle M, s\rangle \models p \iff p \in V(w)$ for all worlds $w \in s$
2. $\langle M, s\rangle \models \bot \iff s = \emptyset$
3. $\langle M, s\rangle \models ?\alpha \iff \langle M, s\rangle \models \alpha$ or $\langle M, s\rangle \models \neg\alpha$
4. $\langle M, s\rangle \models \varphi \wedge \psi \iff \langle M, s\rangle \models \varphi$ and $\langle M, s\rangle \models \psi$
5. $\langle M, s\rangle \models \alpha \to \varphi \iff$ for any $t \subseteq s$, if $\langle M, t\rangle \models \alpha$ then $\langle M, t\rangle \models \varphi$
6. $\langle M, s\rangle \models K_a\varphi \iff$ for any $w \in s$, $\langle M, \sigma_a(w)\rangle \models \varphi$
7. $\langle M, s\rangle \models E_a\varphi \iff$ for any $w \in s$ and for any $t \in \Sigma_a(w)$, $\langle M, t\rangle \models \varphi$

Interpreting support as suggested above, the reader should be able to get an intuitive understanding of the clauses quite straightforwardly, except perhaps for the clauses for implication and the modalities, which will be discussed in more detail below. However, before turning to that, we will take a moment to show how our state based semantics allows us to recover a truth conditional semantics for declaratives. Let us first mention an important monotonicity property of our semantics: support is preserved as information grows.

Fact 1 (Persistency of support)
If $\langle M, s\rangle \models \varphi$ *and* $t \subseteq s$*, then* $\langle M, t\rangle \models \varphi$

The empty state always supports any sentence. We may thus think of the empty state as the *absurd* state.

Fact 2 (The empty state supports everything)
For any M *and any* φ*,* $\langle M, \emptyset\rangle \models \varphi$

Now let us turn to truth. We mentioned that, for a declarative sentence, $\langle M, s\rangle \models \alpha$ may be read as "α is true in any world in s". This intuition suggests the following definition of truth at a world.

Definition 6 (Truth)
We say that a sentence φ is *true* at a world w in a model M, and write $\langle M, w\rangle \models \varphi$, if and only if φ is supported by the state $\{w\}$ in M. In short:

$$\langle M, w\rangle \models \varphi \iff \langle M, \{w\}\rangle \models \varphi$$

One can easily check that restricting the support clauses for the connectives to singleton states we indeed fall back into the usual truth-conditional clauses. Notice however that we let truth be defined for *all* sentences, including interrogatives. What does this mean? When is an interrogative true at a particular world? The answer is, *always.* Thus, truth is trivial for interrogatives. While not very exciting, this will prove handy to state definitions in a uniform way. In terms of truth we define the *truth-set* of a sentence in a model.

Definition 7 (Truth set)
The *truth set* of a sentence φ in a model M, denoted $|\varphi|_M$, is defined as the set of worlds in M where φ is true:

$$|\varphi|_M := \{w \in \mathcal{W} \mid \langle M, w\rangle \models \varphi\}$$

Now that we have a precise notion of truth, our intuitive characterization of support for declaratives becomes a formal claim that can be stated and proved: a declarative is supported by a state just in case it is true at all worlds in that state.

Fact 3 (Truth and support)
For any model M, any state s and any declarative α, the following holds:

$$\langle M, s\rangle \models \alpha \iff \langle M, w\rangle \models \alpha \text{ for all } w \in s$$

This ensures that the meaning of a declarative is still completely determined by its truth conditions w.r.t. single worlds. Given this fact, one may wonder what the benefit is of defining truth in terms of support and not vice versa. To see this, notice that one cannot simply *first* define truth and *then* support. For, given clauses 6 and 7, the truth of some declaratives depends on the support conditions for some interrogatives, which in turn depend on the truth conditions for more basic declaratives. Thus, the alternative would be a simultaneous recursive definition of truth for declaratives and support for interrogatives. This would be a rather cumbersome endeavor, especially when one realizes that it would require two separate clauses for conjunction, two clauses for implication, and two clauses for each modal operator. Our definition assigns a uniform type of meaning to all sentences. On the practical level, this simplifies many definitions that should otherwise go by cases. On the conceptual level, it concurs with the main tenet of inquisitive semantics, which is that informative and inquisitive content should be brought under the umbrella of one unified notion of meaning. Notice that the logical operators that apply to both declaratives and interrogatives—conjunction, implication, and the modals—do so *uniformly*, that is, the same semantic clause takes care of both cases. We believe this points to an interesting semantic uniformity of these operations.[7]

In standard epistemic logic, a sentence is evaluated relative to possible worlds. Accordingly, the proposition that it expresses is a set of worlds, namely, the set of all worlds where the sentence is true. In our framework, instead, a sentence is evaluated relative to states. Thus, the proposition that a sentence expresses is a set of states, namely, the set of all states that support the sentence.

Definition 8 (Propositions)
The proposition $[\varphi]_M$ expressed by a sentence φ in a model M is the set of all states in M that support φ:

$$[\varphi]_M := \{s \subseteq \mathcal{W} \mid s \models \varphi\}$$

By Fact 1 and 2, the proposition expressed by a sentence is always a non-empty, downward closed set of states, i.e., an *issue* in the sense of definition 2.

The truth-set of a sentence always coincides with the union of the proposition that it expresses.

Fact 4 (Propositions and truth-sets)
For any sentence φ and any model M, $|\varphi|_M = \bigcup[\varphi]_M$.

[7] In the case of the connectives, this uniformity is brought out in even fuller generality in the standard hybrid system of inquisitive semantics (see the references in footnote 5).

As in the classical setting, we think of a sentence as providing the information that the actual world is one where the sentence is true. We say that φ is *informative* just in case it has the potential to provide non-trivial information, i.e., just in case there is a model M such that $|\varphi|_M \neq \mathcal{W}_M$. Similarly, we think of a sentence as requesting enough information to reach a state that supports the sentence. We say that a sentence φ is *inquisitive* just in case it has the potential to make a non-trivial request for information, i.e., just in case there is a model M such that $|\varphi|_M \notin [\varphi]_M$. This means that the information that φ itself provides in M is not sufficient to support φ in M. Thus, in this case, further information is required in order to establish a state that supports φ.

Definition 9 (Informativeness and inquisitiveness)

- φ is informative just in case there is a model M such that $|\varphi|_M \neq W_M$
- φ is inquisitive just in case there is a model M such that $|\varphi|_M \notin [\varphi]_M$

As we anticipated earlier, there is a clearcut division of labor between the two types of sentences in our language: declaratives may be informative but are never inquisitive, while interrogatives may be inquisitive but are never informative.

Fact 5 (Division of labor)

- *Declaratives are never inquisitive;*
- *Interrogatives are never informative.*

Equipped with these basic insights about the semantics, let us now turn back to the clauses for implication and the modalities. First, let us point out that the clause for implication, clause 5, is equivalent to the more perspicuous clause 5′ below, which reduces the assessment of a conditional in a state to the assessment of the consequent in a state enhanced with the information provided by the antecedent.

5′. $\langle M, s\rangle \models \alpha \to \varphi \iff \langle M, s \cap |\alpha|_M\rangle \models \varphi$

In words: a conditional $\alpha \to \varphi$ is supported by a state s if and only if the consequent φ is supported by the state resulting from enhancing s with the information provided by the antecedent α. Thus, a conditional declarative $\alpha \to \beta$ is true everywhere in s if and only if β is true in all of the worlds in s where α is true. A conditional interrogative $\alpha \to \mu$, on the other hand, is resolved in s if and only if, by adding the information provided by α to s, we get to a state where μ is resolved.

For concrete illustration, consider the following conditional question:

(1) If Ann goes to the party, will Bill go as well? $p \to ?q$

A state s in a model M supports $p \to ?q$ just in case $s \cap |p|_M$ supports $?q$. This is the case if $s \cap |p|_M$ supports q or if it supports $\neg q$. In the first case s has to support $p \to q$, and in the second case s has to support $p \to \neg q$. Thus, a state supports $p \to ?q$ just in case it supports either $p \to q$ or $p \to \neg q$. The latter two sentences correspond exactly to the two basic answers to our conditional question:[8]

[8] Notice that two states supporting $p \to q$ and $p \to \neg q$, respectively, may *overlap* (they may both contain worlds where p is false). For this reason, conditional questions have always been notoriously problematic for theories of questions that model issues as *partitions* of logical space (e.g. Groenendijk and Stokhof, 1984). This problem no longer arises in inquisitive semantics since its notion of issues is more general than the partition notion. Conditional questions have played an important motivational role in the development of inquisitive semantics (see, e.g., Mascarenhas, 2009; Groenendijk, 2011; Ciardelli et al., 2012a). We will return to this point when comparing our proposal with that of van Benthem and Ştefan Minică (2012) in section 4.

(2) a. Yes, if Ann goes, Bill will go as well. $p \to q$
b. No, if Ann goes, Bill will not go. $p \to \neg q$

Let us now take a closer look at the semantics of the modal operators. First consider the knowledge operator K_a. Since $K_a\varphi$ is always a declarative, fact 3 ensures that in order to understand its meaning we just need to look at its truth conditions at worlds. Now, clause 6 says that $K_a\varphi$ is true at a world w just in case φ is supported in the state $\sigma_a(w)$, which encodes the information available to a in w. Given fact 3, for a declarative α this means that $K_a\alpha$ is true at w iff α is true everywhere in $\sigma_a(w)$, i.e. true in every world compatible with the information of a in w. So, when applied to declaratives, K_a boils down to the familiar knowledge modality of epistemic logic. On the other hand, for an interrogative μ, the clause says that $K_a\mu$ is true in w just in case μ is *resolved* in $\sigma_a(w)$, which means that $K_a\mu$ expresses the fact that a has sufficient information to resolve μ at w. For instance, $K_a?p$ is true at a world w just in case $\sigma_a(w)$ supports either p or $\neg p$, that is, just in case a knows *whether* p holds. Notice that this treatment of interrogatives embedded under the knowledge operator is in no way restricted to polar interrogatives: it applies to more complex interrogatives as well, and it would extend straightforwardly to the first-order setting, allowing us to deal with embedded *wh*-questions.

Now let us consider the entertain operator E_a. Again, since the sentence $E_a\varphi$ is always a declarative, we just need to consider truth conditions. According to clause 7, $E_a\varphi$ is true at w just in case φ is supported by any state $t \in \Sigma_a(w)$. Remember that $\Sigma_a(w)$ encodes the inquisitive state of a at w, and that the elements of $\Sigma_a(w)$ are precisely the enhancements of the information state of a at w where the issues of a are resolved. Thus, $E_a\varphi$ says that as soon as the private issues of a are resolved, φ is supported. If φ is a declarative α, one can prove that this happens just in case α is supported by $\sigma_a(w)$, so $E_a\alpha$ is simply equivalent to $K_a\alpha$. However, if φ is an interrogative μ, then $E_a\mu$ is true just in case resolving the private issues of a entails resolving μ, or, speaking more informally, just in case all the states in which the curiosity of a is satisfied are such that μ is resolved. This is close to saying that a *wonders* about μ, except for one case: if a already has enough information to resolve μ, i.e., if $K_a\mu$ is the case, then obviously any state in which her issues are resolved, being an enhancement of her current state, will also contain enough information to resolve μ, so $E_a\mu$ would be true. But in such a scenario, we would not say that a *wonders* about μ.[9] We can characterize the situation of an agent a wondering about μ as one where the agent does not yet have sufficient information to resolve μ (so that $\neg K_a\mu$ holds) but the states she wants to get to are states that do contain such information (so that $E_a\mu$ holds). In short, a wonders about μ if she does not know about μ but she wants to know about μ. So, we can introduce a defined *wonder* modal operator W_a as follows:

$$W_a\varphi := \neg K_a\varphi \wedge E_a\varphi$$

By means of this operator, we can construct sentences like $W_a(?K_bW_a?p)$, expressing such subtle facts as *a wonders whether b knows that she wonders whether p*.

Notice that if the W_a modality is applied to a declarative, given that $K_a\alpha$ and $E_a\alpha$ are equivalent in this case, it immediately results in a contradiction, in tune with the intuition that one just cannot wonder *that p*.

[9] Of course, we would also not naturally say that *a entertains* μ in that case: although we read $E_a\varphi$ as "a entertains φ", this should be understood as technical terminology.

This concludes our illustration of the clauses for implication and the modalities. Hopefully, the reader has a reasonably clear picture by now of the meanings that the sentences in our system express. We now take a step back to make a few comments on the mathematical workings of the system. We start by noticing that, in the strict sense of modal logic, our 'modal operators' K_a and E_a are in fact not modalities *at all*. That is, they cannot be regarded as quantifiers asserting the truth of their argument at certain worlds—their argument is not even necessarily the sort of thing capable of having a truth value at worlds. However, there is a sense in which these operators work in our system precisely in the way modalities do in standard modal logic.

Recall that in epistemic logic, as we remarked in section 2, the modality K_a can be taken to express a relation between the state $\sigma_a(w)$ and the proposition $|\varphi|_M$ expressed by its argument. In the particular case of K_a, the relation simply amounts to inclusion:

$$\langle M, w\rangle \models K_a\varphi \iff \sigma_a(w) \subseteq |\varphi|_M$$

All modal operators of standard modal logic can be seen as working in this way: they express a relation between two sets of worlds, a set of worlds associated with the world of evaluation, and the proposition expressed by the sentence that the operator takes as its argument.

Our modal operators K_a and E_a work in exactly the same way: they express a relation between the state $\Sigma_a(w)$ and the proposition $[\varphi]_M$. The only difference is that we take these two semantic objects to be of a different type than in standard modal logic. We argued that, in order to capture the issues that agents may entertain and the inquisitive content that sentences may have, both the states assigned to agents and the propositions expressed by sentences should be non-empty downward closed sets of information states. Now, since such entities are more structured than simple sets of worlds, several relations may turn out to carry an intuitive significance. The modalities K_a and E_a express two of these relations, as is brought out more clearly by the following equivalent reformulations of clauses 6 and 7 above.[10]

6′. $\langle M, w\rangle \models K_a\varphi \iff \bigcup \Sigma_a(w) \in [\varphi]_M$
7′. $\langle M, w\rangle \models E_a\varphi \iff \Sigma_a(w) \subseteq [\varphi]_M$

Common knowledge and public issues. Besides the information and issues that are private to each agent, agents also share certain public information and jointly entertain certain issues. In section 2 we saw how the common knowledge construction in epistemic logic allows us to derive a public information map σ_* representing the information that is publicly available to all the agents, starting from the epistemic maps σ_a encoding the information available to the individual agents. The question is whether this construction can be generalized to the present setting. That is, is it possible to derive a public state map Σ_*, encoding public information and issues, from the maps Σ_a describing the information and issues of the individual agents?

One way to go about answering this question is to consider, as we did in the case of common knowledge, the conditions that a public entertain modality E_*

[10] We give *truth* conditions here, rather than *support* conditions, to bring out the analogy with standard modal logic more clearly. Since $K_a\varphi$ and $E_a\varphi$ are declaratives, fact 3 says that they are supported by a state s just in case they are true at every world in s.

associated with the map Σ_* would have to satisfy. Indirectly, this will then put constraints on the definition of Σ_*. So, let us consider what it would mean for a sentence to be *publicly entertained*. In standard epistemic logic, φ is publicly known in case every agent knows that φ, and every agent knows that every agent knows that φ, and so on ad infinitum. Analogously, it seems natural to say that φ is *publicly entertained* in case every agent entertains φ, and every agent knows that every agent entertains φ, and every agent knows that every agent knows, etcetera. Thus, the behavior of the public entertain modality E_* would have to be subject to the following condition:

$$\langle M,s\rangle \models E_*\varphi \iff \langle M,s\rangle \models K_{a_1}\ldots K_{a_{n-1}}E_{a_n}\varphi \text{ for all } a_1\ldots a_n\in\mathcal{A},\ n\geq 0$$

If one finds the alternation of the modalities puzzling, there is no need to worry: since K_a and E_a are equivalent with declarative arguments, and since any sentence that starts with a modality is a declarative, we can simply replace all the K_a's with E_a and obtain the equivalent "homogeneous" condition:

$$\langle M,s\rangle \models E_*\varphi \iff \langle M,s\rangle \models E_{a_1}\ldots E_{a_n}\varphi \text{ for all } a_1\ldots a_n\in\mathcal{A},\ n\geq 0$$

Does this condition put precisely enough constraints on the map Σ_* to define it uniquely? The answer is *yes*. One can verify that the above conditions on E_* will hold for any particular valuation V if and only if the map Σ_* is defined as follows:

$$\begin{aligned}\Sigma_*(w) = \{s \mid\ & \text{there exist } v_0,\ldots,v_n\in\mathcal{W} \text{ and } a_0,\ldots,a_n\in\mathcal{A}\\ & \text{such that } v_0=w,\ v_{i+1}\in\sigma_{a_i}(v_i) \text{ for all } i<n, \text{ and } s\in\Sigma_{a_n}(v_n)\}\end{aligned}$$

Importantly, the public information map σ_* corresponding to the public state map Σ_*, defined as $\sigma_*(w) := \bigcup\Sigma_*(w)$, coincides exactly with the map we would obtain by performing the common knowledge construction on the individual information maps σ_a. Thus, the standard common knowledge construction from epistemic logic generalizes smoothly and elegantly to a 'public state' construction which encompasses both information and issues.

Given this construction, we can add modalities K_* and E_* to our logical language, and interpret them as follows:

8. $\langle M,s\rangle \models K_*\varphi \iff$ for any $w\in s$, $\sigma_*(w)\in[\varphi]_M$
9. $\langle M,s\rangle \models E_*\varphi \iff$ for any $w\in s$, $\Sigma_*(w)\subseteq[\varphi]_M$

If μ is an interrogative, then $K_*\mu$ says that enough information is publicly available to resolve μ; in other words, μ is *publicly settled* in the exchange. In terms of K_* and E_*, a public wonder modality can be defined as well. A group of agents $\mathcal{A}$ jointly wonders about φ if they publicly entertain φ and φ is not yet publicly settled.

$$W_*\varphi := E_*\varphi \wedge \neg K_*\varphi$$

Like the modality K_*, the modality W_* plays a crucial role in describing the state of an information exchange: while K_* talks about what is settled in the exchange, W_* talks about what the group as a whole is wondering about, that is, what the *open issues* are in the exchange.

Interestingly, $W_*\mu$ itself does not entail $W_a\mu$ for any particular agent a: if $W_*\mu$ holds, then φ is publicly entertained but not publicly settled, that is, the common knowledge of the group does not settle μ. But it may well be that this is the case while a's *private* knowledge does settle μ. This does not prevent μ from being an open issue in the conversation, so long as a's private information is not made publicly available. In fact, $W_*\mu$ might even be the case while *every* individual agent can resolve μ, but this fact is just not common knowledge.

Finally, notice that by means of the common knowledge map, the public state map admits of a rather straightforward characterization:

$$\Sigma_*(w) = \bigcup_{v \in \sigma_*(w), a \in \mathcal{A}} \Sigma_a(v)$$

This entails the following connection between the public entertain modality and common knowledge: φ is publicly entertained just in case every agent entertains φ and this fact is common knowledge. That is, if $\mathcal{A} = \{a_1, \dots, a_n\}$ then:

$$\langle M, w \rangle \models E_*\varphi \iff \langle M, w \rangle \models K_*(E_1\varphi \wedge \dots \wedge E_n\varphi)$$

3.2 Dynamics

So far we have designed new models to represent situations where private and public issues are present, alongside private and public information, and we have provided a suitable logical language to talk about such situations. Now the time has come to dynamify this picture, describing how situations change when agents perform certain actions, and how our language provides the means for such actions.

As we did in our overview of standard DEL, we will limit our discussion here to just one, basic type of action: public announcement. Recall that in DEL, the effect of a public announcement of φ is to establish φ as common knowledge. Our enriched picture includes both information and issues, and both may be affected by agents making public announcements. Publicly announcements may now involve both declarative and interrogative sentences. The public announcement of a declarative sentence will have the effect of establishing common knowledge, while the public announcement of an interrogative sentence will have the effect of raising a public issue. Both of these effects may be seen as instances of a uniform principle: the effect of a public announcement of a sentence φ is to make φ publicly entertained. Technically, we achieve this by letting a public announcement of φ have the effect of eliminating from the model all worlds where φ is false, and restricting the epistemic maps in such a way that φ comes to be entertained by all agents at all worlds.[11] That is, a public announcement of φ transforms an inquisitive epistemic model $M = (\mathcal{W}, V, \Sigma_{\mathcal{A}})$ into the model $M^\varphi = (\mathcal{W}^\varphi, V^\varphi, \Sigma_{\mathcal{A}}{}^\varphi)$ defined as follows:

- $\mathcal{W}^\varphi = \mathcal{W} \cap |\varphi|_M$
- $V^\varphi = V\restriction_{\mathcal{W}^\varphi}$
- $\Sigma_{\mathcal{A}}{}^\varphi = \{\Sigma_a{}^\varphi \mid a \in \mathcal{A}\}$, where for every $w \in \mathcal{W}^\varphi$: $\Sigma_a{}^\varphi(w) = \Sigma_a(w) \cap [\varphi]_M$

One can verify that indeed, a public announcement of φ leads to φ being publicly entertained: that is, for any model M and any sentence φ, the resulting model M^φ is such that $\langle M^\varphi, w \rangle \models E_*\varphi$ at any world w. If φ is a declarative, then $E_*\varphi$ is equivalent to $K_*\varphi$, and so, just as in the standard setting, the public utterance of φ makes φ common knowledge.

[11] As in standard DEL, removal of worlds from the model is needed here in order to ensure that the epistemic maps of the resulting model keep satisfying the factivity condition. Were we to lift this condition, removal of worlds from the model would not be necessary.

Now suppose that φ is an interrogative. Consider a model M and a world w. If φ is not publicly settled prior to the announcement, that is, if $\langle M, w\rangle \models \neg K_*\varphi$, then an announcement of φ will obviously not lead to φ being settled (it is easy to see that a public announcement of an interrogative does not enhance any information state). Thus, in the resulting model M^φ we will have that $\langle M^\varphi, w\rangle \models \neg K_*\varphi \wedge E_*\varphi$, which means, by definition, that $\langle M^\varphi, w\rangle \models W_*\varphi$. Thus, if an interrogative φ is not yet settled in the exchange, publicly announcing it leads to a state where φ is an open issue. Summing up: public announcements of declarative sentences establish information, while public announcements of interrogative sentences raise issues.

One very convenient terminological convention is to talk of *asserting* for the act of announcing a declarative sentence, and of *asking* for the act of announcing an interrogative sentence. This makes it possible to phrase things in a very intuitive way. However, it is important to note that on our account, asserting and asking are not two intrinsically different kinds of speech act, but rather one and the same speech act performed with two different kinds of sentences.[12]

Now let us turn to the logical language. As in standard DEL, we will enrich the language by adding a dynamic modality $[\varphi]$ to our language to talk about what is the case after a public announcement of φ. The modality $[\varphi]$ can be applied to a formula ψ of either category—declarative or interrogative—to yield a formula of the same category. That is, we extend the syntax of our language with the following clause:

- if $\varphi \in \mathcal{L}_\circ$ and $\psi \in \mathcal{L}_\bullet$, then $[\varphi]\psi \in \mathcal{L}_\bullet$, where $\circ, \bullet \in \{!, ?\}$

Semantically, assessing a sentence $[\varphi]\psi$ at a pair $\langle M, s\rangle$ amounts to assessing ψ at the pair $\langle M^\varphi, s \cap |\varphi|_M\rangle$ consisting of the model resulting from the announcement of φ and the state resulting from enhancing s with the information provided by φ.

$$\langle M, s\rangle \models [\varphi]\psi \iff \langle M^\varphi, s \cap |\varphi|_M\rangle \models \psi$$

It follows from the preceding discussion that the following sentences are all logical validities of the resulting system.

- $[\varphi]E_*\varphi$
- $[\alpha]K_*\alpha$, where α is a declarative
- $\neg K_*\mu \to [\mu]W_*\mu$, where μ is an interrogative

In terms of the announcement operator we can give an alternative, dynamic characterization of informative and inquisitive sentences. A sentence is informative in case an announcement of it has the potential to establish new common knowledge, and inquisitive in case an announcement of it has the potential to raise new public issues. More formally, a sentence φ is informative in case we can find a model M and a state s in M such that φ is not yet common knowledge in s, but does become common knowledge once it is announced, that is, $\langle M, s\rangle \models \neg K_*\varphi \wedge [\varphi]K_*\varphi$. Similarly, φ is inquisitive in case we can find a model M and a state s such that φ is not yet a public issue in s, but becomes so after φ is announced, $\langle M, s\rangle \models \neg W_*\varphi \wedge [\varphi]W_*\varphi$.

Fact 6 (Dynamic characterization of informative and inquisitive sentences)

- *φ is informative iff for some M and s, $\langle M, s\rangle \models \neg K_*\varphi \wedge [\varphi]K_*\varphi$*

[12] We will return to this important point when comparing our proposal with that of van Benthem and Ştefan Minică (2012) in section 4.

– *φ is inquisitive iff for some M and s,* $\langle M, s\rangle \models \neg W_* \varphi \wedge [\varphi] W_* \varphi$

Of course, many kinds of actions besides public announcements can and should be considered as well in order to model real scenarios of information exchange. For a start, our proposal could be refined to model various sorts of private announcements. Once the possibility of false information and disagreement is admitted, acceptance and rejection actions should also be made available for the addressees of an announcement. We will leave such refinements to future work. Our main goal here was to develop a basic inquisitive dynamic epistemic logic, in which information and issues are treated on a par, and to illustrate some fundamental aspects of the workings of such a system. In the next section, we will compare our approach with a recent alternative proposal.

4 Related work

Though questions only played a very marginal role in early work on dynamic epistemic logic (with Baltag, 2001, as a notable exception), they did receive considerable attention in more recent work (Unger and Giorgolo, 2008; van Eijck and Unger, 2010; Peliš and Majer, 2010, 2011; Ågotnes et al., 2011; van Benthem, 2011; Minică, 2011; van Benthem and Ştefan Minică, 2012). One prominent framework that has emerged from this line of work is the *dynamic epistemic logic with questions* (DELQ) of van Benthem and Ştefan Minică (2012). In this section we will provide an overview of DELQ and compare it to our own approach. We start in section 4.1 with the static component of DELQ. In section 4.2 we turn to its dynamic component, and in section 4.3 to the comparison with our proposal.

4.1 Epistemic logic with issues

The semantic structures that van Benthem and Minică consider are standard epistemic models enriched with a set of issues, one for each agent. Following Groenendijk and Stokhof (1984), issues are modeled as equivalence relations $\approx$ on the set of worlds. Such an equivalence relation may be equivalently regarded as a partition $\pi_\approx$ of the logical space, whose cells correspond to the possible answers that the issue admits of. For any world w, $\pi_\approx(w)$ is used to denote the unique cell of the partition containing w. Intuitively, $\pi_\approx(w)$ is the information state that results from minimally and truthfully resolving the issue $\approx$ in w.

To be faithful to the presentation of van Benthem and Minică, we also shift to the standard presentation of epistemic models which uses epistemic accessibility relations instead of epistemic maps.

Definition 10 (Epistemic issue models)
An epistemic issue model M is a quadruple $\langle \mathcal{W}, V, \sim_\mathcal{A}, \approx_\mathcal{A}\rangle$, where:

- $\mathcal{W}$ is a set whose elements are called possible worlds
- $V : \mathcal{W} \to \wp(\mathcal{P})$ is a valuation function
- $\sim_\mathcal{A} = \{\sim_a \mid a \in \mathcal{A}\}$ is a set of equivalence relations on $\mathcal{W}$, called *epistemic relations*
- $\approx_\mathcal{A} = \{\approx_a \mid a \in \mathcal{A}\}$ is a set of equivalence relations on $\mathcal{W}$, called *issue relations*

The language that van Benthem and Minică use to describe their epistemic issue models is the standard language of epistemic logic enriched with a universal modality U, as well as a question modality Q_a and a resolution modality R_a for any agent a. These modalities are interpreted as follows.

1. $\langle M, w\rangle \models U\varphi$ iff $\langle M, v\rangle \models \varphi$ for all $v \in \mathcal{W}$
2. $\langle M, w\rangle \models Q_a\varphi$ iff $\langle M, v\rangle \models \varphi$ for all $v \in \mathcal{W}$ such that $w \approx_a v$
3. $\langle M, w\rangle \models R_a\varphi$ iff $\langle M, v\rangle \models \varphi$ for all $v \in \mathcal{W}$ such that $w \sim_a v$ and $w \approx_a v$

The universal modality, a standard tool in modal logic, talks about what is true at all worlds in the model. The question modality Q_a talks about what is true in all worlds $\approx_a$-equivalent to the evaluation world w, that is, all worlds in the state $\pi_{\approx_a}(w)$. We said above that this state represents the information state that would result from correctly resolving the issue $\approx_a$ at w. Thus, the question modality Q_a talks about what would be established if the issue entertained by a were correctly resolved.

The resolution modality R_a, on the other hand, talks about what is true at all the worlds which are both $\sim_a$-equivalent and $\approx_a$-equivalent to w. These are the worlds that make up the information state resulting from pooling together the private information available to a at w and the information that would result from correctly resolving a's issues at w. Thus, the resolution modality R_a talks about what agent a would know if her current issue were truthfully resolved.

Combining the modalities U and Q_a we can express facts about the issues that agent a entertains. Consider the formula:

$$U(Q_a\varphi \vee Q_a\neg\varphi)$$

This formula says that any world w is such that, if a's private issues were resolved correctly at w, either φ or $\neg\varphi$ would be established. Thus, it says that resolving a's private issues necessarily involves establishing an answer to the question whether φ is the case. We can take this to be a description of what it means for a to entertain the issue whether φ.

4.2 Dynamics

We have seen how van Benthem and Minică's models, just like ours, include a description of private issues and information. Here, too, both components may be affected by agents performing certain actions. Van Benthem and Minică consider a number of actions. We will focus our attention on two of these, the most fundamental ones: the action of *publicly announcing* that φ is the case, denoted $\varphi!$, and the action of *publicly asking* whether φ is the case, denoted $\varphi?$. Recall that in our proposal there are two types of sentences—declaratives and interrogatives—and only one type of action—public announcement. By contrast, in DELQ there is only one type of sentences—declaratives—but there are two types of actions—announcing and asking. Let us see how these operations work.

A public announcement of φ transforms a model M into the model $M^{\varphi!}$ which differs from M only in the agents' epistemic relations. The new epistemic relation ${\sim_a}^{\varphi}$ for agent a is $\sim_a \cap \equiv^{\varphi}$, where $\equiv^{\varphi}$ is the relation holding between two worlds just in case φ has the same truth value in both worlds. Thus, a public announcement of φ has the effect of making it common knowledge whether φ holds.[13] Publicly asking whether φ is the case has a similar effect, but on the issue component of the model. That is, it tranforms a model M into the model $M^{\varphi ?}$ which differs from M only in the agents' issue relations. The new issue relation $\approx_a^{\varphi}$ for agent a is $\approx_a \cap \equiv^{\varphi}$, where $\equiv^{\varphi}$ is as before. Thus, a public question whether φ has the effect of making it an open issue for all agents whether φ.

As customary, the system provides dynamic modalities $[\varphi !]$ and $[\varphi ?]$ corresponding to these actions, whose semantics is given by the familiar scheme.

1. $\langle M, w\rangle \models [\varphi !]\psi \iff \langle M^{\varphi !}, w\rangle \models \psi$
2. $\langle M, w\rangle \models [\varphi ?]\psi \iff \langle M^{\varphi ?}, w\rangle \models \psi$

This concludes our essential tour of DELQ. We now turn to a comparison of the two proposals.

4.3 Comparison

As we saw, DELQ is very much in the same spirit as our inquisitive dynamic epistemic logic (IDEL for short): both systems are designed to model information exchange as a dynamic process in which agents request and provide information according to what they know and what they want to know. However, there are also a number of crucial differences between the two systems.

4.3.1 A non-difference

Let us start our comparison with something which is *not* a difference between the two proposals. As mentioned above, the most standard implementation of inquisitive semantics, which is referred to as InqB, assumes no syntactic division between declarative and interrogative sentences. Rather, it is based on a plain propositional language. Semantically, there is no strict division of labor: unlike in the system we gave here, sentences may be *hybrid*, that is, both informative *and* inquisitive. The logic arising from this system is an *intermediate* logic, that is, a logic stronger than intuitionistic logic but weaker than classical logic (Ciardelli and Roelofsen, 2011). In discussing the relation between DELQ and inquisitive semantics, van Benthem and Ştefan Minică (2012, p.666) claim that, in spite of similarities between the two systems,

> "[...] there is also a major difference. The 'inquisitive logic' matching inquisitive semantics is an intermediate logic with some intuitionistic, rather than classical features. By contrast, our dynamic logics are conservative extensions of classical propositional logic with new dynamic modalities for issue-changing actions."

13 Notice that on this approach, a public announcement never removes any world from the model. This has the puzzling consequence that in a $\neg\varphi$-world, announcing that φ has the effect of making $\neg\varphi$ common knowledge. This treatment of public announcements of declarative sentences is different from the one we gave. However, this difference is not so essential, since both systems are compatible with either account of public announcement for declaratives.

This passage correctly points out a difference between DELQ and the standard inquisitive semantics system InqB. But, as a general approach to meaning, inquisitive semantics is not committed to the particular treatment of the logical constants adopted in InqB. In the present paper, we opted for a formulation of inquisitive semantics in which there is a clearcut division of labor between declaratives and interrogatives. Indeed, in the system that we have been developing, the declarative fragment of the language entirely obeys the laws of classical logic. Thus, just like DELQ, IDEL is a conservative extension of classical propositional logic. Removing this source of divergence between DELQ and the inquisitive approach will allow us to focus on what we take to be the real fundamental differences between the two: as we will see, there are at least three such differences.

4.3.2 Local versus global issues

The development of our semantic picture was driven by a simple but powerful idea: possible worlds represent states of affairs; when we consider an information exchange, a state of affairs encompasses not just the external facts which constitute the basic topic of the exchange, but also any feature of the exchange itself which is relevant for the purpose at hand.[14] The formal model should reflect this idea, equipping each world with a description of all the relevant features. In propositional logic, one does not consider an information exchange *at all*, but only certain facts. Thus, for the purposes of propositional logic, a world can be characterized by a valuation determining which of the basic facts are true and which are false. In epistemic logic, one is also interested in the knowledge that the agents have. Accordingly, a world comes equipped with a description of the agents' information states. In our inquisitive epistemic logic, a third feature of the exchange entered the picture, namely, the issues that the agents entertain. Thus, in our setting worlds also come equipped with a description of the agents' issues.

In DELQ, a world does not come with a description of the issues that the agents entertain at that world. Rather, an epistemic issue model comes with just one issue for each agent, which is not relativized to any particular world. Thus, while the information available to the agents may differ from world to world, the issues that the agents entertain are fixed and independent of the world under consideration.

Conceptually, it is difficult to see how this asymmetry could be motivated. Certainly, a particular distribution of issues among the participants partly determines what a world is like, just like a particular distribution of information. Moreover, it is natural to assume that agents may entertain different issues at different worlds.

These conceptual concerns also have important practical consequences. In particular, the asymmetric treatment of information and issues puts significant limitations on the descriptive power of DELQ. Just like in IDEL, agents may have incomplete knowledge about other agents' knowledge in DELQ, and if they do, they may indeed wonder what the other agents know. However, one would also like to be able to describe situations where agents have incomplete knowledge and wonder about the *issues* that the other agents entertain. In IDEL, such situations can be described straightforwardly. Indeed, the language of IDEL contains sentence such as $K_a W_b \mu$, expressing the fact that *a knows that b wonders about* μ, and $W_a ?W_b \mu$, expressing the fact that *a wonders whether b wonders about* μ.

[14] This point has been argued forcefully by Stalnaker (1998).

In DELQ, such situations cannot be modeled appropriately. To see this, recall that the formula $U(Q_a\varphi \vee Q_a\neg\varphi)$ is used in DELQ to describe situations in which agent a entertains the issue whether φ is the case or not. Thus, the formula $K_bU(Q_a\varphi \vee Q_a\neg\varphi)$ is used to describe situations in which agent b *knows* that agent a entertains the issue whether φ holds or not. Now suppose that M is a model and w a world such that $\langle M, w\rangle \models U(Q_a\varphi \vee Q_a\neg\varphi)$. That is, b entertains the issue whether φ in w. Then, since the universal modality U ranges over all worlds in M, we must also have for any world $v \neq w$ in M that $\langle M, v\rangle \models U(Q_a\varphi \vee Q_a\neg\varphi)$. But then we must certainly have that $\langle M, w\rangle \models K_bU(Q_a\varphi \vee Q_a\neg\varphi)$. That is, if one agent entertains a certain issue, then all the other agents automatically this. Thus, it is impossible to model situations where the agents have incomplete information about the other agents' issues, let alone situations where the agents wonder about the other agents' issues.

This limitation is not the only price that DELQ pays for its non world-based treatment of issues. The other significant limitation that it encounters concerns the construction of a public issue state. Both the models of IDEL and those of DELQ contain in their definition only a description of individual issues. Of course, *public issues* play a crucial role in information exchange. Van Benthem and Minică are well aware of the importance of public notions. For instance, when discussing further research directions (p. 663), they say:

> "We need extensions of our systems to group actions of information and issue management, including common knowledge, and group issue modalities."

In section 3, we showed that IDEL elegantly deals with the challenge of constructing a public state map which describes public issues and allows us to suitably interpret the public entertain modality E_*. The public state map is constructed from the maps encoding individual states, and it is completely determined by the requirement that something be publicly entertained iff it is common knowledge that everyone entertains it. This solution is not available in DELQ, since it requires the model to represent what agents know about the issues that other agents entertain, what they know about what other agents know about the issues that other agents entertain, etcetera. This information, as we saw, is not represented in the models of DELQ. It follows that, if we want public issues to enter the picture in DELQ, they will have to be specified as an independent component of the models. But this would miss the fundamental relation existing between public issues and the individual states. As a consequence, in the dynamics we would be forced to postulate a special maintenance rule for the public issue state, independent of the maintenance rules for the individual states. This is not necessary in IDEL, where public issues automatically change *as a result* of changes in the agents' private states.

4.3.3 Different notions of issues

Issues play a central role in the models of both DELQ and IDEL. However, the systems are based on two different formal notions of issues. In DELQ, an issue is an equivalence relation $\approx$ on the set of worlds. As we saw, such an equivalence relation corresponds to a partition $\pi_\approx$ of the logical space, whose blocks correspond to the basic *answers* to the issue. In IDEL, on the other hand, an issue I is defined as a

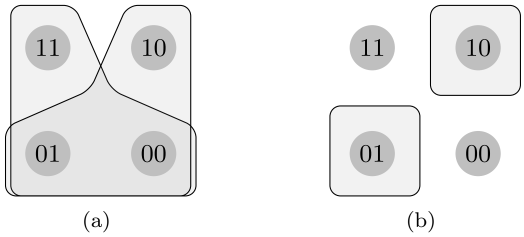

Fig. 2 Two issues that do not correspond to a partition of the logical space.

non-empty downward closed set of information states, to be thought of intuitively as those states that contain enough information to resolve the issue. Basic answers can be taken to correspond to the minimal pieces of information that resolve the issue, that is, to the maximal elements of I with respect to the $\subseteq$-ordering. Such maximal elements do not necessarily form a partition of the logical space: there may be worlds which are contained in no maximal element, precisely one, or more than one.

The question is: are there natural examples of issues in the sense of IDEL whose basic answers do *not* correspond to the blocks of a partition of the logical space? The answer is *yes*. First, the basic answers to an issue need not be mutually exclusive. To see this, consider the conditional interrogative $p \to ?q$ discussed on page 13 and a model consisting of just four worlds, 11, 10, 01, and 00, where 11 is a world that makes both p and q true, 10 is a world that makes p true and q false, etcetera. The proposition expressed by $p \to ?q$ in this model, which is indeed an issue in the sense of IDEL, is depicted in figure 2(a). We have depicted only the maximal elements of the proposition, which correspond to the two basic answers, $p \to q$ and $p \to \neg q$. The figure immediately reveals that these basic answers overlap, and therefore do not form a partition of the logical space.

Second, issues need not cover the entire logical space. As mentioned before, an issue may assume certain information. Consider an agent who knows that precisely one of the atomic sentences p and q is true, and who entertains the issue which of p and q is true. Consider the same model as above, consisting of the four worlds 11, 10, 01, and 00. Then, the agent's information state is $\{10, 01\}$ and her inquisitive state is $\{\{10\}, \{01\}, \emptyset\}$, as depicted in figure 2(b). Note that only the two maximal elements of the agent's inquisitive state are depicted. These maximal elements correspond to the two basic answers to the issue that the agent entertains, which are $p \wedge \neg q$ and $q \wedge \neg p$. Clearly, these basic answers do in general *not* exhaust the logical space. The issue assumes the information that exactly one of p and q is true, and thus it cannot be resolved—and it does not have an answer—in any world where this assumption is not met.

Thus, we have seen two examples of issues that do not correspond to partitions of the logical space: issues which assume certain information, whose answers do not cover the whole space; and conditional issues, whose answers are not mutually exclusive. If we were to move to the first-order setting, we would encounter another important class of issues whose answers are not mutually exclusive. These are issues expressed by so-called *mention-some* questions, such as (3):

(3) Where can I buy an Italian newspaper?

Evidently, such questions admit of several non-mutually exclusive answers.

Summing up, the notion of issues adopted in IDEL is strictly more general than the one adopted in DELQ. Every issue in the sense of DELQ, modeled by an equivalence relation $\approx$, immediately translates to an issue in IDEL, namely the issue:

$$I_{\approx} = \{t \mid w \approx w' \text{ for all } w, w' \in t\}$$

consisting of all states which are included in one block of the partition induced by $\approx$. However, the converse is not the case: there are many issues in the sense of IDEL that do not correspond to any issue in the sense of DELQ, namely, all those issues whose basic answers do not form a partition of the logical space. And, as we argued, important types of issues fall within this class. We conclude that the notion of issues adopted in DELQ, while natural and formally well-behaved, is not rich enough to deal with several types of issues that play a significant role in information exchange.[15]

4.3.4 Questions as interrogative sentences versus questions as speech acts

So far we identified two important differences between the *epistemic issue models* of DELQ and the *inquisitive epistemic models* of IDEL. One difference concerns the way issues are modeled, the other the way issues are embedded into the framework of epistemic logic. A third crucial difference concerns the treatment of questions.

In DELQ, the static language consists entirely of declarative sentences. No sentence is syntactically interrogative or semantically inquisitive. Questions only come into the picture in the dynamic component of the system, as a particular kind of speech act. As we saw, the effect of a question involving a sentence α is to raise the issue whether α holds.

In IDEL, questions enter the picture already at the level of the static language, in the form of interrogative sentences. Just like declaratives, interrogative sentences have a semantic value, which captures their inquisitive content. This semantic value enters the compositional process, allowing us to compositionally assign meanings to sentences where interrogatives are embedded under modal operators. It also allows us to keep the dynamic component of the system simpler: we only need a single action of announcing a sentence, be it declarative or interrogative, rather than two distinct actions for announcing and questioning. It is the content of the sentence that is being announced which determines whether the announcement brings about a change in information or in issues.

We will argue that there are good reasons to prefer the latter approach. In DELQ, all questions have the effect of raising a polar issue, namely the issue whether a certain declarative sentence α holds. The same effect is obtained in IDEL by an announcement of the polar interrogative $?\alpha$. Thus, the effect of a question action in DELQ may be simulated by the announcement of an interrogative in IDEL. However, the converse is problematic. Not all interrogatives that may be asked in IDEL express polar issues. Consider for instance the conditional interrogative $p \to ?q$, whose meaning was depicted in figure 2(a) above. The effect of such an interrogative cannot be modeled in DELQ since, as we saw, the notion of issues

[15] Similar arguments, not addressing DELQ directly but rather the partition theory of questions that it is based on (Groenendijk and Stokhof, 1984), have been made by Mascarenhas (2009), Groenendijk (2011), and Ciardelli et al. (2012a).

adopted in DELQ is not rich enough. But suppose this problem were amended. Then DELQ would still be in trouble, since asking $p \to ?q$ does not correspond in any way to asking whether a certain declarative sentence is true or not. In order to address this problem, DELQ may be extended with an additional, more complex action of *conditional questioning*, which would involve two declarative sentences, one serving as the antecedent and one serving as the consequent of the question. But one can of course easily imagine more and more complex question types, which force DELQ to postulate a richer and richer repertoire of question actions.

Whether or not DELQ might eventually succeed in making its repertoire of actions rich enough, its treatment of questions as speech acts faces another difficulty as well. In IDEL, as mentioned above, an interrogative sentence is assigned a semantic value, which does not only determine the effect of announcing that interrogative, but also the meaning of more complex expressions in which the interrogative may be *embedded*. In particular, these more complex expressions may be declaratives, whose truth-conditions depend on the issue expressed by the embedded interrogative. Concretely, the basic way to construct a declarative from an interrogative μ is to embed μ under a modal operator, such as K_a, E_a, W_a, or their public counterparts K_*, E_*, W_*, all of which allow for an interrogative complement. In this way, we can construct declaratives such as $K_a\mu$, which expresses the fact that a can resolve μ; $W_a\mu$, which expresses the fact that a wonders about μ; $K_*\mu$, which expresses the fact that μ is publicly settled among the agents; and $W_*\mu$, which expresses the fact that μ is an open issue among the agents.

DELQ does not allow the construction of sentences that involve an interrogative embedded under a modal operator. The possibility of expressing the corresponding facts depends on the possibility of analyzing claims about interrogatives in terms of claims concerning declaratives. In some cases such analyses are indeed possible. For instance, consider a polar interrogative $?\alpha$. In DELQ, *a knows whether* α may be analyzed as $K_a\alpha \vee K_a\neg\alpha$, and *a wonders whether* α may be analyzed as $U(Q_a\alpha \vee Q_a\neg\alpha)$, as we saw. In general, if an interrogative μ has a finite set of predetermined answers $\alpha_1, \ldots, \alpha_n$, then *a knows* μ may be analyzed as $K_a\alpha_1 \vee \cdots \vee K_a\alpha_n$, and *a wonders about* μ may be analyzed as $U(Q_a\alpha_1 \vee \cdots \vee Q_a\alpha_n)$.

However, this strategy has two drawbacks. First, analyzing sentences involving an interrogative μ in this way requires knowledge of the set of answers to μ. Thus, in order to express facts about a question, DELQ needs to outsource the analysis of the question to some theory that predicts what its answers are. Our semantics, on the contrary, *includes* such a theory of questions. Equivalences such as:

$$K_a?\alpha \leftrightarrow (K_a\alpha \vee K_a\neg\alpha)$$

characterizing the knowledge that it takes to resolve a certain question, are obtained as *logical validities* of the theory, not merely assumed as definitions.

Second, the 'paraphrase' strategy considered here may be feasible for the propositional case, where questions have a finite, predetermined set of answers. However, if we move to the first order setting, things look very different. Many types of questions—such as *wh*-questions (*Who attended the party?*), *which*-questions (*Which students attended the party?*), and quantified questions (*Which party did every student attend?*)—have a set of answers that is neither predetermined—it depends on the domain of interpretation—nor necessarily finite. The particular paraphrase strategy sketched above cannot be applied to questions of these kinds. Perhaps for any particular type of question some paraphrase in terms of declaratives may be found. However, it is very unlikely that a uniform analysis of embedded questions in terms of declaratives exists. For any particular question, we will have to come up with a 'custom-made' translation.

Our compositional strategy, on the other hand, carries over straightforwardly to the first-order case. Drawing on ideas from first-order inquisitive semantics (Ciardelli, 2009; Roelofsen, 2011a), we could define a first-order language in which a broad range of issues can be expressed, including those corresponding to the question types above. For instance, our language would contain interrogatives of the form $?x.\alpha(x)$, corresponding to *wh*-questions such as *Who attended the party?*. Just like in the propositional case, such interrogatives may be embedded under modal operators to yield sentences such as $K_a?x.\alpha(x)$ and $W_a?x.\alpha(x)$, expressing that *a knows who attended the party* and that *a wonders who attended the party*, respectively.[16]

5 Conclusion

We proposed an inquisitive dynamic epistemic logic in which the issues that the agents entertain are treated on a par with the information that they have, as an integral component of the state of affairs in each world, thus preserving the general philosophy of standard epistemic logic. We imported from inquisitive semantics a notion of issues which is more general than the traditional partition-notion. Moreover, we enriched the logical language with interrogative sentences, and generalized the semantics in order to treat declaratives and interrogatives in a uniform way, moving from single worlds to information states as points of evaluation, while still being able to derive the natural, truth-conditional interpretation of declaratives relative to single worlds. We specified a natural public state construction, analogous to the familiar common knowledge construction, which allows us to derive the public information and issues from the description of the private ones. Finally, we provided a basic dynamics for actions of public announcement. This results in a system in which a rich spectrum of facts concerning public and private knowledge as well as public and private issues can be modeled and reasoned about.

Several directions for future work naturally suggest themselves. Perhaps most urgently, the logic that the system gives rise to needs to be investigated. In previous work on (non-dynamic, non-epistemic implementations of) inquisitive semantics, a natural notion of entailment has been introduced, which applies uniformly to declaratives and interrogatives. One sentence φ is defined to entail another sentence ψ just in case every state that supports φ also supports ψ. When φ and ψ are both declaratives, this notion of entailment amounts to the classical notion of entailment; if φ and ψ are both interrogatives then $\varphi \models \psi$ means that every piece of information that resolves φ also resolves ψ, that is, the issue that φ expresses is

[16] In the liguistic literature, the point that a proper treatment of questions, especially embedded questions, requires inquisitiveness to enter the picture at the semantic level, and not just at the speech act level, has been made in much detail by Groenendijk and Stokhof (1997). At the time, it was directed mostly at the speech act treatment of questions proposed by Searle (1969) and Vanderveeken (1990), and at the imperative-epistemic treatment of questions proposed by Åqvist (1965) and Hintikka (1976, 1983). The argument we just gave is essentially the same, but now directed specifically at the speech act treatment of questions in DELQ.

at least as demanding as the issue that ψ expresses; if φ is a declarative and ψ an interrogative, then $\varphi \models \psi$ means that φ resolves ψ; finally, if φ is an interrogative and ψ a declarative, then we can only have that $\varphi \models \psi$ if ψ is a tautology. This uniform notion of entailment is also suitable in the context of IDEL. Evidently, we would like to characterize this notion axiomatically. Previous work on inquisitive logic (Ciardelli, 2009; Ciardelli and Roelofsen, 2011) provides a useful starting point for such a characterization.

In this paper, our goal has been to show that information and issues are amenable to a uniform treatment in a natural extension of the basic DEL framework, and that inquisitive semantics provides the right tools for this enterprise. We illustrated this by building a system which is minimal in many respects. This basic system may be extended in several directions, incorporating insights from the existing literature on both dynamic epistemic logic and inquisitive semantics. For instance, on the dynamic epistemic logic side, besides the very strong notion of knowledge that we assumed here, which is characterized by factivity and full introspection, we may also consider the dynamics of weaker notions of knowledge and belief (see, e.g., van Ditmarsch, 2005; van Benthem, 2007). On the inquisitive semantics side, besides the notion of issues adopted here, which captures inquisitive content, we may also import semantic structures that capture *attentive* content (see, e.g., Ciardelli et al., 2009; Roelofsen, 2011b). To make our account more faithful to actual linguistic exchange, we may also consider sentences involving *presuppositions* (Ciardelli et al., 2012b), such as alternative questions, which were not considered here. Finally, it may be interesting to see how our framework can be applied in the analysis of *question-answer games*, as investigated previously based on DELQ by Ågotnes et al. (2011). For now, we hope to have shown that the basic machinery of dynamic epistemic logic can be extended in a natural and principled way so as to allow for a more inclusive logical analysis of information exchange, encompassing both informative and inquisitive aspects.

Acknowledgements To be inserted after reviewing process.

References

Thomas Ågotnes, Johan van Benthem, Hans van Ditmarsch, and Ştefan Minică. Question-answer games. *Journal of Applied Non-Classical Logics*, 21(3-4):265–288, 2011. doi: 10.3166/jancl.21.265-288.

L. Åqvist. *A new approach to the logical theory of interrogatives*. University of Uppsala, 1965.

Alexandru Baltag. Logics for insecure communication. In *Proceedings of Theoretical Aspects of Rationality and Knowledge (TARK) VIII*, pages 111–122, 2001.

Alexandru Baltag, Lawrence S. Moss, and Slawomir Solecki. The logic of public announcements, common knowledge, and private suspicions. In *Proceedings of the 7th Conference on Theoretical Aspects of Rationality and Knowledge*, pages 43–56. Morgan Kaufmann Publishers, 1998.

Johan van Benthem. Dynamic logic for belief revision. *Journal of Applied Non-Classical Logics*, 17(2):129–155, 2007.

Johan van Benthem. *Logical dynamics of information and interaction*. Cambridge University Press, 2011.

Johan van Benthem and Ştefan Minică. Toward a Dynamic Logic of Questions. *Journal of Philosophical Logic*, 41(4):633–669, 2012. doi: 10.1007/s10992-012-9233-7.

Ivano Ciardelli. Inquisitive semantics and intermediate logics. Master Thesis, University of Amsterdam, 2009.

Ivano Ciardelli and Floris Roelofsen. Inquisitive logic. *Journal of Philosophical Logic*, 40(1):55–94, 2011.

Ivano Ciardelli, Jeroen Groenendijk, and Floris Roelofsen. Attention! *Might* in inquisitive semantics. In Satoshi Ito and Ed Cormany, editors, *Proceedings of Semantics and Linguistic Theory (SALT XIX)*. CLC Publications, 2009.

Ivano Ciardelli, Jeroen Groenendijk, and Floris Roelofsen. Inquisitive semantics: a new notion of meaning. Forthcoming in *Language and Linguistics Compass.*, 2012a.

Ivano Ciardelli, Jeroen Groenendijk, and Floris Roelofsen. Inquisitive semantics. NASSLLI lecture notes, 2012b. URL www.illc.uva.nl/inquisitive-semantics.

Hans van Ditmarsch. *Knowledge Games*. PhD thesis, University of Groningen, 2000.

Hans van Ditmarsch. Prolegomena to dynamic logic for belief revision. *Synthese*, 147(2):229–275, 2005.

Hans van Ditmarsch, Wiebe van der Hoek, and Barteld Kooi. *Dynamic Epistemic Logic*. Springer, 2007.

Ronald Fagin, Joseph Y. Halpern, Yoram Moses, and Moshe Y. Vardi. *Reasoning about knowledge*. MIT Press, Cambridge, MA, 1995.

Jelle Gerbrandy and Willem Groeneveld. Reasoning about information change. *Journal of logic, language and information*, 6(2):147–169, 1997.

Jeroen Groenendijk. Inquisitive semantics: Two possibilities for disjunction. In Peter Bosch, David Gabelaia, and Jérôme Lang, editors, *Seventh International Tbilisi Symposium on Language, Logic, and Computation*. Springer-Verlag, 2009.

Jeroen Groenendijk. Erotetic languages and the inquisitive hierarchy. In a Festschrift for Martin Stokhof, 2011. URL www.vddoes.net/Martin/articles/Jeroen.pdf.

Jeroen Groenendijk and Floris Roelofsen. Inquisitive semantics and pragmatics. Presented at the Workshop on Language, Communication, and Rational Agency at Stanford, May 2009, 2009. URL www.illc.uva.nl/inquisitive-semantics.

Jeroen Groenendijk and Martin Stokhof. *Studies on the Semantics of Questions and the Pragmatics of Answers*. PhD thesis, University of Amsterdam, 1984.

Jeroen Groenendijk and Martin Stokhof. Questions. In Johan van Benthem and Alice ter Meulen, editors, *Handbook of Logic and Language*, pages 1055–1124. Elsevier, 1997.

Jaako Hintikka. The semantics of questions and the questions of semantics. *Acta Philosophica Fennica*, 1976.

Jaako Hintikka. New foundations for a theory of questions and answers. In *Questions and Answers*, pages 159–190. Reidel, Dordrecht, 1983.

Salvador Mascarenhas. Inquisitive semantics and logic. Master Thesis, University of Amsterdam, 2009.

Ştefan Minică. *Dynamic Logic of Questions*. PhD thesis, ILLC, University of Amsterdam, 2011.

Michal Peliš and Ondrej Majer. Logic of questions from the viewpoint of dynamic epistemic logic. In Michal Peliš, editor, *The Logica Yearbook*, pages 157–172.

College Publications, 2010.

Michal Peliš and Ondrej Majer. Logic of questions and public announcements. In Nick Bezhanishvili, Sebastian Löbner, K. Schwabe, and L. Spada, editors, *Logic, Language and Computation: Selected Revised Papers from the Eighth International Tbilisi Symposium*, pages 145–157. Springer, 2011.

Jan Plaza. Logics of public communications. In M.L. Emrich, M.S. Pfeifer, M. Hadzikadic, and Z.W. Ras, editors, *Proceedings of the Fourth International Symposium on Methodologies for Intelligent Systems*, pages 201–216. Oak Ridge National Laboratory, 1989. Reprinted as Plaza (2007).

Jan Plaza. Logics of public communications. *Synthese*, 158(2):165–179, 2007.

Floris Roelofsen. Algebraic foundations for inquisitive semantics. In Hans van Ditmarsch, Jerome Lang, and Ju Shier, editors, *Proceedings of the Third International Conference on Logic, Rationality, and Interaction*, pages 233–243. Springer-Verlag, 2011a.

Floris Roelofsen. Information and attention. Manuscript, ILLC University of Amsterdam, 2011b.

John R. Searle. *Speech acts: an essay in the philosophy of language*. Cambridge University Press, 1969.

Robert Stalnaker. On the representation of context. *Journal of logic, language, and information*, 7(1):3–19, 1998.

Christina Unger and Gianluca Giorgolo. Interrogation in dynamic epistemic logic. In *ESSLLI Student Session*, pages 195–202, 2008.

Jan van Eijck and Christina Unger. *Computational semantics with functional programming*. Cambridge University Press, 2010.

Daniel Vanderveeken. *Meaning and speech acts: principles of language use*. Cambridge University Press, 1990.