Inquisitive dynamic epistemic logic*

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Abstract

Information exchange can be seen as a dynamic process of raising and resolving issues. The goal of this paper is to provide a logical framework to model and reason about this process. We develop an inquisitive dynamic epistemic logic, IDEL, which enriches the standard framework of dynamic epistemic logic, incorporating insights from recent work on inquisitive semantics. At a static level, IDEL does not only allow us to model the *information* available to a set of agents, like standard epistemic logic, but also the *issues* that the agents entertain. At a dynamic level, IDEL does not only allow us to model the effects of public announcements that provide new information, like standard DEL, but also the effects of actions that *raise new issues*. Thus, IDEL provides the fundamental tools needed to analyze information exchange as a dynamic process of raising and resolving issues.

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Contents

1 Introduction

Much attention has been paid in recent years to the development of logical tools to describe and reason about information exchange through communication. One prominent framework that has emerged from this work is the framework of *dynamic epistemic logic* (DEL) (van Ditmarsch et al., 2007; van Benthem, 2011, among many others). In its basic incarnation, this framework allows us to formally describe and reason about the information available to a number of agents, and how this information changes when an assertion is made, providing new information.

However, in modeling information exchange it is not only important to keep track of the information that is available to the agents involved, but also of the information that they would like to obtain, i.e., the *issues* that they entertain. Moreover, while assertions evidently play an important role in the process of exchanging information, an equally important role is played by *questions*. Information exchange is a collaborative process of raising and resolving issues. Agents raise issues by asking questions, and resolve these issues by making assertions.

This paper develops an *inquisitive dynamic epistemic logic*, IDEL, incorporating insights from recent work on inquisitive semantics (Ciardelli, 2009; Groenendijk and Roelofsen, 2009; Ciardelli et al., 2012, 2013a, among others). At a static level, IDEL does not only allow us to model the information available to a set of agents, like standard epistemic logic, but also the issues that the agents entertain. And similarly, at a dynamic level, IDEL does not only allow us to capture the effects of assertions, which provide new information, but also the effects of questions, which raise new issues.

We will compare our approach in some detail with the *dynamic epistemic logic with questions* (DELQ) recently developed by van Benthem and Minică (2012). Although the two proposals are very much in the same spirit, we will argue that IDEL has some crucial advantages.

The paper is organized as follows. Section 2 provides an overview of standard dynamic epistemic logic. In section 3 we present our inquisitive dynamic epistemic logic, and in section 4 we compare our framework with DELQ. Finally, section 5 concludes with some directions for future work.

2 Dynamic epistemic logic

We start with an overview of the standard, most basic incarnation of dynamic epistemic logic. Our exposition will be quite detailed, which will allow us later on to draw very explicit parallels between the treatment of issues in IDEL and the treatment of information in DEL. As is usually done, we will present DEL in two stages. First, in section 2.1, we will discuss the essential elements of epistemic logic, which forms the static component of DEL. Then, in section 2.2, we will discuss the dynamic component of the system.

2.1 Epistemic logic

Epistemic logic is designed to formally describe and reason about a certain set of facts together with what certain agents know about these facts and about one another's knowledge. Epistemic situations are represented by models that are based on the notion of a possible world. Intuitively, a possible world represents a possible state of affairs. When a certain set of facts is all that is at stake, a state of affairs may well be identified with a valuation that tells us which facts are true and which are false. In this way we get worlds characterized uniquely by a propositional valuation, which are suitable models for classical propositional logic. However, if we do not just want to take the facts themselves into consideration, but also what certain agents know about these facts, then we need a richer characterization of states of affairs, reflecting all the additional features that are relevant for the purpose at hand. Thus, in an epistemic model, a possible world w is laid out by specifying two things: (i) a valuation for the facts under discussion and (ii) an *information state* for each agent. Formally, an information state is modeled as a set of possible worlds, to be conceived of as those worlds that are compatible with the available information. We thus arrive at the following definition.

Definition 2.1 (Epistemic models)**.** An epistemic model for a set $\mathcal{P}$ of atomic sentences and a set $\mathcal{A}$ of agents is a tuple $M = \langle \mathcal{W}, V, \sigma_{\mathcal{A}} \rangle$ where:

- $\mathcal{W}$ is a set, whose elements are called *possible worlds*.
- $V : \mathcal{W} \to \wp(\mathcal{P})$ is a *valuation map* that specifies for every world w which atomic sentences are true at w.

- $\sigma_{\mathcal{A}} = \{\sigma_a \,|\, a \in \mathcal{A}\}$ is a set of *epistemic maps* from $\mathcal{W}$ to $\wp(\mathcal{W})$, each of which assigns to any world w an information state $\sigma_a(w)$ in accordance with:

  Factivity : for any $w \in \mathcal{W}$, $w \in \sigma_a(w)$

  Introspection : for any $w, v \in \mathcal{W}$, if $v \in \sigma_a(w)$, then $\sigma_a(v) = \sigma_a(w)$

Together, the valuation map V and the epistemic maps in $\sigma_{\mathcal{A}}$ equip every world w in the model with a complete specification of the state of affairs that it represents, given that we take a state of affairs to be completely determined in this setting by what the facts are like and what the agents know.

The epistemic maps need to satisfy two conditions. The factivity condition requires that the information available to agents be truthful, so that the information state of an agent is indeed a knowledge state, and not merely a belief state. The introspection condition requires that agents know their knowledge state, so that if the information state of a in w differs from her state in v, then a can tell the worlds w and v apart.[1]

The epistemic maps $\sigma_a : \mathcal{W} \to \wp(\mathcal{W})$ can be equivalently regarded as binary relations $\sim_a \subseteq \mathcal{W} \times \mathcal{W}$, where for any w and v: $w \sim_a v$ iff $v \in \sigma_a(w)$. The factivity and introspection conditions on σ_a then translate to the requirement that $\sim_a$ be an equivalence relation. While the presentation of epistemic models that uses equivalence relations rather than functions is more common in the literature, the functional notation has an important advantage for our current purposes: it brings out more clearly how the maps σ_a are one of the ingredients that, together with the valuation V, characterize the state of affairs associated with each possible world. This suggests that, if we wanted to take into account more aspects of a state of affairs than just the information available to all the agents involved, we could add more maps to our models to describe these additional aspects. This is indeed the approach we will take in section 3.

The logical language used to talk about epistemic models is a propositional language enriched with modal operators K_a for each $a \in \mathcal{A}$. The interpretation of the modality K_a relies on the epistemic map σ_a. In a model M and at a world w, $K_a\varphi$ is true iff any world compatible with a's information at w is one where φ is true:

$$\langle M, w\rangle \models K_a\varphi \iff \text{for any } v \in \sigma_a(w),\ \langle M, v\rangle \models \varphi$$

In other words, $K_a\varphi$ is true at w if the truth of φ follows from the information available to a at w, that is, if a knows that φ at w.

The *proposition* expressed by a sentence φ in an epistemic model M, which we will denote as $|\varphi|_M$, is the set of worlds in M where φ is true. Notice that the modality K_a can be regarded as making a claim about the relation between

[1] Either of these conditions may be dropped or weakened to model scenarios of false information or not fully introspective agents (see, for instance, Fagin et al., 1995). The system considered here is usually taken to be the most basic variant of epistemic logic. For this reason we take it as a point of departure here, but we do not expect to encounter particular difficulties in adapting our proposal to weaker variants.

two sets of worlds, the information state of a at the evaluation world and the proposition expressed by its argument, as the following reformulation of the clause for K_a shows:

$$\langle M, w\rangle \models K_a\varphi \iff \sigma_a(w) \subseteq |\varphi|_M$$

This perspective will help us understand how modalities generalize beyond standard modal logic in the richer semantic picture that we will introduce.

Besides the agents' individual knowledge, notions of group knowledge also play an important role in the analysis of information exchange. One notion that is of particular importance is that of *common knowledge*, i.e., the information that is publicly available to all the agents. One might think that treating this notion would require enriching our models with a map σ_* that specifies, for each world w, an information state $\sigma_*(w)$ embodying the information that is publicly available to all the agents in w. We could then expand our language with a corresponding modality K_*, interpreted as follows:

$$\langle M, w\rangle \models K_*\varphi \iff \text{for any } v \in \sigma_*(w),\ \langle M, v\rangle \models \varphi$$

However, common knowledge is very closely tied to the agents' individual knowledge: in fact, it is *determined* by it. A sentence φ is common knowledge if and only if every agent a knows that φ, and every agent a knows that every agent b knows that φ, and every agents a knows that every agent b knows that every agent c knows that φ, and so on ad infinitum. Thus, whether the sentence $K_*\varphi$ is true or false at a world should be completely determined by the following condition:

$$\langle M, w\rangle \models K_*\varphi \iff \langle M, w\rangle \models K_{a_1}K_{a_2}\dots K_{a_n}\varphi \text{ for any } a_1, \dots, a_n \in \mathcal{A},\ n \geq 0$$

One can show that, in order to guarantee this equivalence for any particular valuation V, the common knowledge map σ_* must be defined precisely as follows:

$$\begin{aligned}\sigma_*(w) = \{v \mid\ & \text{there exist } u_0, \dots, u_{n+1} \in \mathcal{W} \text{ and } a_0, \dots, a_n \in \mathcal{A} \\ & \text{such that } u_0 = w,\ u_{n+1} = v, \text{ and for } i \leq n,\ u_{i+1} \in \sigma_{a_i}(u_i)\}\end{aligned}$$

This means that the common knowledge map σ_* is uniquely determined by the set of individual epistemic maps $\sigma_{\mathcal{A}}$, and need not be added to our models as an additional component.

2.2 Dynamics

Epistemic logic allows us to describe the information available to a group of agents in a particular state of affairs. *Dynamic* epistemic logic allows us to describe how such a state of affairs may *change* when certain actions are performed.

In the most basic version of DEL, only one kind of action is considered, namely, the *public announcement* of a sentence φ.[2] Such an announcement is taken to have the effect of making it common knowledge that φ is true at the moment of utterance. That is, as a result of a public announcement of φ, all agents learn that the actual world lies in the proposition expressed by φ at the moment of utterance, and they learn that everyone else now knows this, and that everyone knows that everyone knows, and so on ad infinitum. Technically, this is achieved by letting a public announcement of φ have the effect of eliminating from the model all worlds where φ is false, and restricting the epistemic maps of the agents accordingly.[3] That is, a public announcement of φ transforms an epistemic model $M = \langle \mathcal{W}, V, \sigma_{\mathcal{A}} \rangle$ into the model $M^{\varphi} = \langle \mathcal{W}^{\varphi}, V^{\varphi}, \sigma_{\mathcal{A}}{}^{\varphi} \rangle$, where:

- $\mathcal{W}^{\varphi} = \mathcal{W} \cap |\varphi|_M$
- $V^{\varphi} = V \restriction_{\mathcal{W}^{\varphi}}$
- $\sigma_{\mathcal{A}}{}^{\varphi} = \{\sigma_a{}^{\varphi} \mid a \in \mathcal{A}\}$, where for every $w \in \mathcal{W}^{\varphi}$: $\sigma_a{}^{\varphi}(w) = \sigma_a(w) \cap |\varphi|_M$

If the sentence φ that is being announced is *factive*, i.e., if it does not contain any epistemic operators, then after the announcement, $K_*\varphi$ will hold. However, this is not the case in general: while a public announcement creates the common knowledge that the sentence was true at the moment of the announcement, the sentence may not be common knowledge, and indeed may no longer be true, after the announcement has taken place.[4] For, although the update will not change truth values of proposition letters at any world, it may very well change the epistemic states of some agents at some worlds. As a consequence, the truth values of epistemic sentences at a given world in the updated model may differ from the truth values that these sentences received at the same world in the original model, and this may result in sentences expressing different propositions than they previously did.

To illustrate this, consider the sentence $\varphi := p \wedge \neg K_* p$. Let M be an epistemic model and w a world in M where φ is true. Then it is easy to see that after the update, p will have become common knowledge, $\langle M^{\varphi}, w \rangle \models K_* p$, which implies that $\langle M^{\varphi}, w \rangle \not\models \varphi$. Thus, in this case the announcement of a true sentence leads to a state where the sentence is false.

For any type of action A that one may want to consider, the language of epistemic logic could be enriched with a corresponding dynamic modality $[A]$ that talks about what will be the case in the model after A is performed. In the

[2]Public announcement logic was first proposed by Plaza (1989) and was further developed by Gerbrandy and Groeneveld (1997), Baltag et al. (1998), and van Ditmarsch (2000), among others. Recent overviews of the system and its role in the general dynamic epistemic logic landscape are provided by van Ditmarsch et al. (2007) and van Benthem (2011).

[3]Actually, restricting the epistemic maps to φ-worlds would be all we need to model the intended change, which is a merely epistemic one. The only reason why the $\neg\varphi$-worlds also have to be eliminated from the model is that, if we did not eliminate them, the resulting model would no longer be an epistemic model in the sense of our definition, since the new epistemic maps would not validate the factivity requirement in the $\neg\varphi$-worlds.

[4]We thank an anonymous reviewer for emphasizing this.

case of basic public announcement logic, a dynamic modality $[\varphi]$ is introduced that talks about what is the case after a public announcement of φ. Now suppose we want to evaluate the sentence $[\varphi]\psi$ in a model M at a world w. If $\langle M, w\rangle \models \neg\varphi$, then the public announcement removes w from the model, and there is no fact of the matter as to what holds at w after the announcement. In this basic framework, such non-truthful announcements are treated as a case of inconsistency: if $\langle M, w\rangle \models \neg\varphi$, then one lets $\langle M, w\rangle \models [\varphi]\psi$ for all ψ. On the other hand, if $\langle M, w\rangle \models \varphi$, then w survives the public announcement of φ, and one lets $\langle M, w\rangle \models [\varphi]\psi$ hold if and only if $\langle M^\varphi, w\rangle \models \psi$ holds. Summing up:

$$\langle M, w\rangle \models [\varphi]\psi \iff w \notin |\varphi|_M \text{ or } \langle M^\varphi, w\rangle \models \psi$$

Of course, it is possible to consider many more actions than just public announcements. To mention just one important case, private announcements, directed only to a subset of the agents involved, have received much attention in the literature (see, for instance, Baltag et al., 1998). However, we will restrict our attention here to the most basic system, with public announcements only, in order to explicate our proposal in a more perspicuous way.

One may worry that the given treatment of public announcements is too strong, in that it does not give the addressees the option to reject the proposed informational update. This is clearly unrealistic if our goal is to model an actual conversation, where disagreement may occur. However, recall that we are working here under the assumption that an agent's information is always truthful. In such a setting, disagreement cannot occur, since any two agents always have compatible information states. Assuming a Gricean pragmatic rule that requires agents to only announce what they know, a situation in which one of the addressees has a reason to reject a public announcement can never arise. Of course, a more realistic picture in which knowledge is replaced by belief, and disagreement may occur, will be more interesting from a conversational perspective. We leave the investigation of the details of such a picture for future work.

3 Inquisitive dynamic epistemic logic

In a nutshell, the picture of information exchange assumed in DEL is that of a group of agents, each equipped with a certain body of information, sharing some of their individual knowledge with the other participants by making informative announcements. Something crucial is missing from this picture. When agents enter an information exchange, they are not just equipped with a certain body of information, but they also entertain certain issues that they would like to see resolved. In many cases, the desire to resolve these issues actually constitutes the motivation for the agents to engage in the exchange in the first place. Furthermore, the exchange itself does not merely consist in a sequence of informative announcements. Rather, it is an interactive process of raising and resolving issues. Agents ask questions to raise new issues, and they make assertions to resolve these issues.

In order to do justice to this more comprehensive picture of information exchange, we need to make room for issues in our logical framework. Just like agents are modeled as having certain information and are given the ability to share this information in the exchange by making informative announcements, they should also be modeled as entertaining certain issues, and they should be given the ability to raise these issues in the exchange by making inquisitive announcements, i.e., by asking questions.

3.1 Inquisitive epistemic logic

Our first task is to add an inquisitive dimension to epistemic logic. That is, we will develop a framework in which it is not only possible to model the information available to a set of agents, but also the issues that they entertain.

3.1.1 Semantic structures

While in epistemic logic a possible world w was laid out by specifying (i) a valuation for the atomic sentences in the language, and (ii) an information state for each agent, we now also need to specify (iii) an *inquisitive state* for each agent, encoding the issues that the agent entertains in w. But what kind of formal object should these inquisitive states be? In other words, what is a good mathematical representation of an issue? We will adopt the formal notion of issues that has been developed in recent work on *inquisitive semantics*.[5] The fundamental idea is to lay out an issue by specifying what it takes for the issue to be resolved. That is, an issue is identified with a set of information states: those information states that contain enough information to resolve the issue.

We assume that every issue can be resolved in at least one way, which means that issues should be identified with *non-empty* sets of information states. Moreover, a set of information states can only suitably embody an issue if it is *downward closed*. That is, if t is an information state in an issue I, then any $u \subseteq t$ should be in I as well. After all, if $t \in I$, then t contains enough information to resolve I; but then any $u \subseteq t$ clearly also contains enough information to resolve I, and should therefore be included in I as well. Thus, issues are defined as non-empty, downward closed sets of information states.[6]

[5] A detailed exposition of inquisitive semantics, in particular the notion of issues that we will adopt here, can be found in Ciardelli et al. (2012, 2013a); Roelofsen (2013a). Earlier expositions of the framework can be found in Groenendijk and Roelofsen (2009); Ciardelli (2009); Ciardelli and Roelofsen (2011). The notion of issues that will play a crucial role here is already implicit in these earlier expositions, but is not explicitly defined and motivated there. Yet earlier expositions of the framework can be found in Groenendijk (2009); Mascarenhas (2009). However, the notion of issues that is implicit in this early work is really different from the one adopted here, and, as argued in Ciardelli (2009); Ciardelli and Roelofsen (2011); Ciardelli et al. (2013b), not general enough to suitably capture the issues that are expressed by certain types of questions in natural language (e.g., disjunctive questions and mention-some *wh*-questions).

[6] Notice that this means that the empty information state, $\emptyset$, is an element of every issue. Intuitively, $\emptyset$ models the *absurd*, inconsistent information state, in which any candidate world is discarded. In this limit state, any piece of information is established, and any issue is

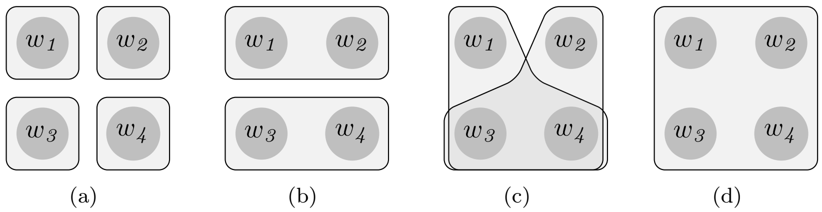

Figure 1: Issues over the state $\{w1, w2, w3, w4\}$.

Definition 3.1 (Issues)**.**
An *issue* is a non-empty, downward closed set of information states.
We say that an information state t *settles* an issue I in case $t \in I$.

It is only possible to truthfully resolve an issue I if the actual world is contained in at least one $t \in I$, i.e., if the actual world is contained in $\bigcup I$. Therefore, we say that an issue I *assumes* the information that the actual world is located in $\bigcup I$. Moreover, if s is an information state, then we say that I is an issue *over* s just in case $\bigcup I = s$. The set of all issues over a state s is denoted by Π_s, and the set $\bigcup_{s \subseteq \mathcal{W}} \Pi_s$ of issues over some state is denoted by Π. Finally, if I is an issue and s a state, then we define the *restriction* of I to s as the issue $I\!\restriction_s := \{t \in I \mid t \subseteq s\}$.

Figure 1 depicts some issues over the state $s = \{w_1, w_2, w_3, w_4\}$. In order to keep the figures neat, we have depicted only the maximal elements of these issues. The issue in (a) can only be settled by specifying precisely which world in s is the actual one. The issue in (b) can be settled either by locating the actual world in $\{w_1, w_2\}$, or by locating it in $\{w_3, w_4\}$. The issue in (c) can be settled either by locating the actual world in $\{w_1, w_3, w_4\}$, or by locating it in $\{w_2, w_3, w_4\}$. Finally, the issue in (d) is the *trivial* issue over s, which is already settled by s itself.

This notion of issues is precisely what we need to give epistemic logic an inquisitive dimension. Recall that in epistemic logic, every agent a is assigned an information state $\sigma_a(w)$ in every world w, determining the range of worlds that she considers possible candidates for the actual one. Now, every agent will also be assigned an inquisitive state $\Sigma_a(w)$, which will be modeled as an issue over $\sigma_a(w)$, reflecting the agent's desire to locate the actual world more precisely inside her information state.

Since $\Sigma_a(w)$ will be modeled as an issue over $\sigma_a(w)$, we will always have that $\sigma_a(w) = \bigcup \Sigma_a(w)$. This means that from the inquisitive state $\Sigma_a(w)$ of an agent a in a world w, we can always derive the information state $\sigma_a(w)$ of that agent in that world, simply by taking the union of $\Sigma_a(w)$. Thus, in effect, $\Sigma_a(w)$ encodes both the information available to a and the issues entertained by a at w. This means that the map Σ_a suffices as a specification of the state

resolved. This may be regarded as a generalization to issues of the usual *ex falso quodlibet* principle.

of the agent a at each world, encompassing both information and issues. We do not have to list σ_a explicitly as an independent component in the definition of an inquisitive epistemic model: we can simply use $\sigma_a(w)$ as an abbreviation for $\bigcup \Sigma_a(w)$, keeping in mind that this set of worlds represents the information state of agent a in w. We thus arrive at the following definition.

Definition 3.2 (Inquisitive epistemic models)**.**
An inquisitive epistemic model is a triple $M = \langle \mathcal{W}, V, \Sigma_{\mathcal{A}} \rangle$ where:

- $\mathcal{W}$ is a set, whose elements will be called possible worlds.
- $V : \mathcal{W} \to \wp(\mathcal{P})$ is a *valuation map* that specifies for every world w which atomic sentences are true at w.
- $\Sigma_{\mathcal{A}} = \{\Sigma_a \mid a \in \mathcal{A}\}$ is a set of *state maps* $\Sigma_a : \mathcal{W} \to \Pi$, each of which assigns to any world w an issue $\Sigma_a(w)$, in accordance with:

  Factivity : for any $w \in \mathcal{W}$, $w \in \sigma_a(w)$

  Introspection : for any $w, v \in \mathcal{W}$, if $v \in \sigma_a(w)$, then $\Sigma_a(v) = \Sigma_a(w)$

  where $\sigma_a(w) := \bigcup \Sigma_a(w)$ represents the *information state* of agent a in w.

Of the two conditions placed on the state maps, the factivity condition is just as before, ensuring that the agents' information states are truthful. The introspection condition now concerns both information and issues: agents must be introspective in that they must know not only what information they have, but also what issues they entertain. That is, if the state of a in world w differs from the state of a in v, either in information or in issues, then a must be able to tell w and v apart.

As the reader will have noticed, there is a striking similarity between inquisitive epistemic models and standard epistemic models. In both frameworks, a model consists simply of a set of worlds, each equipped with (i) a valuation for atomic sentences and (ii) a state for each agent. The only difference is that while in standard epistemic logic the agents' states describe just their information, in the present setting they encompass both their information and their issues.

3.1.2 Logical language

So far we have introduced and motivated a notion of inquisitive epistemic models, to serve as semantic structures for our framework. The next step is to define a logical language that will enable us to talk about such models. As issues play a prominent role in our semantic picture, we want to endow our language with sentences whose meaning is *inquisitive*, i.e., can be identified not with a piece of information, but with an issue. We will do this by extending the usual declarative language of epistemic logic with sentences of a new syntactic category, the category of *interrogatives*.[7]

[7]This is not the only way in which inquisitive sentences can be introduced in the picture. In inquisitive semantics, it is actually common practice to assume a language that

As will be immediately clear from the syntax of our language, interrogatives will not only play a role in their own right, but they will also play a role as components of larger sentences, as they may be embedded under various modal operators. Therefore, due to the presence of interrogatives, the declarative fragment of our language will also be richer than usual. The set $\mathcal{L}_!$ of declaratives and the set $\mathcal{L}_?$ of interrogatives are laid out by simultaneous recursion.

Throughout the paper, we will adopt the following convention concerning meta-variables: α, β, γ (possibly with an index) range over declaratives, μ, ν, λ range over interrogatives, and φ, ψ, χ range over the whole language.

Definition 3.3 (Syntax)**.**
Let $\mathcal{P}$ be a set of atomic sentences and let $\mathcal{A}$ be a set of agents.

1. For any $p \in \mathcal{P}$, $p \in \mathcal{L}_!$
2. $\bot \in \mathcal{L}_!$
3. If $\alpha_1, \ldots, \alpha_n \in \mathcal{L}_!$, then $?\{\alpha_1, \ldots, \alpha_n\} \in \mathcal{L}_?$
4. If $\varphi \in \mathcal{L}_\circ$ and $\psi \in \mathcal{L}_\circ$, then $\varphi \wedge \psi \in \mathcal{L}_\circ$, where $\circ \in \{!, ?\}$
5. If $\alpha \in \mathcal{L}_!$ and $\varphi \in \mathcal{L}_\circ$, then $\alpha \to \varphi \in \mathcal{L}_\circ$, where $\circ \in \{!, ?\}$
6. If $\varphi \in \mathcal{L}_\circ$ for $\circ \in \{!, ?\}$ and $a \in \mathcal{A}$, then $K_a\varphi \in \mathcal{L}_!$
7. If $\varphi \in \mathcal{L}_\circ$ for $\circ \in \{!, ?\}$ and $a \in \mathcal{A}$, then $E_a\varphi \in \mathcal{L}_!$
8. Nothing else belongs to either $\mathcal{L}_!$ or $\mathcal{L}_?$

We start out by classifying atomic sentences and the falsum as declarative sentences. The third clause allows the construction of a basic interrogative $?\{\alpha_1, \ldots, \alpha_n\}$ from an arbitrary sequence $\alpha_1, \ldots, \alpha_n$ of declarative sentences. Notice that the declaratives $\alpha_1, \ldots, \alpha_n$ do not have to be atomic; in particular, they may in turn contain interrogatives as sub-constituents.

The fourth clause allows us to conjoin two sentences of the same category to obtain a sentence of the same category.[8] The fifth clause states that a sentence of

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does *not* make a categorical distinction between declaratives and interrogatives (see, e.g., Ciardelli, 2009; Groenendijk and Roelofsen, 2009; Ciardelli and Roelofsen, 2011; Ciardelli et al., 2012). However, the inquisitive semantic framework can equally naturally be applied to a bi-categorical language (Groenendijk, 2011; Ciardelli et al., 2013b). A detailed comparison of the two approaches, as well as meaning-preserving translations between the two resulting formal systems, are provided in Ciardelli et al. (2013b). Here, we choose to spell out our proposal for a bi-categorial language for two reasons. First, it seems that the intuitions are somewhat easier to get across this way. And second, assuming a distinction between declaratives and interrogatives makes it easier to compare our proposal to others, in particular that of van Benthem and Minică (2012), which will be done in section 4.

[8] Since our system assumes a strict partition of sentences into declaratives and interrogatives, hybrid conjunctions like $p \wedge ?q$ are not included in our logical language. Such conjunctions do in fact occur quite widely in natural language, both as standalone sentences and embedded under modal operators (e.g., *Ann is coming, but is Bill coming as well?*, *I know that Ann is coming and whether Bill is coming as well*), and can be handled straightforwardly in the standard hybrid system of inquisitive semantics (see the references in footnote 7).

either category may be conditionalized by a declarative antecedent, resulting in a conditional sentence of the same category. As usual, for any two declaratives α and β, we will use $\neg\alpha$ as an abbreviation of $\alpha \to \bot$ and $\alpha \vee \beta$ as an abbreviation of $\neg(\neg\alpha \wedge \neg\beta)$. Furthermore, a basic interrogative of the form $?\{\alpha, \neg\alpha\}$ will be referred to as a *polar interrogative* and will be abbreviated as $?\alpha$.

The last two clauses introduce the two modalities that we will consider. Since agents are equipped with both information and issues, we will consider a *knowledge* modality K_a, which will allow us to talk about the agents' information, and an *entertain* modality E_a, which will allow us to talk about the issues that the agents entertain. Notice that both modalities are allowed to embed sentences of either category. Our knowledge modality will coincide with its standard counterpart when its complement is a declarative. However, it is more flexible, since its complement may also be an interrogative. This enables us to construct sentences like $K_a?p$, expressing the fact that *a knows whether p*. The entertain modality E_a on the other hand, is specifically designed to talk about issues, and as such does not have a counterpart in standard epistemic logic. Once the semantics of our language has been laid out, we will see that the entertain modality, in combination with the knowledge modality, allows us to express facts such as *agent a wonders whether p*.

Finally, let us point out once more how the definitions of declaratives and interrogatives are intertwined. The interrogative operator ? forms basic interrogatives out of declaratives, from which more complex interrogatives may then be constructed. On the other hand, the modalities form declaratives out of sentences of either kind, including interrogatives. This allows us to construct sentences such as $K_a?K_b?p$, expressing complex facts like *a knows whether b knows whether p*.

3.1.3 Semantics

Our next task is to provide an interpretation for the sentences of our logical language. This is a crucial step in our enterprise, since this is where we need to go beyond the usual techniques of epistemic logic, relying fundamentally on insights from inquisitive semantics.

In epistemic logic, like in any other modal logic, sentences are interpreted relative to a world in a model. The semantics recursively specifies the conditions under which a sentence is true at a given world. This is a suitable approach as long as we consider only declarative sentences. But our language also contains interrogatives, for which a truth-conditional semantics just does not seem viable. To know the meaning of an interrogative sentence is not to understand under what conditions it is true, but rather to understand what information it takes to resolve it. Thus, the natural evaluation points for interrogatives are information states, rather than worlds, and the meaning of an interrogative should be taken to consist in its *resolution conditions* rather than its truth conditions.[9]

[9]We will see in a moment that, while we can make sense of the notion of truth for interrogatives, the truth conditions of an interrogative sentence—unlike those of a declarative sentence—do not completely determine its semantics.

At first sight, it may seem that this forces us to develop a double-face semantics in which declaratives are evaluated at worlds and interrogatives at states. However, there is a solution which is both conceptually more elegant and formally much more efficient. Namely, as is commonly done in inquisitive semantics, we will lift the interpretation of declaratives from the level of worlds to the level of information states as well, in a way that will allow us to promptly recover truth conditions if needed. This will enable us to provide a uniform semantic treatment of all sentences in our language. The following definition specifies recursively when a sentence is *supported* by a state s. Intuitively, for declaratives being supported amounts to being *established*, or *true everywhere* in s, while for interrogatives it amounts to being *resolved* in s.

Definition 3.4 (Semantics)**.**
Let M be an inquisitive epistemic model and let s be an information state in M.

1. $\langle M, s\rangle \models p \iff p \in V(w)$ for all worlds $w \in s$
2. $\langle M, s\rangle \models \bot \iff s = \emptyset$
3. $\langle M, s\rangle \models ?\{\alpha_1, \ldots, \alpha_n\} \iff \langle M, s\rangle \models \alpha_i$ for some index $1 \leq i \leq n$
4. $\langle M, s\rangle \models \varphi \land \psi \iff \langle M, s\rangle \models \varphi$ and $\langle M, s\rangle \models \psi$
5. $\langle M, s\rangle \models \alpha \to \varphi \iff$ for any $t \subseteq s$, if $\langle M, t\rangle \models \alpha$ then $\langle M, t\rangle \models \varphi$
6. $\langle M, s\rangle \models K_a\varphi \iff$ for any $w \in s$, $\langle M, \sigma_a(w)\rangle \models \varphi$
7. $\langle M, s\rangle \models E_a\varphi \iff$ for any $w \in s$ and for any $t \in \Sigma_a(w)$, $\langle M, t\rangle \models \varphi$

Before turning to a detailed explanation of the support clauses, we will first highlight some general properties of the semantics and define, in terms of support, the proposition expressed by a sentence, as well as appropriate notions of truth, entailment, and equivalence. First, we note that the given semantics ensures that support is *persistent*: if a state s supports a sentence φ, then any more informed state $t \subseteq s$ also supports φ.

Fact 3.5 (Persistency of support)**.**
If $\langle M, s\rangle \models \varphi$ and $t \subseteq s$, then $\langle M, t\rangle \models \varphi$

Second, we note that the absurd state $\emptyset$ always supports any sentence.

Fact 3.6 (The empty state supports everything)**.**
For any M and any φ, $\langle M, \emptyset\rangle \models \varphi$

The support conditions for negation, disjunction, and polar interrogatives, which were defined in terms the other operators, are as follows:

Fact 3.7 (Support for negation, disjunction, and polar interrogatives)**.**

8. $\langle M, s\rangle \models \neg\alpha \iff$ for any non-empty $t \subseteq s$, $\langle M, t\rangle \not\models \alpha$

9. $\langle M, s\rangle \models \alpha \vee \beta \iff$ there exist t_1, t_2 s.t. $s = t_1 \cup t_2$, $\langle M, t_1\rangle \models \alpha$ and $\langle M, t_2\rangle \models \beta$

10. $\langle M, s\rangle \models ?\alpha \iff \langle M, s\rangle \models \alpha$ or $\langle M, s\rangle \models \neg\alpha$

Now let us show that from our support based semantics we can straightforwardly recover a notion of truth, which is entirely classical as far as the declarative fragment of our language is concerned. We mentioned above that for any declarative sentence α, $\langle M, s\rangle \models \alpha$ may be read as "α is true in any world in s". This intuition suggests the following definition of truth at a world.

Definition 3.8 (Truth)**.**
We say that a sentence φ is *true* at a world w in a model M, and write $\langle M, w\rangle \models \varphi$, if and only if φ is supported by the state $\{w\}$ in M. In short:

$$\langle M, w\rangle \models \varphi \iff \langle M, \{w\}\rangle \models \varphi$$

The support conditions for singleton states, then, tell us how the truth of a complex sentence depends on the support conditions of its immediate constituents.

Fact 3.9 (Truth-conditions)**.**

1. $\langle M, w\rangle \models p \iff p \in V(w)$
2. $\langle M, w\rangle \not\models \bot$
3. $\langle M, w\rangle \models ?\{\alpha_1, \ldots, \alpha_n\} \iff \langle M, w\rangle \models \alpha_i$ for some index $1 \leq i \leq n$
4. $\langle M, w\rangle \models \varphi \wedge \psi \iff \langle M, w\rangle \models \varphi$ and $\langle M, w\rangle \models \psi$
5. $\langle M, w\rangle \models \alpha \to \varphi \iff \langle M, w\rangle \not\models \alpha$ or $\langle M, w\rangle \models \varphi$
6. $\langle M, w\rangle \models \neg\alpha \iff \langle M, w\rangle \not\models \alpha$
7. $\langle M, w\rangle \models \alpha \vee \beta \iff \langle M, w\rangle \models \alpha$ or $\langle M, w\rangle \models \beta$
8. $\langle M, w\rangle \models K_a\varphi \iff \langle M, \sigma_a(w)\rangle \models \varphi$
9. $\langle M, w\rangle \models E_a\varphi \iff$ for any $t \in \Sigma_a(w)$, $\langle M, t\rangle \models \varphi$

Notice that the truth-functional behavior of the propositional connectives is entirely classical (even when their arguments are interrogatives). Moreover, in the case of the propositional connectives, the truth conditions of complex sentences are fully determined by the truth conditions of their immediate constituents. This is no longer the case for the modalities, whose truth conditions crucially depend on the support conditions of the embedded sentence.

In terms of truth we define the *truth-set* of a sentence in a model.

Definition 3.10 (Truth set)**.**
The *truth set* of a sentence φ in a model M, denoted $|\varphi|_M$, is defined as the set of worlds in M where φ is true:

$$|\varphi|_M := \{w \in \mathcal{W} \mid \langle M, w\rangle \models \varphi\}$$

In general, the truth-conditions of a sentence do not determine its support conditions. To see this, notice that the two interrogatives $?p$ and $?q$ have the same truth conditions (namely, both are true at all worlds) but different support conditions: $?p$ is supported by a state s in case $s \subseteq |p|_M$ or $s \subseteq |\neg p|_M$, while $?q$ is supported by s in case $s \subseteq |q|_M$ or $s \subseteq |\neg q|_M$.

However, if we restrict ourselves to declarative sentences, then truth-conditions *do* completely determine support conditions. Indeed, the previously stated characterization of support for declaratives can now be formally stated and proved: a declarative is supported by a state just in case it is true at all worlds in that state.

Fact 3.11 (Truth and support)**.**
For any model M, any state s and any declarative α, the following holds:

$$\langle M, s\rangle \models \alpha \iff \langle M, w\rangle \models \alpha \text{ for all } w \in s$$

Proof. The left-to-right implication follows directly from persistence. The other direction can be established by induction on the complexity of α, where in the base case α is taken to be either atomic or headed by one of the modalities. $\square$

This connection between truth and support ensures that the meaning of a declarative is still completely determined by its truth conditions w.r.t. single worlds. Given this fact, one may wonder what the benefit is of defining truth in terms of support, rather than vice versa. Notice, however, that it is not possible to simply define truth *independently* of support, since the truth-conditions for certain declaratives depend on the support conditions for interrogatives occurring within them, which in turn are not fully determined by their truth-conditions.

Thus, the alternative to our uniform support semantics would be a simultaneous recursive definition of truth for declaratives and support for interrogatives. This would be a rather cumbersome endeavor, especially when one realizes that it would require two separate clauses for conjunction, two clauses for implication, and two clauses for each modal operator. Our definition assigns a uniform type of meaning to all sentences. On the practical level, this simplifies many definitions that should otherwise go by cases. On the conceptual level, it concurs with the main tenet of inquisitive semantics, which is that informative and inquisitive content should be brought under the umbrella of one unified notion of meaning. Notice that the logical operators that apply to both declaratives and interrogatives—conjunction, implication, and the modalities—do so *uniformly*, that is, the same semantic clause takes care of both cases. We believe that this points to an interesting semantic uniformity of these operations.[10]

In standard epistemic logic, a sentence is evaluated relative to possible worlds. Accordingly, the proposition that it expresses is a set of worlds, namely, the set of all worlds where the sentence is true. In our framework, instead, a sentence is evaluated relative to states. Thus, the proposition that a sentence expresses is a set of states, namely, the set of all states that support the sentence.

[10] In the case of the connectives, this uniformity is brought out in even fuller generality in the standard hybrid system of inquisitive semantics (see the references in footnote 7).

Definition 3.12 (Propositions)**.**
The proposition $[\varphi]_M$ expressed by a sentence φ in a model M is the set of all states in M that support φ:

$$[\varphi]_M := \{s \subseteq \mathcal{W} \mid s \models \varphi\}$$

By Fact 3.5 and 3.6, the proposition expressed by a sentence is always a non-empty, downward closed set of states, i.e., an *issue* in the sense of definition 3.1. The truth-set of a sentence always coincides with the union of the proposition that the sentence expresses.

Fact 3.13 (Propositions and truth-sets)**.**
For any sentence φ and any model M, $|\varphi|_M = \bigcup[\varphi]_M$.

Proof. Suppose $w \in |\varphi|_M$. This means that $\langle M, w\rangle \models \varphi$ which, by definition of truth, amounts to $\langle M, \{w\}\rangle \models \varphi$. Thus, $\{w\} \in [\varphi]_M$, whence we get $w \in \bigcup[\varphi]_M$. Conversely, suppose $w \in \bigcup[\varphi]_M$, that is, suppose $w \in s$ for some state s such that $\langle M, s\rangle \models \varphi$. Since $\{w\} \subseteq s$, the persistency of support gives $\langle M, \{w\}\rangle \models \varphi$, which amounts to $\langle M, w\rangle \models \varphi$, i.e., to $w \in |\varphi|_M$. $\square$

We have defined the notion of truth for all sentences, declaratives and interrogatives. But, intuitively, what does it mean for an interrogative to be *true* at a world? The previous fact allows us to answer this question. We said that, for interrogatives, support conditions amount to resolution conditions: an interrogative μ is supported at a state s in case s contains enough information to resolve μ. The previous fact asserts that a world w makes a sentence φ true just in case it belongs to a state that supports the sentence. Thus, a world w makes an interrogative μ true just in case it belongs to some state s where μ is resolved. So, the worlds where an interrogative is true are precisely those at which the interrogative can be (truthfully) resolved.[11]

To every interrogative μ we can associate a declarative π_μ which shares the same truth conditions. We can regard this declarative as encoding the information needed to guarantee that μ can be truthfully resolved, or, in the terminology introduced above, as encoding the information *assumed* by the issue $[\mu]_M$. For reasons that will become clear later on, we refer to π_μ as the *presupposition* of μ.

Definition 3.14 (Presupposition of an interrogative)**.**

- $\pi_{?\{\alpha_1,\ldots,\alpha_n\}} = \alpha_1 \vee \cdots \vee \alpha_n$
- $\pi_{\mu\wedge\nu} = \pi_\mu \wedge \pi_\nu$
- $\pi_{\alpha\to\mu} = \alpha \to \pi_\mu$

A straightforward induction suffices to check that, indeed, the presupposition of an interrogative always has the same truth-conditions as the interrogative itself.

[11] This complies with the suggestion that Belnap ended his 1966 paper *Questions, answers, and presuppostions* with: "I should like in conclusion to propose the following linguistic reform: that we all start calling a question 'true' just when some direct answer thereto is true."

Fact 3.15. For any interrogative μ and any model M, $|\mu|_M = |\pi_\mu|_M$.

The following fact identifies a large class of interrogatives which are always true at every world (and whose presupposition is in fact a classical tautology). This class contains all polar interrogatives and is closed under conjunction and conditionalization.[12] The proof of this fact, which amounts to checking truth conditions, is omitted.

Fact 3.16. For any model M, the following holds:

- for any declarative α, $|?\alpha|_M = \mathcal{W}$
- if $|\mu|_M = \mathcal{W}$ and $|\nu|_M = \mathcal{W}$, then $|\mu \wedge \nu|_M = \mathcal{W}$
- if $|\mu|_M = \mathcal{W}$, then $|\alpha \to \mu|_M = \mathcal{W}$

Entailment, validity, and equivalence between sentences are defined in the natural way. Notice that these notions now apply uniformly to sentences of either category, allowing us to regard declarative entailment, interrogative entailment, and mixed entailments as special instances of one and the same notion.[13]

Definition 3.17 (Entailment)**.**
We say that a sentence φ *entails* another sentence ψ, notation $\varphi \models \psi$, just in case for all models M and states s, if $\langle M, s\rangle \models \varphi$ then $\langle M, s\rangle \models \psi$.

Definition 3.18 (Validity)**.**
We say that a sentence φ is *valid* in case it is supported by all states in all models.

Definition 3.19 (Equivalence)**.**
We say that two sentences φ and ψ are *equivalent*, notation $\varphi \equiv \psi$, just in case for all models M and states s, $\langle M, s\rangle \models \varphi \iff \langle M, s\rangle \models \psi$.

With these basic semantic notions in place, let us now turn back to the support clauses of our semantics. We will go through the clauses one by one and, where necessary, explain the underlying ideas in more detail. Clause 1 and 2 are rather self-explanatory: the former says that an atomic declarative sentence is established in a state just in case it is true at all worlds in that state, while the latter says that the absurd declarative $\bot$ is established only in the empty, inconsistent state.

Clause 3 says that a basic interrogative $?\{\alpha_1, \ldots, \alpha_n\}$ is resolved in a state just in case one of $\alpha_1, \ldots, \alpha_n$ is established in that state. Thus, the interrogative $?\{\alpha_1, \ldots, \alpha_n\}$ can be thought of as requesting enough information to establish at least one of $\alpha_1, \ldots, \alpha_n$. As a simple example consider the polar interrogative $?p$. A state s in a model M supports $?p$ just in case it supports either p or $\neg p$.

[12] If we would *restrict* the interrogative fragment of our system to this class of interrogatives, we would arrive at (a modal extension of) a system known as InqC, whose properties are discussed in some detail in Ciardelli et al. (2013b).

[13] For a simple propositional language augmented with interrogatives, this cross-categorial notion of entailment has been investigated in detail and axiomatized in Ciardelli et al. (2013b).

The former holds iff p is true in all worlds in s, the latter iff p is false in all worlds in s.

Clause 4 is again quite transparent: a conjunction is established (resolved) in a state just in case both conjuncts are established (resolved).

Clause 5 for implication deserves some more attention. We first note that this clause is equivalent to the more perspicuous Clause 5′ below, which reduces the assessment of a conditional in a state to the assessment of the consequent in a state enhanced with the information that the antecedent is true.

5′. $\langle M, s\rangle \models \alpha \to \varphi \iff \langle M, s \cap |\alpha|_M\rangle \models \varphi$

In words: a conditional $\alpha \to \varphi$ is established (resolved) in a state s if and only if the consequent φ is established (resolved) in the state resulting from enhancing s with the information that the antecedent α is true. For concrete illustration, consider the following conditional question:

(1) If Ann invites Bill to the party, will he go? $p \to ?q$

A state s in a model M supports $p \to ?q$ just in case $s \cap |p|_M$ supports $?q$. This is the case if $s \cap |p|_M$ supports q or if it supports $\neg q$. In the first case s has to support $p \to q$, and in the second case s has to support $p \to \neg q$. Thus, a state supports $p \to ?q$ just in case it supports either $p \to q$ or $p \to \neg q$. The latter two sentences correspond exactly to the two basic answers to our conditional question:[14]

(2) a. Yes, if Ann invites Bill, he will go. $p \to q$
b. No, if Ann invites Bill, he will not go. $p \to \neg q$

Finally, let us take a closer look at the semantics of the modal operators. First consider the knowledge operator K_a. Since $K_a\varphi$ is always a declarative, Fact 3.11 ensures that in order to understand its meaning we just need to look at its truth conditions at worlds. Now, Clause 6 says that $K_a\varphi$ is true at a world w just in case φ is supported by the state $\sigma_a(w)$, which encodes the information available to a in w. Given Fact 3.11, for a declarative α this means that $K_a\alpha$ is true at w iff α is true everywhere in $\sigma_a(w)$, i.e. true in every world compatible with the information of a in w. So, when applied to declaratives, K_a boils down to the familiar knowledge modality of epistemic logic. On the other hand, for an interrogative μ, the clause says that $K_a\mu$ is true in w just in case μ is *resolved* in $\sigma_a(w)$, which means that $K_a\mu$ expresses the fact that a has sufficient information to resolve μ at w. For instance, $K_a?p$ is true at a

[14] Notice that two states supporting $p \to q$ and $p \to \neg q$, respectively, may *overlap* (they may both contain worlds where p is false). For this reason, conditional questions have always been notoriously problematic for theories of questions that model issues as *partitions* of the logical space (e.g. Groenendijk and Stokhof, 1984). This problem no longer arises in inquisitive semantics since its notion of issues is more general than the partition notion. Conditional questions have played an important motivational role in the development of inquisitive semantics (see, e.g., Mascarenhas, 2009; Groenendijk, 2011; Ciardelli et al., 2013a). We will return to this point when comparing our proposal with that of van Benthem and Minică (2012) in section 4.

world w just in case $\sigma_a(w)$ supports either p or $\neg p$, that is, just in case a knows *whether* p holds. Notice that this treatment of interrogatives embedded under the knowledge operator is in no way restricted to polar interrogatives: it applies to more complex interrogatives as well, such as the conditional interrogative in (1) above, and it would extend straightforwardly to the first-order setting, allowing us to deal with embedded *wh*-questions.

Now let us consider the entertain operator E_a. Again, since the sentence $E_a\varphi$ is always a declarative, we just need to consider truth conditions. According to Clause 7, $E_a\varphi$ is true at w just in case φ is supported by any state $t \in \Sigma_a(w)$. Remember that $\Sigma_a(w)$ encodes the inquisitive state of a at w, and that the elements of $\Sigma_a(w)$ are precisely those enhancements of the information state of a at w where the issues of a are resolved. Thus, $E_a\varphi$ says that as soon as the private issues of a are resolved, φ is supported.

Note that for any φ, $K_a\varphi$ entails $E_a\varphi$, since whenever $\langle M, \sigma_a(w)\rangle \models \varphi$ we also have that $\langle M, t\rangle \models \varphi$ for any $t \in \Sigma_a(w)$, by persistence of support and the fact that any $t \in \Sigma_a(w)$ is a subset of $\sigma_a(w) = \bigcup \Sigma_a(w)$.

Fact 3.20. For any sentence φ, $K_a\varphi \models E_a\varphi$.

Further note that for any declarative α, $K_a\alpha$ and $E_a\alpha$ are in fact equivalent. To see that $E_a\alpha$ entails $K_a\alpha$, recall that a declarative α is supported by a state s just in case it is true in every world in s. This implies that if α is supported by every $t \in \Sigma_a(w)$ then it is also true in any world $w \in \bigcup \Sigma_a(w) = \sigma_a(w)$, and thus supported by $\sigma_a(w)$.

Fact 3.21. For any declarative α, $E_a\alpha \equiv K_a\alpha$.

So, for declaratives, E_a simply boils down to K_a. However, things become interesting when E_a is applied to an interrogative μ. According to Clause 7, $E_a\mu$ is true just in case resolving the private issues of a entails resolving μ, or, speaking more informally, just in case all the states in which the curiosity of a is satisfied are states where μ is resolved. This is close to saying that *a wonders* about μ, except for one case: if a *already* has enough information to resolve μ, i.e., if $K_a\mu$ holds, then by Fact 3.20, $E_a\mu$ will be true as well. But in such a scenario, we would not say that *a wonders* about μ.[15] We can characterize the situation of an agent a wondering about μ as one where the agent does not yet have sufficient information to resolve μ (so that $\neg K_a\mu$ holds) but the states she wants to get to are states that do contain such information (so that $E_a\mu$ holds). In short, a wonders about μ if she does not know about μ but she wants to know about μ. So, we can introduce a defined *wonder* modality W_a as follows:

$$W_a\varphi := \neg K_a\varphi \land E_a\varphi$$

By means of this operator, we can construct sentences like $W_a?K_bW_a?p$, expressing subtle facts like *a wonders whether b knows that she wonders whether p*.

[15] Of course, we would also not naturally say that *a entertains* μ in that case: although we read $E_a\varphi$ as "a entertains φ", this should be understood as technical terminology.

Notice that if the W_a modality is applied to a declarative, given that $K_a\alpha$ and $E_a\alpha$ are equivalent in this case, it immediately results in a contradiction, in tune with the intuition that one just cannot wonder *that* p.

Since $K_a\varphi$ entails $E_a\varphi$, the operator E_a is expressible in terms of K_a and W_a.

Fact 3.22. For any φ, $E_a\varphi \equiv K_a\varphi \vee W_a\varphi$.

This equivalence gives us a way to "read" the meaning of the sentence $E_a\mu$ where μ is an interrogative: $E_a\mu$ holds just in case a either already has, or would like to obtain, enough information to resolve μ.

This concludes our illustration of the support clauses. Hopefully, the reader has a reasonably clear picture by now of the meanings that are assigned to sentences in our system. We now take a step back to make a few comments on the mathematical workings of the system. We start by noticing that our modal operators K_a and E_a are not standard, Kripke modalities. That is, they cannot be regarded as quantifiers asserting the truth of their argument at certain worlds. However, there is a sense in which these operators work in our system precisely the way Kripke modalities do in standard modal logic.

Recall that in standard epistemic logic, as we remarked in section 2, the modality K_a can be taken to express a relation between the state $\sigma_a(w)$ and the proposition $|\varphi|_M$ expressed by its argument. In the particular case of K_a, the relation simply amounts to inclusion:

$$\langle M, w\rangle \models K_a\varphi \iff \sigma_a(w) \subseteq |\varphi|_M$$

All modal operators of standard modal logic can be seen as working in this way: they express a relation between two sets of worlds, a set of worlds associated with the world of evaluation, and the proposition expressed by the sentence that the operator takes as its argument.

Our modal operators K_a and E_a work in exactly the same way: they express a relation between the state $\Sigma_a(w)$ and the proposition $[\varphi]_M$. The only difference is that we take these two semantic objects to be of a different type than in standard modal logic. We argued that, in order to capture the issues that agents may entertain and the inquisitive content that sentences may have, both the states assigned to agents and the propositions expressed by sentences should be non-empty downward closed sets of information states. Now, since such entities are more structured than simple sets of worlds, several relations may turn out to carry an intuitive significance. The modalities K_a and E_a express two of these relations, as is brought out more clearly by the following equivalent reformulations of Clauses 6 and 7 above.[16]

6′. $\langle M, w\rangle \models K_a\varphi \iff \bigcup \Sigma_a(w) \in [\varphi]_M$

[16] We give *truth* conditions here, rather than *support* conditions, to bring out the analogy with standard modal logic more clearly. Since $K_a\varphi$ and $E_a\varphi$ are declaratives, Fact 3.11 ensures that they are supported by a state s just in case they are true at every world in s.

7′. $\langle M, w\rangle \models E_a\varphi \iff \Sigma_a(w) \subseteq [\varphi]_M$

We conclude this section by remarking that the inquisitive epistemic logic proposed here is a *conservative extension* of standard epistemic logic. To see this, notice that any inquisitive epistemic model $M = \langle W, V, \Sigma_{\mathcal{A}}\rangle$ immediately induces a standard epistemic model $M^e = \langle W, V, \sigma_{\mathcal{A}}\rangle$, obtained simply by forgetting about issues, and replacing each state map Σ_a by the information state map σ_a that it determines (i.e., for any w, $\sigma_a(w) = \bigcup \Sigma_a(w)$). Now, it can be shown that the truth-conditions that our semantics assigns to any sentence in the language of standard epistemic logic in a model M boil down precisely to the truth-conditions assigned to this sentence by standard epistemic logic in the model M^e.

Fact 3.23.
Let M be an inquisitive epistemic model, w a world. If α is a sentence in the language of standard EL, then α is also a declarative in the language of IEL, and we have:

$$\langle M, w\rangle \models \alpha \iff \langle M^e, w\rangle \models \alpha$$

Proof. By induction on the syntax of sentences in EL. For illustration, let us spell out the induction step for the epistemic modality K_a (the other steps are immediate). By induction hypothesis, α is true in a world w in M iff it is true in w in M^e, that is, we have $|\alpha|_M = |\alpha|_{M^e}$. Applying Facts 3.9 and 3.11, as well as our induction hypothesis, we have: $\langle M, w\rangle \models K_a(\alpha) \iff \langle M, \sigma_a(w)\rangle \models \alpha \iff \sigma_a(w) \in [\alpha]_M \iff \sigma_a(w) \subseteq |\alpha|_M \iff \sigma_a(w) \subseteq |\alpha|_{M^e} \iff \langle M^e, w\rangle \models \alpha$. □

Thus, as far as the language of standard epistemic logic is concerned, IEL coincides with EL. At the same time, IEL allows us to talk not only about the things that an agent knows (e.g., $K_a p$), but also about the issues that an agent can resolve (e.g. $K_a ?p$) and the issues that an agent entertains (e.g., $W_a ?p$). Moreover, as we will see, the presence of interrogatives in our language will provide the fundamental tools for issue-raising actions in a dyamics of information exchange.

3.1.4 Common knowledge and public issues

Besides the information and issues that are private to each agent, agents also share certain public information and jointly entertain certain issues. In section 2 we saw how the common knowledge construction in epistemic logic allows us to derive a public information map σ_* representing the information that is publicly available to all the agents, starting from the epistemic maps σ_a encoding the information available to the individual agents. The question is whether this construction can be generalized to the present setting. That is, is it possible to derive a public state map Σ_*, encoding public information and issues, from the maps Σ_a describing the information and issues of the individual agents?

One way to go about answering this question is to consider, as we did in the case of common knowledge, the conditions that a public entertain modality E_*

associated with the map Σ_* would have to satisfy. Indirectly, this will then put constraints on the definition of Σ_*. So, let us consider what it would mean for a sentence to be *publicly entertained*. In standard epistemic logic, φ is publicly known in case every agent knows that φ, and every agent knows that every agent knows that φ, and so on ad infinitum. Analogously, it seems natural to say that φ is *publicly entertained* in case every agent entertains φ, and every agent knows that every agent entertains φ, and every agent knows that every agent knows, etcetera. Thus, the behavior of the public entertain modality E_* would have to be subject to the following condition:

$$\langle M, s\rangle \models E_*\varphi \iff \langle M, s\rangle \models K_{a_1} \dots K_{a_{n-1}} E_{a_n}\varphi \text{ for all } a_1 \dots a_n \in \mathcal{A},\ n \geq 0$$

If one finds the alternation of the modalities puzzling, there is no need to worry: since K_a and E_a are equivalent with declarative arguments, and since any sentence that starts with a modality is a declarative, we can simply replace all the K_a's with E_a and obtain the equivalent "homogeneous" condition:

$$\langle M, s\rangle \models E_*\varphi \iff \langle M, s\rangle \models E_{a_1} \dots E_{a_n}\varphi \text{ for all } a_1 \dots a_n \in \mathcal{A},\ n \geq 0$$

Does this condition put precisely enough constraints on the map Σ_* to characterize it uniquely? The answer is *yes*. One can verify that the above condition on E_* holds for any particular valuation V if and only if the map Σ_* is defined as follows:

$$\begin{aligned}\Sigma_*(w) = \{s \quad | \quad & \text{there exist } v_0, \dots, v_n \in \mathcal{W} \text{ and } a_0, \dots, a_n \in \mathcal{A} \\ & \text{such that } v_0 = w,\ v_{i+1} \in \sigma_{a_i}(v_i) \text{ for all } i < n, \text{ and } s \in \Sigma_{a_n}(v_n)\}\end{aligned}$$

Importantly, the public information map σ_* corresponding to the public state map Σ_*, defined as $\sigma_*(w) := \bigcup \Sigma_*(w)$, coincides exactly with the map we would obtain by performing the common knowledge construction on the individual information maps σ_a. Thus, the standard common knowledge construction from epistemic logic generalizes smoothly and elegantly to a 'public state' construction which encompasses both information and issues.

Given this construction, we can add modalities K_* and E_* to our logical language, and interpret them as follows:

8. $\langle M, s\rangle \models K_*\varphi \iff$ for any $w \in s$, $\sigma_*(w) \in [\varphi]_M$
9. $\langle M, s\rangle \models E_*\varphi \iff$ for any $w \in s$, $\Sigma_*(w) \subseteq [\varphi]_M$

If α is a declarative, then $K_*\alpha$ gets its standard meaning, expressing that α is common knowledge. It is then easy to see that Fact 3.23 extends to this richer language: just like IEL is a conservative extension of EL, IEL enriched with public state modalities is a conservative extension of EL enriched with the common knowledge modality.

On the other hand, in our setting K_* also applies to interrogatives: if μ is an interrogative, then $K_*\mu$ says that enough information is publicly available to resolve μ or, in other words, that μ is *publicly settled* in the exchange. Moreover, our language is equipped with a public entertain modality E_* which, in combination with K_*, allows us to define a public wonder modality. A group of agents $\mathcal{A}$ jointly wonder about φ if they publicly entertain φ and φ is not yet publicly settled.

$$W_*\varphi := \neg K_*\varphi \wedge E_*\varphi$$

Like the modality K_*, the modality W_* plays a crucial role in describing the state of affairs in an information exchange: while K_* talks about what is settled in the exchange, W_* talks about what the group as a whole is wondering about, that is, what the *open issues* are in the exchange.

Interestingly, $W_*\mu$ does not entail $W_a\mu$ for any particular agent a. While this may come as a surprise at first, it is just as it should be: if $W_*\mu$ holds, then φ is publicly entertained but not publicly settled, that is, the common knowledge of the group does not settle μ. It may well be that there is some agent a whose *private* knowledge does settle μ. This does not prevent μ from being an open issue in the conversation, so long as a's private information is not made publicly available. In fact, $W_*\mu$ might even be the case while *every* individual agent can resolve μ, but the information needed to resolve μ has not been made common knowledge: although the issue is settled for each individual agent in this case, it is still open for the group as a whole.

Finally, notice that by means of the common knowledge map, the public state map admits of a rather straightforward characterization:

$$\Sigma_*(w) = \bigcup\nolimits_{v\in\sigma_*(w)\,,\,a\in\mathcal{A}} \Sigma_a(v)$$

This entails the following connection between the public entertain modality and common knowledge: φ is publicly entertained just in case every agent entertains φ and this fact is common knowledge. That is, if $\mathcal{A} = \{a_1, \ldots, a_n\}$ then:

$$E_*\varphi \equiv K_*(E_{a_1}\varphi \wedge \cdots \wedge E_{a_n}\varphi)$$

3.2 Dynamics

So far we have designed new models to represent situations where private and public issues are present, alongside private and public information, and we have provided a suitable logical language to talk about such situations. Now the time has come to dynamify this picture, describing how situations change when certain actions are performed, and how our language provides the means for such actions.

3.2.1 Public announcements

As we did in our overview of standard DEL, we will limit our discussion here to just one, basic type of action: public announcement.

In DEL, public announcements establish new common knowledge. More specifically, given a model M, the public announcement of a sentence φ is taken to have the effect of making the proposition $|\varphi|_M$ expressed by φ in M common knowledge.[17]

In our system, both information and issues are present. By publicly announcing a sentence, an agent may establish new common knowledge and raise new public issues. These two effects fall out of a uniform principle, which is a natural generalization of the treatment of public announcements in DEL: the effect of publicly announcing a sentence φ in a model M is to make the proposition $[\varphi]_M$ expressed by φ in M publicly entertained.

Technically, we achieve this by intersecting all the inquisitive states $\Sigma_a(w)$ with $[\varphi]_M$. This means that the information state of each agent is enhanced with the information that the actual world lies in $|\varphi|_M$, while their inquisitive state is enhanced with the goal to reach one of the states in $[\varphi]_M$.[18] Worlds where φ was false to begin with should be removed from the model in order to ensure that the resulting maps will satisfy the factivity condition, and the valuation map should be restricted accordingly.[19] Hence, a public announcement of φ will transform an inquisitive epistemic model $M = (\mathcal{W}, V, \Sigma_\mathcal{A})$ into the model $M^\varphi = (\mathcal{W}^\varphi, V^\varphi, \Sigma_\mathcal{A}{}^\varphi)$ defined as follows:

- $\mathcal{W}^\varphi = \mathcal{W} \cap |\varphi|_M$
- $V^\varphi = V \restriction_{\mathcal{W}^\varphi}$
- $\Sigma_\mathcal{A}{}^\varphi = \{\Sigma_a{}^\varphi \mid a \in \mathcal{A}\}$, where for every $w \in \mathcal{W}^\varphi$: $\Sigma_a{}^\varphi(w) = \Sigma_a(w) \cap [\varphi]_M$

The following fact says that the information state $\sigma_a{}^\varphi(w)$ of an agent at a world in the updated model is obtained just like in standard DEL, by restricting the original epistemic state $\sigma_a(w)$ to the proposition $|\varphi|_M$ expressed by φ in the original model.

Fact 3.24. For any sentence φ, agent a, model M and world w we have that:

$$\sigma_a{}^\varphi(w) = \sigma_a(w) \cap |\varphi|_M$$

Proof. Notice that if S is a downward closed set of information states, $w \in \bigcup S \iff \{w\} \in S$. Since $\Sigma_a(w), \Sigma_a{}^\varphi(w)$ and $[\varphi]_M$ are all downward closed, we have:

$$\begin{aligned} v \in \sigma_a{}^\varphi(w) \quad &\iff \quad \{v\} \in \Sigma_a{}^\varphi(w) \iff \{v\} \in \Sigma_a(w) \cap [\varphi]_M \\ &\iff \quad \{v\} \in \Sigma_a(w) \text{ and } \{v\} \in [\varphi]_M \\ &\iff \quad v \in \sigma_a(w) \text{ and } v \in |\varphi|_M \iff v \in \sigma_a(w) \cap |\varphi|_M \end{aligned}$$

[17]This means that, after the announcement, it is common knowledge that the world is located in $|\varphi|_M$. Technically, for any world w in the model M^φ resulting from the announcement, we will have $\sigma_*{}^\varphi(w) \subseteq |\varphi|_M$. As discussed in section 2, this does *not* mean that the formula $K_*\varphi$ necessarily holds in the updated model. For, after the update, φ may come to express a *different* proposition than it previously did, that is, we do not necessarily have $|\varphi|_{M^\varphi} = |\varphi|_M \cap W^\varphi$.

[18]For the update of a single inquisitive state, understood as the public state, this dynamic picture is put forward and motivated in detail in Ciardelli et al. (2013a).

[19]Notice that, like in standard DEL, an update does not change the truth value of atomic sentences at worlds, which reflects the fact that atoms are intended to model facts that are not themselves epistemic in nature.

□

We can use this fact to ensure that our update operation turns inquisitive epistemic models into inquisitive epistemic models.

Fact 3.25. For any inquisitive epistemic model M and any sentence φ, M^φ is an inquisitive epistemic model.

Proof. First, notice that ${\Sigma_a}^\varphi(w)$ is a non-empty, downward closed set of information states, as it should be. For, $\Sigma_a\varphi(w) = \Sigma_a(w) \cap [\varphi]_M$, and the intersection of two non-empty, downward closed sets of information states is itself non-empty and downward closed.

To see that the updated maps ${\Sigma_a}^\varphi$ satisfy the factivity condition, consider a world $w \in \mathcal{W}^\varphi = \mathcal{W} \cap |\varphi|_M$. Since the original map satisfies factivity by assumption, we have $w \in \sigma_a(w)$, and since $w \in |\varphi|_M$ we can conclude $w \in \sigma_a(w) \cap |\varphi|_M = {\sigma_a}^\varphi(w)$.

Finally, we have to check that the updated maps ${\Sigma_a}^\varphi$ satisfy the introspection condition. Suppose $v \in {\sigma_a}^\varphi(w) = \sigma_a(w) \cap |\varphi|_M$ Since the original map satisfies introspection by assumption, $v \in \sigma_a(w)$ implies $\Sigma_a(v) = \Sigma_a(w)$, whence ${\Sigma_a}^\varphi(v) = \Sigma_a(v) \cap [\varphi]_M = \Sigma_a(w) \cap [\varphi]_M = {\Sigma_a}^\varphi(w)$. □

In order to familiarize ourselves with the effects of public announcements, let us consider in turn the announcement of a declarative and of an interrogative. First consider a declarative α. We know from Fact 3.11 that $[\alpha]_M = \{s \subseteq \mathcal{W} \mid s \subseteq |\alpha|_M\}$. Thus, recalling from page 9 that the *restriction* of an issue I to an information state s is defined as $I\restriction_s := \{t \in I \mid t \subseteq s\}$, we have that:

$$ {\Sigma_a}^\alpha(w) = \Sigma_a(w) \cap [\alpha]_M = \{s \in \Sigma_a(w) \mid s \subseteq |\alpha|_M\} = \Sigma_a(w)\restriction_{|\alpha|_M} $$

That is, the inquisitive state of an agent at a world after the announcement of α is nothing but the restriction of the original state to the set of worlds in M where α is true. Thus, there is nothing more to the announcement of a declarative than there used to be in standard DEL: as a consequence of the announcement of α, worlds where α was false are removed from the model, and all the agents' states are restricted accordingly.

Now consider the case in which the announced sentence is an interrogative μ. As before, worlds where μ is false are eliminated from the model. In this respect, an announcement of μ behaves just like an announcement of its presupposition π_μ. However, now this is not all that there is to the announcement. For any agent a and world w, the announcement enhances the inquisitive state $\Sigma_a(w)$ by intersecting it with $[\mu]_M$, that is, it makes it a goal for a to reach a state where μ is resolved. This, then, is how the announcement of an interrogative can raise new issues.[20]

In this way, our uniform semantics of declaratives and interrogatives is put to use in a dynamics that allows for a unified treatment of providing information

[20] In section 3.2.4 we will develop a more realistic dynamic picture, in which the announcement of an interrogative does not *provide* the information that its presupposition is true, but rather requires such information to be publicly established prior to the announcement.

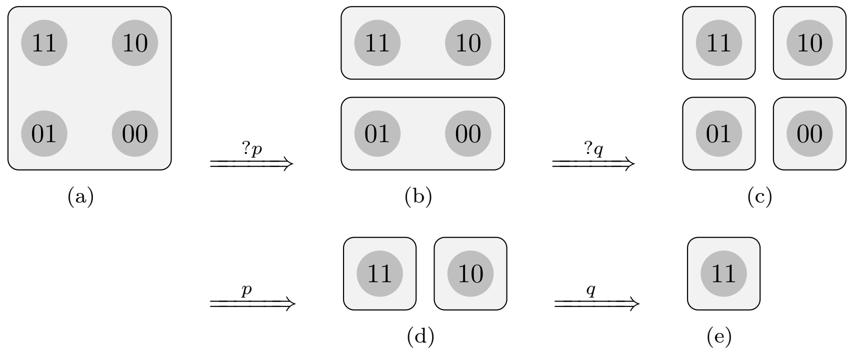

Figure 2: The effects of a series of simple announcements on a state.

and raising issues. Although it may be convenient to talk of *asserting* for the act of announcing a declarative sentence, and of *asking* for the act of announcing an interrogative sentence, asserting and asking are in this view not two intrinsically different kinds of speech act, but rather one and the same speech act performed with two different kinds of sentences.[21]

Let us illustrate the effects of public announcements graphically for some very simple cases. Consider a language with just two atomic sentences, p and q, and a model consisting of just four worlds, $W = \{11, 10, 01, 00\}$, such that 11 makes both p and q true, 10 makes p true and q false, 01 makes p false and q true, and 00 makes both p and q false. Suppose that there is just one agent, a, and suppose that initially, in any $w \in W$, a's inquisitive state $\Sigma_a(w)$ is embodied by the trivial issue over W, which is depicted in Figure 2(a). Throughout this example, a's inquisitive state will always be the same for any $w \in W$, so we will simply denote it by Σ_a. We visualize inquisitive states by depicting only their maximal elements (just as we did for issues in Figure 1 on page 9). In this case, since Σ_a is embodied by the trivial issue over W, there is just one maximal element, which is W itself. This means that initially a has no (non-trivial) information and no (non-trivial) issues.

Now suppose that a polar interrogative $?p$ is publicly announced. To capture the effect of this announcement, Σ_a needs to be intersected with the proposition expressed by $?p$, which consist of all information states that support either p or $\neg p$. The resulting inquisitive state is depicted in Figure 2(b).

Next suppose that another polar interrogative, $?q$, is publicly announced. To capture the effect of this announcement, Σ_a needs to be further intersected with the proposition expressed by $?q$, which consists of all information states that support either q or $\neg q$. The resulting inquisitive state is depicted in Figure 2(c). Notice that a's *information* state, i.e., $\sigma_a = \bigcup \Sigma_a$, has not changed:

[21] We will return to this important point when comparing our proposal with that of van Benthem and Minică (2012) in section 4.

no worlds have been eliminated, which means that no information has been gained. However, a's inquisitive state has been enhanced in a non-trivial way, capturing the fact that two issues have been raised: the issue whether p, and the issue whether q. The resulting inquisitive state consists precisely of those information states that resolve both these issues.

Now suppose that the declarative sentence p is publicly announced. The effect of this announcement is depicted in Figure 2(d). Now a's information state σ_a *does* change: all worlds where p is false are eliminated. This resolves one of the open issues, i.e., whether p. However, the resulting inquisitive state reflects that the other public issue, whether q, is still open.

Finally, suppose that the declarative sentence q is publicly announced. This leads to the inquisitive state in Figure 2(e). Again, σ_a changes through this announcement: all worlds where q is false are eliminated. This resolves the issue whether q, and leads to a situation in which a no longer entertains any (non-trivial) issues.

3.2.2 Extending the logical language

To be able to talk about the effects of a public announcement not just in the meta-language but also in the object language, we extend the latter, as is commonly done in DEL, with a dynamic modal operator which allows us to talk about what is the case after a public announcement has been performed. For any sentence φ, we introduce a corresponding dynamic modality $[\varphi]$ that can be applied to a sentence of either category—declarative or interrogative—to yield a sentence of that same category. That is, we extend the syntax of our language with the following clause:

- if $\varphi \in \mathcal{L}_! \cup \mathcal{L}_?$ and $\psi \in \mathcal{L}_\bullet$, then $[\varphi]\psi \in \mathcal{L}_\bullet$, where $\bullet \in \{!, ?\}$

Semantically, assessing a sentence $[\varphi]\psi$ at a pair $\langle M, s\rangle$ amounts to assessing ψ at the pair $\langle M^\varphi, s \cap |\varphi|_M\rangle$ consisting of the model resulting from the announcement of φ and the information state resulting from enhancing s with the information provided by φ.

$$\langle M, s\rangle \models [\varphi]\psi \iff \langle M^\varphi, s \cap |\varphi|_M\rangle \models \psi$$

From this support clause we recover as a special case the truth conditions which are familiar form our discussion of standard DEL.

$$\langle M, w\rangle \models [\varphi]\psi \iff w \notin |\varphi|_M \text{ or } \langle M^\varphi, w\rangle \models \psi$$

A straightforward check reveals that what we established so far about declaratives (essentially Fact 3.11 and its consequences, such as the identity ${\Sigma_a}^\alpha(w) = \Sigma_a(w) \restriction_{|\alpha|_M}$) remains valid for declaratives in the language enriched with dynamic modalities. Since support for declaratives is still determined by truth conditions, the semantics of a declarative $[\varphi]\alpha$ can be understood in terms of truth conditions, which are the standard ones we encountered in our overview of standard DEL in Section 2.2: $[\varphi]\alpha$ is true in w in case a truthful announcement of φ in w is impossible, because φ is false in w, or in case such an announcement is possible and α is true after the announcement has taken place. In particular, this means that our system equipped with dynamic modalities is a conservative extension of the standard public announcement logic discussed in Section 2.2.

However, in our system the dynamic modality can be prefixed not only to declaratives, like in standard DEL, but also to interrogatives. If μ is an interrogative, then $[\varphi]\mu$ should be thought of as a "dynamically conditionalized" interrogative, which asks to resolve μ under the assumption not just that φ were true, but that φ were actually announced. Thus, for instance, $[p]?K_aq$ encodes the question: "if p were announced, would a know that q?". Contrast this with the simple conditional interrogative $p \to ?K_aq$, which encodes the question "if p is true, does a know that q?". To know that $K_a(p \to q)$, for instance, is sufficient information to resolve the former interrogative, but not the latter.

Notice that the definition of the presupposition of an interrogative needs to be extended to cover interrogatives of the form $[\varphi]\mu$ as well. If we let $\pi_{[\varphi]\mu} = [\varphi]\pi_\mu$, it is immediate to verify that Fact 3.15 above extends to the language enriched with dynamic modalities.

3.2.3 Factive announcements

We said that after an announcement of φ, the proposition that φ expressed in the original model comes to be publicly entertained: for any world w of the updated model we have $\Sigma_a{}^\varphi(w) \subseteq [\varphi]_M$, which also implies that for any world, $\Sigma_*{}^\varphi(w) \subseteq [\varphi]_M$ holds. However, this does not in general mean that $E_*\varphi$ will hold in the updated model. For, like in standard DEL, the update may change the states associated with each world, and so in the updated model φ can come to express a different proposition than it did in the original model. Since, as we saw, the announcement of a declarative works just like in standard DEL, the declarative $p \wedge \neg K_*p$ discussed above is still a case in point: after being publicly announced, this sentence becomes false, and thus not publicly entertained.

However, we will see that this phenomenon does not arise with sentences that do not contain any occurrence of the modalities K_a, E_a, K_* and E_*. We will refer to such sentences as *factive* sentences. The following fact establishes that a factive sentence is supported by a state in an updated model iff it was supported by the same state in the original model.

Fact 3.26. Let φ be any sentence and let ψ be a factive sentence. Then for any state $s \subseteq \mathcal{W}^\varphi$:

$$\langle M^\varphi, s\rangle \models \psi \iff \langle M, s\rangle \models \psi$$

Proof. For sentences not containing the dynamic modality, a straightforward induction on the complexity of ψ suffices to establish the claim. Moreover, any factive sentence can be proven to be equivalent to one that does not contain any dynamic modality. To see this, consider a sentence $[\varphi]\psi$, where φ and ψ are factive sentences which do not contain a dynamic modality. First suppose φ is a declarative α: since we already know that the equivalence we are proving holds for ψ, we have $\langle M, w\rangle \models [\alpha]\psi \iff \langle M^{\alpha}, \{w\} \cap |\alpha|_M\rangle \models \psi \iff \langle M, \{w\} \cap |\alpha|_M\rangle \models \psi \iff \langle M, w\rangle \models \alpha \to \psi$. Thus, $[\alpha]\psi$ is equivalent with $\alpha \to \psi$. On the other hand, if φ is not a declarative but an interrogative μ, then a similar argument shows that $[\mu]\psi$ is equivalent with $\pi_\mu \to \psi$. Using these equivalences inductively, one can turn an arbitrary factive sentence into an equivalent one without dynamic modalities, which in turn shows that our claim holds for all factive sentences. □

The previous Fact tells us that for any model M, sentence φ and factive sentence ψ, we have $[\psi]_{M^\varphi} = [\psi]_M \restriction_{|\varphi|}$. In particular, when $\varphi = \psi$, $[\varphi]_M \restriction_{|\varphi|}$ boils down to $[\varphi]_M$, and so we have the following fact.

Fact 3.27. If φ is factive, then for any model M, $[\varphi]_{M^\varphi} = [\varphi]_M$.

Now consider the model M^φ resulting from the announcement of a factive φ. At any world $w \in \mathcal{W}^\varphi$ we have by construction that $\Sigma_a{}^\varphi(w) \subseteq [\varphi]_M$ and thus by the previous fact $\Sigma_a{}^\varphi(w) \subseteq [\varphi]_{M^\varphi}$. And since this is true for any agent a and for any world $w \in \mathcal{W}^\varphi$, it follows by definition of $\Sigma_*{}^\varphi$ that we must also have $\Sigma_*{}^\varphi(w) \subseteq [\varphi]_{M^\varphi}$ at each world w. But this means that $E_*\varphi$ will be supported by any state (and thus, in particular, true at any world) in the updated model.

Fact 3.28. If φ is a factive sentence, then for any $s \subseteq \mathcal{W}^\varphi$:

$$\langle M^\varphi, s\rangle \models E_*\varphi$$

So, for factive sentences φ it does hold that a public announcement of φ has the effect of making φ publicly entertained. As a consequence, the following is a validity for any factive φ:

$$[\varphi]E_*\varphi$$

Now, if φ is a declarative α, then we know that $E_*\alpha \equiv K_*\alpha$. Thus, announcing a factive declarative has the effect of making it common knowledge, just like in standard DEL.

Fact 3.29. If α is a factive declarative, then for any $s \subseteq \mathcal{W}^\alpha$:

$$\langle M^\alpha, s\rangle \models K_*\alpha$$

As a consequence, the following sentence is valid for any factive declarative α:

$$[\alpha]K_*\alpha$$

Obviously, the same principle does not apply to factive interrogatives: a mere public announcement of μ does not in general lead to a state where μ is publicly settled. What one may expect, rather, is that after being publicly announced, a factive interrogative becomes an open issue, that is, one may expect that $W_*\mu$ would hold in the updated model. However, this is not always the case: for, although Fact 3.28 ensures that μ will be publicly entertained after the announcement, nothing ensures that μ will not be publicly settled. That could come about in two ways: first, μ may be already publicly settled prior to the announcement, in which case, since μ is factive, it will remain settled after the announcement. Second, μ could be publicly settled by the very information that it provides. As an example, consider a pair $\langle M, w\rangle$ such that $\langle M, w\rangle \models q \wedge \neg K_* q \wedge K_* \neg p$, and let μ be the factive interrogative $?\{p \wedge \neg q, \neg p \wedge q\}$. It is easy to see that $\langle M, w\rangle \models \neg K_* \mu \wedge [\mu] K_* \mu$: for, the announcement of μ establishes common knowledge that exactly one of p and q is true, which, together with the previous common knowledge that $\neg p$ (which, being factive, is not affected by the announcement) creates common knowledge of $\neg p \wedge q$, which is sufficient to resolve μ.

However, if the common knowledge of the group prior to the announcement is sufficient to establish the presupposition of a certain factive interrogative μ, but is not sufficient to resolve μ, then indeed announcing μ has the effect of making μ an open issue. In order to see this, we will first prove the following fact, which holds for any sentence φ: if φ is already common knowledge at a world, then a public announcement of φ does not enhance any information state at that world.

Fact 3.30.
Let M be a model, w a world in M and φ a sentence.
If $\sigma_*(w) \subseteq |\varphi|_M$, then ${\sigma_a}^\varphi(w) = \sigma_a(w)$ for any $a \in \mathcal{A}$, and ${\sigma_*}^\varphi(w) = \sigma_*(w)$.

Proof. First, if $\sigma_*(w) \subseteq |\varphi|_M$, then since $w \in \sigma_*(w)$ we have $w \in |\varphi|_M$, which means that w is also a world in the updated model M^φ. Consider first the case of a private state ${\sigma_a}^\varphi(w)$. Since $\sigma_a(w) \subseteq \sigma_*(w) \subseteq |\varphi|_M$, Fact 3.24 yields

$${\sigma_a}^\varphi(w) = \sigma_a(w) \cap |\varphi|_M = \sigma_a(w)$$

Now consider common knowledge. Obviously, ${\sigma_*}^\varphi(w) \subseteq \sigma_*(w)$. Viceversa, suppose $v \in \sigma_*(w)$: then there is a sequence $u_0, \ldots, u_{n+1}$ of worlds and a sequence $a_1, \ldots, a_n$ of agents with $u_0 = w$, $u_{n+1} = v$ and $u_{i+1} \in \sigma_{a_i}(u_i)$ for $0 \leq i \leq n$. But it is easy to prove by induction that at each u_i we must have $\sigma_*(u_i) \subseteq |\varphi|_M$, whence by what we just established for private maps, $\sigma_{a_i}(u_i) = {\sigma_{a_i}}^\varphi(u_i)$. So, we have a sequence $u_0, \ldots, u_{n+1}$ of worlds and a sequence $a_1, \ldots, a_n$ of agents with $u_0 = w$, $u_{n+1} = v$ and $u_{i+1} \in {\sigma_{a_i}}^\varphi(u_i)$ for $0 \leq i \leq n$, which means that $v \in {\sigma_*}^\varphi(w)$. $\square$

With this fact in place we are ready to prove the fact mentioned before: if the presupposition of a factive interrogative μ is common knowledge, but μ is not publicly settled, then announcing μ results in μ becoming an open issue.

Fact 3.31. If μ is a factive interrogative, then the following is a validity:

$$(K_* \pi_\mu \wedge \neg K_* \mu) \rightarrow [\mu] W_* \mu$$

Proof. Since the implication is a declarative, by Fact 3.11 we just have to make sure that it is true in any world of any model. So, suppose $\langle M, w\rangle \models K_* \pi_\mu \wedge \neg K_* \mu$. Since $\langle M, w\rangle \models K_* \pi_\mu$, we must have that $\sigma_*(w) \in [\pi_\mu]_M$, which by

Fact 3.11 amounts to $\sigma_*(w) \subseteq |\pi_\mu|_M = |\mu|_M$. But, by factivity, $w \in \sigma_*(w)$, and thus $w \in |\mu|_M$, which means that w is a world in M^μ.

Now, since μ is factive, by Fact 3.28 we know that $\langle M^\mu, w\rangle \models E_*\mu$. Moreover, since $\sigma_*(w) \subseteq |\mu|_M$, the previous Fact yields ${\sigma_*}^\mu(w) = \sigma_*(w)$. On the other hand, the assumption $\langle M, w\rangle \models \neg K_*\mu$ means that $\sigma_*(w) \notin [\mu]_M$, whence it follows ${\sigma_*}^\mu(w) \notin [\mu]_M$. Finally, since $[\mu]_M = [\mu]_{M^\mu}$ by Fact 3.27, we obtain ${\sigma_*}^\mu(w) \notin [\mu]_{M^\mu}$, which means that $\langle M^\mu, w\rangle \models \neg K_*\mu$. Putting things together, we have $\langle M^\mu, w\rangle \models \neg K_*\mu \wedge E_*\mu$, that is, $\langle M^\mu, w\rangle \models W_*\mu$, which implies $\langle M, w\rangle \models [\mu]W_*\mu$, as we set out to show. □

Notice that Fact 3.16 tells us that factive *polar* interrogatives, as well as complex interrogatives that are obtained from them by conjunction and conditionalization, have tautological presuppositions. In this case, the condition $K_*\pi_\mu$ is trivially satisfied, and the above validity can be simplified to $\neg K_*\mu \to [\mu]W_*\mu$, which expresses that, unless μ is already publicly settled, announcing μ will result in μ becoming an open issue in the exchange.

3.2.4 Division of labor

In the system we described so far, the announcement of a sentence may both provide new information, and raise new issues. However, in many natural languages (including English) these communicative tasks seem to be quite rigidly divided between declaratives and interrogatives: generally, information is provided by uttering declaratives, and issues are raised by uttering interrogatives. To make IDEL faithful to actual linguistic exchange in this respect, announcements may be constrained according to a principle that we may call *division of communicative labor*: announcing a declarative is appropriate in a given context only if it provides new information and does not raise any new issues; viceversa, announcing an interrogative is appropriate only if it raises new issues and does not provide any new information.

To formulate this principle more precisely, we need to specify when a sentence is informative and when it is inquisitive. We will say that a sentence φ is *informative* in a world w in case an announcement of φ in w enhances the public information state in w.

Definition 3.32 (Informativeness)**.**
A sentence φ is *informative* in a world w in case ${\sigma_*}^\varphi(w) \subset \sigma_*(w)$.

Second, we will say that a sentence φ is *inquisitive* in a world w in case the public issues after an announcement of φ are not just the restriction of the old public issues to the new public information state resulting from the announcement. In other words, φ is inquisitive at a world in case an announcement of φ would not just enhance the public information state (if it does at all) but also makes it harder to reach a state where the public issues are settled.

Definition 3.33 (Inquisitiveness)**.**
A sentence φ is *inquisitive* in a world w in case ${\Sigma_*}^\varphi(w) \subset \Sigma_*(w)\restriction_{{\sigma_*}^\varphi(w)}$.

To understand these notions better, we will prove two facts that characterize when an announcement of φ brings about a change in common knowledge, or in the public state in general. The first of these facts says that an announcement of φ establishes new common knowledge at w in case it is not already common knowledge that φ is true.

Fact 3.34. φ is informative in $w \iff \sigma_*(w) \not\subseteq |\varphi|_M$.

Proof. If $\sigma_*(w) \not\subseteq |\varphi|_M$ then since ${\sigma_*}^{\varphi}(w) \subseteq \mathcal{W}^{\varphi} = |\varphi|_M$ we cannot have ${\sigma_*}^{\varphi}(w) = \sigma_*(w)$, and since obviously ${\sigma_*}^{\varphi}(w) \subseteq \sigma_*(w)$, we must have ${\sigma_*}^{\varphi}(w) \subset \sigma_*(w)$. The converse direction was proved as part of Fact 3.30. $\square$

The second fact says that an announcement of φ enhances the public state at w just in case φ is not already publicly entertained at w.

Fact 3.35. ${\Sigma_*}^{\varphi}(w) \subset \Sigma_*(w) \iff \Sigma_*(w) \not\subseteq [\varphi]_M$.

Proof. If $\Sigma_*(w) \not\subseteq [\varphi]_M$, then since ${\Sigma_*}^{\varphi}(w) \subseteq [\varphi]_M$ by construction of the updated model, ${\Sigma_*}^{\varphi}(w)$ cannot be equal to $\Sigma_*(w)$, and so we must have ${\Sigma_*}^{\varphi}(w) \subset \Sigma_*(w)$. Viceversa, suppose $\Sigma_*(w) \subseteq [\varphi]_M$. Then $\sigma_*(w) = \bigcup \Sigma_*(w) \subseteq \bigcup [\varphi]_M = |\varphi|_M$, so by the previous lemma ${\sigma_*}^{\varphi}(w) = \sigma_*(w)$. But if $s \in \Sigma_*(w)$, then there exists a $v \in \sigma_*(w)$ such that $s \in \Sigma_a(v)$ for some agent a. Now, since ${\sigma_*}^{\varphi}(w) = \sigma_*(w)$, we also have $v \in {\sigma_*}^{\varphi}(w)$. Moreover, since $v \in \sigma_*(w)$, it follows from the definition of the public state map that $\Sigma_a(v) \subseteq \Sigma_*(w)$, so our assumption implies $\Sigma_a(v) \subseteq [\varphi]_M$, whence ${\Sigma_a}^{\varphi}(v) = \Sigma_a(v) \cap [\varphi]_M = \Sigma_a(v)$. Putting these pieces together, we have $v \in {\sigma_*}^{\varphi}(w)$ and $s \in {\Sigma_a}^{\varphi}(v)$, which means that $s \in {\Sigma_*}^{\varphi}(w)$. $\square$

The notions of informativeness and inquisitiveness allow us to articulate our principle of division of labor more precisely.[22]

Definition 3.36 (Principle of division of communicative labor)**.**

- The announcement of a declarative α is appropriate w.r.t. a world w in case α is informative and not inquisitive in w.
- The announcement of an interrogative μ is appropriate w.r.t. a world w in case μ is inquisitive and not informative in w.

Let us now examine what these conditions amount to for declaratives and interrogatives. First, we will show that a declarative is never inquisitive.

Fact 3.37. A declarative sentence is never inquisitive in a world.

[22] Notice that the principle as formulated here incorporates a *non-redundancy* requirement: an announcement is only appropriate at a world in case it enhances the public state. We could choose to separate out non-redundancy from division of labor proper, which would then amount simply to the following: the announcement of a declarative α is appropriate in a world only if α is non-inquisitive, while the announcement of an interrogative μ is appropriate in a world only if μ is non-informative.

Proof. We have to show that, for arbitrary model M and world w, ${\Sigma_*}^\alpha(w)$ cannot be a proper subset of $\Sigma_*(w)\upharpoonright_{{\sigma_*}^\alpha(w)}$. To show this, take a state $s \in \Sigma_*(w)\upharpoonright_{{\sigma_*}^\alpha(w)}$, that is, $s \in \Sigma_*(w)$ and $s \subseteq {\sigma_*}^\alpha(w)$. We want to prove that $s \in {\Sigma_*}^\alpha(w)$.

If $s = \emptyset$, then $s \in {\Sigma_*}^\alpha(w)$ is trivially true, so we may assume without loss of generality that s is non-empty. Now, $s \in \Sigma_*(w)$ means that there is a world $v \in \sigma_*(w)$ and an agent a such that $s \in \Sigma_a(v)$. Now, let u be any world in s: since $s \in \Sigma_a(v)$, we have $u \in \sigma_a(v)$, whence by introspection we get $\Sigma_a(u) = \Sigma_a(v)$. But then $s \in \Sigma_a(u)$, and since we also have that $s \subseteq {\sigma_*}^\alpha(w) \subseteq |\alpha|_M$, it must be the case that $s \in {\Sigma_a}^\alpha(u) = \Sigma_a(u)\upharpoonright_{|\alpha|_M}$. Finally, since $u \in s$ and $s \subseteq {\sigma_*}^\alpha(w)$, we have that $u \in {\sigma_*}^\alpha(w)$. From $u \in {\sigma_*}^\alpha(w)$ and $s \in {\Sigma^\alpha}_a(u)$ it follows that $s \in {\Sigma_*}^\alpha(w)$, as required. □

All that our principle requires of a declarative α, then, is that it be informative in the world where it is uttered, which by Fact 3.34 amounts to $\sigma_*(w) \not\subseteq |\alpha|_M$. In turn, Fact 3.11 ensures that for any state s and declarative α, $s \subseteq |\alpha|_M \iff s \in [\alpha]_M$. So, the condition that α should be informative in w boils down to $\sigma_*(w) \notin [\alpha]_M$, that is, $\langle M, w\rangle \models \neg K_*\alpha$. In conclusion, announcing a declarative α is appropriate just in case α is not already common knowledge.

Fact 3.38 (Appropriateness for declaratives)**.**
Announcing a declarative α is appropriate w.r.t. a world w iff $\langle M, w\rangle \models \neg K_*\alpha$.

Now let us consider what it takes for the utterance of an interrogative μ to be appropriate w.r.t. a world w. First, μ should not be informative in w. This means that we should have ${\sigma_*}^\mu(w) = \sigma_*(w)$. By Fact 3.34, this is equivalent to $\sigma_*(w) \subseteq |\mu|_M = |\pi_\mu|_M$, where the last equality comes from Fact 3.15. Using Fact 3.11 once again, we have $\sigma_*(w) \subseteq |\pi_\mu|_M \iff \sigma_*(w) \in [\pi_\mu]_M \iff \langle M, w\rangle \models K_*\pi_\mu$. Thus, the first appropriateness condition for μ is that π_μ be common knowledge. This also explains why it is fitting to refer to π_μ as the *presupposition* of μ: π_μ encodes the information that has to be publicly established in a context in order for an announcement of μ to be appropriate.

Moreover, the principle requires that μ be inquisitive in w. This means that we should have ${\Sigma_*}^\mu(w) \subset \Sigma_*(w)\upharpoonright_{{\sigma_*}^\mu(w)}$. But, assuming that the first condition is met, that is, μ is not informative in w, we have ${\sigma_*}^\mu(w) = \sigma_*(w)$, and thus $\Sigma_*(w)\upharpoonright_{{\sigma_*}^\mu(w)} = \Sigma_*(w)\upharpoonright_{\sigma_*(w)} = \Sigma_*(w)$. The requirement that μ be inquisitive in w can then be simplified as ${\Sigma_*}^\mu(w) \subset \Sigma_*(w)$. By Fact 3.35, this holds if and only if $\Sigma_*(w) \not\subseteq [\mu]_M$, that is, if and only if $\langle M, w\rangle \models \neg E_*\mu$.

Thus, we have found that announcing an interrogative μ is appropriate w.r.t. a world w just in case (i) the presupposition π_μ of μ is common knowledge in w and (ii) μ is not already publicly entertained in w.

Fact 3.39 (Appropriateness for interrogatives)**.**
Announcing an interrogative μ is appropriate w.r.t. w iff $\langle M, w\rangle \models K_*\pi_\mu \wedge \neg E_*\mu$.

Since $E_*\mu \equiv K_*\mu \vee W_*\mu$, the second conjunct of the appropriateness condition can also be rewritten as $\neg K_*\mu \wedge \neg W_*\mu$: in order for an announcement of μ to be appropriate, μ should be neither already settled, nor already an open issue in the exchange.

3.2.5 A pragmatically sensitive announcement operator

Now let us consider how our logical language may be enriched to talk about the effects of public announcements in a way that takes appropriateness conditions into account. Presently, our logical language contains expressions of the form $[\varphi]\psi$, involving a dynamic modal operator that allows us to talk about the effects of a public announcement in a given situation. However, this operator does not take into account whether the announcement under consideration is appropriate in the given situation. To overcome this limitation, we further extend our logical language with a second, 'pragmatically sensitive' announcement operator, $[.]^p$. Just like the dynamic modality $[\varphi]$, its pragmatically sensitive version $[\varphi]^p$ may be prefixed to a sentence of either category, resulting in a new sentence of the same category.

- if $\varphi \in \mathcal{L}_! \cup \mathcal{L}_?$ and $\psi \in \mathcal{L}_\bullet$, then $[\varphi]^p\psi \in \mathcal{L}_\bullet$, where $\bullet \in \{!, ?\}$

Semantically, $[\varphi]^p\psi$ differs from $[\varphi]\psi$ in that the former does not only conditionalize ψ to the fact that φ is announced, but also to the assumption that the announcement of φ is appropriate in the first place. To formulate this support condition precisely, let us define the *appropriateness set* of a sentence in a model.

Definition 3.40 (Appropriateness set)**.**
The appropriateness set of φ in a model M, denoted $\lfloor\varphi\rfloor_M$, is the set of worlds in M where the announcement of φ is appropriate.

In order to check whether a pair $\langle M, s\rangle$ supports $[\varphi]\psi$, we first enhance s with the supposition that φ can indeed be appropriately announced, obtaining $s \cap \lfloor\varphi\rfloor_M$, and then check whether the announcement of φ on $\langle M, s \cap \lfloor\varphi\rfloor_M\rangle$ leads to a pair that supports ψ. Thus, the support conditions for $[\varphi]^p\psi$ can be concisely formulated as follows:

$$\langle M, s\rangle \models [\varphi]^p\psi \iff \langle M, s \cap \lfloor\varphi\rfloor_M\rangle \models [\varphi]\psi$$

Or, spelling out the support conditions for $[\varphi]\psi$:

$$\langle M, s\rangle \models [\varphi]^p\psi \iff \langle M^\varphi, s \cap \lfloor\varphi\rfloor_M \cap |\varphi|_M\rangle \models \psi$$

This clause yields the following truth conditions for $[\varphi]^p\psi$:

$$\langle M, w\rangle \models [\varphi]^p\psi \iff w \notin \lfloor\varphi\rfloor_M \cap |\varphi|_M \text{ or } \langle M^\varphi, w\rangle \models \psi$$

Thus, $[\varphi]^p\psi$ is considered trivially true in case the announcement of φ is not appropriate or (just like in standard DEL) not truthful. If the announcement of

φ is truthful and appropriate, then we go on to check whether ψ is true after the announcement has taken place.

Notice that, by persistency, the support conditions for $[\varphi]^p\psi$ are strictly less demanding than those for $[\varphi]\psi$, so that the following implication is valid:

$$[\varphi]\psi \to [\varphi]^p\psi$$

To illustrate the difference in logical behavior between $[.]^p$ and $[.]$, recall from above (Fact 3.31) that the following is valid for every factive interrogative μ:

$$(K_*\pi_\mu \wedge \neg K_*\mu) \to [\mu]W_*\mu$$

In words: if the presupposition of μ is common knowledge but μ is not yet publicly settled, then announcing μ results in μ being a common open issue. Now, since announcing μ is only *appropriate* if the presupposition of μ is indeed common knowledge and μ is not yet publicly settled, any appropriate announcement of a factive interrogative μ results in μ being a common open issue. That is, for every factive interrogative μ, the following is valid:

$$[\mu]^p W_*\mu$$

This completes our discussion of the dynamic component of our system. Of course, many kinds of actions besides public announcements can and should be considered as well in order to model real scenarios of information exchange. For a start, our proposal could be refined to model various sorts of private announcements. Once the possibility of false information and disagreement is admitted, acceptance and rejection actions should also be made available for the addressees of an announcement. We will leave such refinements to future work. Our main goal here was to develop a basic inquisitive dynamic epistemic logic, in which information and issues are treated on a par, and to illustrate some fundamental aspects of the workings of such a system. In the next section, we will compare our approach with a recent alternative proposal.

4 Related work

Though questions only played a very marginal role in early work on dynamic epistemic logic (with Baltag, 2001, as a notable exception), they did receive considerable attention in more recent work (Unger and Giorgolo, 2008; van Eijck and Unger, 2010; Peliš and Majer, 2010, 2011; Ågotnes et al., 2011; van Benthem, 2011; Minică, 2011; van Benthem and Minică, 2012; Liu and Wang, 2013). One prominent framework that has emerged from this line of work is the *dynamic epistemic logic with questions* (DELQ) of van Benthem and Minică (2012). In this section we will provide an overview of DELQ and compare it to our own approach. We start in section 4.1 with the static component of DELQ. In section 4.2 we turn to its dynamic component, and in section 4.3 to the comparison with our proposal.

4.1 Epistemic logic with issues

The semantic structures that van Benthem and Minică consider are standard epistemic models enriched with a set of issues, one for each agent. Following Groenendijk and Stokhof (1984), issues are modeled as equivalence relations $\approx$ on the set of worlds. Such an equivalence relation may be equivalently regarded as a partition $\pi_{\approx}$ of the logical space, whose cells correspond to the possible answers that the issue admits of. For any world w, $\pi_{\approx}(w)$ is used to denote the unique cell of the partition containing w. Intuitively, $\pi_{\approx}(w)$ is the information state that results from minimally and truthfully resolving the issue $\approx$ in w.

To be faithful to the presentation of van Benthem and Minică, we shift here to the standard presentation of epistemic models which uses epistemic accessibility relations instead of epistemic maps.

Definition 4.1 (Epistemic issue models)**.**
An epistemic issue model M is a quadruple $\langle \mathcal{W}, V, \sim_{\mathcal{A}}, \approx_{\mathcal{A}} \rangle$, where:

- $\mathcal{W}$ is a set whose elements are called possible worlds
- $V : \mathcal{W} \to \wp(\mathcal{P})$ is a valuation function
- $\sim_{\mathcal{A}} = \{\sim_a \mid a \in \mathcal{A}\}$ is a set of equivalence relations on $\mathcal{W}$, called *epistemic relations*
- $\approx_{\mathcal{A}} = \{\approx_a \mid a \in \mathcal{A}\}$ is a set of equivalence relations on $\mathcal{W}$, called *issue relations*

The language that van Benthem and Minică use to describe their epistemic issue models is the standard language of epistemic logic enriched with a universal modality U, as well as a question modality Q_a and a resolution modality R_a for every agent a. These modalities are interpreted as follows.

1. $\langle M, w \rangle \models U\varphi$ iff $\langle M, v \rangle \models \varphi$ for all $v \in \mathcal{W}$
2. $\langle M, w \rangle \models Q_a\varphi$ iff $\langle M, v \rangle \models \varphi$ for all $v \in \mathcal{W}$ such that $w \approx_a v$
3. $\langle M, w \rangle \models R_a\varphi$ iff $\langle M, v \rangle \models \varphi$ for all $v \in \mathcal{W}$ such that $w \sim_a v$ and $w \approx_a v$

The universal modality, a standard tool in modal logic, talks about what is true at all worlds in the model. The question modality Q_a talks about what is true in all worlds $\approx_a$-equivalent to the evaluation world w, that is, all worlds in the state $\pi_{\approx_a}(w)$. We said above that this state represents the information state that would result from correctly resolving the issue $\approx_a$ at w. Thus, the question modality Q_a talks about what would be established if the issue entertained by a were (minimally and truthfully) resolved.

The resolution modality R_a, on the other hand, talks about what is true at all the worlds which are both $\sim_a$-equivalent and $\approx_a$-equivalent to w. These are the worlds that make up the information state resulting from pooling together the private information available to a at w and the information that would result from resolving a's issues at w. Thus, the resolution modality R_a talks about what agent a would know if her current issue were resolved.

Combining the modalities U and Q_a we can express facts about the issues that agent a entertains. For instance, consider the formula:

$$U(Q_a\varphi \vee Q_a\neg\varphi)$$

This formula says that any world w is such that, if a's private issues were resolved correctly at w, either φ or $\neg\varphi$ would be established. Thus, it says that resolving a's private issues necessarily involves establishing an answer to the question whether φ is the case. We can take this to be a description of what it means for a to entertain the issue whether φ.

4.2 Dynamics

We have seen how van Benthem and Minică's models, just like ours, include a description of private issues and information. Here, too, both components may be affected by agents performing certain actions. Van Benthem and Minică consider a number of actions. We will focus our attention on two of these, the most fundamental ones: the action of *publicly announcing* that φ is the case, denoted $\varphi!$, and the action of *publicly asking* whether φ is the case, denoted $\varphi?$. Recall that in our proposal there are two types of sentences—declaratives and interrogatives—and only one type of action—public announcement. By contrast, in DELQ there is only one type of sentence—declaratives—but there are two types of actions—announcing and asking. Let us see how these operations work.

A public announcement of φ transforms a model M into the model $M^{\varphi!}$ which differs from M only in the agents' epistemic relations. The new epistemic relation ${\sim_a}^{\varphi}$ for agent a is ${\sim_a} \cap \equiv^{\varphi}$, where $\equiv^{\varphi}$ is the relation holding between two worlds just in case φ has the same truth value in both worlds. Thus, a public announcement of φ has the effect of making it common knowledge whether φ holds.[23] Publicly asking whether φ is the case has a similar effect, but on the issue component of the model. That is, it tranforms a model M into the model $M^{\varphi?}$ which differs from M only in the agents' issue relations. The new issue relation ${\approx_a}^{\varphi}$ for agent a is ${\approx_a} \cap \equiv^{\varphi}$, where $\equiv^{\varphi}$ is as before. Thus, a public question whether φ has the effect of making it an open issue for all agents whether φ.

As customary, the system provides dynamic modalities $[\varphi!]$ and $[\varphi?]$ corresponding to these actions, whose semantics is given by the familiar scheme.

1. $\langle M, w\rangle \models [\varphi!]\psi \iff \langle M^{\varphi!}, w\rangle \models \psi$

[23] Notice that on this approach, a public announcement never removes any world from the model. This has the puzzling consequence that in a $\neg\varphi$-world, announcing φ has the effect of making $\neg\varphi$ common knowledge. This treatment of public announcements of declarative sentences is clearly different from the one we gave. However, since both systems are in principle compatible with either account of public announcements of declaratives, we do not take this difference to reflect an essential discrepancy between the two approaches.

2. $\langle M, w\rangle \models [\varphi?]\psi \iff \langle M^{\varphi?}, w\rangle \models \psi$

This concludes our essential tour of DELQ. We now turn to a comparison of the two proposals.

4.3 Comparison

As we saw, DELQ is very much in the same spirit as our inquisitive dynamic epistemic logic (IDEL for short): both systems are designed to model information exchange as a dynamic process in which agents request and provide information according to what they know and what they want to know. However, the two systems also present several important differences.

4.3.1 A non-difference

Let us start our comparison with something which is *not* a difference between the two proposals. As mentioned above, the most standard implementation of inquisitive semantics, which is referred to as InqB, assumes no syntactic distinction between declarative and interrogative sentences. Rather, it is based on a plain propositional language, whose connectives are associated with the natural algebraic operations on the space of inquisitive propositions. In particular, in InqB disjunction expresses the join operation on inquisitive propositions, and as such it has the sort of issue-raising behavior that, in the present setting, we have assigned to the question operator. The logic arising from this system is a (non-substitution closed) intermediate logic, that is, a logic stronger than intuitionistic logic but weaker than classical logic (Ciardelli and Roelofsen, 2011). In discussing the relation between DELQ and inquisitive semantics, van Benthem and Minică (2012, p.666) write that, in spite of similarities between the two systems,

> "[...] there is also a major difference. The 'inquisitive logic' matching inquisitive semantics is an intermediate logic with some intuitionistic, rather than classical features. By contrast, our dynamic logics are conservative extensions of classical propositional logic with new dynamic modalities for issue-changing actions."

This passage correctly points out a difference between DELQ and the standard inquisitive semantics system InqB. But, as a general approach to meaning, inquisitive semantics is not committed to the particular treatment of the connectives adopted in InqB. In the present paper, we opted for a formulation of inquisitive semantics in which inquisitive disjunction does not appear, and its role is delegated to sentences of a new syntactic category, the category of interrogatives. As a result, the declarative fragment of the system that we have developed has an entirely classical behavior. Thus, just like DELQ, IDEL is a conservative extension of classical propositional logic. Having neutralized this alleged source of divergence between DELQ and the inquisitive approach allows us to focus on what we take to be the real fundamental differences between the two: as we will see, there are at least three such differences.

4.3.2 Local versus global issues

The development of our semantic picture was driven by a simple but powerful idea: possible worlds represent states of affairs; when we consider an information exchange, a state of affairs encompasses not just the external facts which constitute the basic topic of the exchange, but also any feature of the exchange itself which is relevant for the purpose at hand.[24] The formal model should reflect this idea, equipping each world with a description of all the relevant features. In propositional logic, one does not consider an information exchange *at all*, but only certain facts. Thus, for the purposes of propositional logic, a world can be characterized by a valuation determining which of the basic facts are true and which are false. In epistemic logic, one is also interested in the knowledge that the agents have. Accordingly, a world comes equipped with a description of the agents' information states. In our inquisitive epistemic logic, a third feature of the exchange entered the picture, namely, the issues that the agents entertain. Thus, in our setting worlds also come equipped with a description of the agents' issues.

In DELQ, a world does not come with a description of the issues that the agents entertain at that world. Rather, an epistemic issue model comes with just one issue for each agent, which is not relativized to any particular world. Thus, while the information available to the agents may differ from world to world, the issues that the agents entertain are fixed and independent of the world under consideration.

Conceptually, it is difficult to see how this asymmetry could be motivated. Certainly, a particular distribution of issues among the participants partly determines what a world is like, no less than a particular distribution of information does. Moreover, it is natural to assume that agents may entertain different issues at different worlds.

These conceptual concerns also have important practical consequences. In particular, the asymmetric treatment of information and issues puts significant limitations on the descriptive power of DELQ. In DELQ, just like in IDEL, agents may have incomplete knowledge about other agents' knowledge, and if they do, they may indeed wonder what the other agents know. However, one would also like to be able to describe situations where agents have incomplete knowledge and wonder about the *issues* that the other agents entertain. In IDEL, such situations can be described straightforwardly. Indeed, the language of IDEL contains sentences such as $K_a W_b \mu$, expressing the fact that *a knows that b wonders about* μ, and $W_a ?W_b \mu$, expressing the fact that *a wonders whether b wonders about* μ.

In DELQ, such situations cannot be modeled appropriately. To see this, recall that the formula $U(Q_a\varphi \vee Q_a\neg\varphi)$ is used in DELQ to describe situations in which agent a entertains the issue whether φ is the case or not. Thus, the formula $K_b U(Q_a\varphi \vee Q_a\neg\varphi)$ is used to describe situations in which agent b *knows* that agent a entertains the issue whether φ holds or not. Now suppose that M is a model and w a world such that $\langle M, w\rangle \models U(Q_a\varphi \vee Q_a\neg\varphi)$. That

[24] This point has been argued forcefully by Stalnaker (1998).

is, a entertains the issue whether φ in w. Then, since the universal modality U ranges over all worlds in M, we must also have for any world $v \neq w$ in M that $\langle M, v\rangle \models U(Q_a\varphi \vee Q_a\neg\varphi)$. But then we must certainly have that $\langle M, w\rangle \models K_bU(Q_a\varphi \vee Q_a\neg\varphi)$. That is, if one agent entertains a certain issue, then all the other agents automatically know this. Conversely, if a does *not* entertain the issue, that is, $\langle M, w\rangle \models \neg U(Q_a\varphi \vee Q_a\neg\varphi)$, then $U(Q_a\varphi \vee Q_a\neg\varphi)$ must be false at all worlds, and thus we must also have $\langle M, w\rangle \models K_b\neg U(Q_a\varphi \vee Q_a\neg\varphi)$. Thus, it is impossible to model situations where the agents have incomplete information about other agents' issues, let alone situations where the agents wonder about other agents' issues.

This limitation is not the only price that DELQ pays for its non-local treatment of issues. The other significant limitation that it encounters concerns the construction of a public issue state. Both the models of IDEL and those of DELQ contain in their definition only a description of individual issues. Of course, *public issues* play a crucial role in information exchange. Van Benthem and Minică are well aware of the importance of public notions. For instance, when discussing further research directions (p. 663), they say:

> "We need extensions of our systems to group actions of information and issue management, including common knowledge, and group issue modalities."

In section 3, we showed that IDEL elegantly deals with the challenge of constructing a public state map which describes public issues and allows us to suitably interpret the public entertain modality E_*. The public state map is constructed from the maps encoding individual states, and it is completely determined by the requirement that something be publicly entertained iff it is common knowledge that everyone entertains it. This solution is not available in DELQ, since it requires the model to represent what agents know about the issues that other agents entertain, what they know about what other agents know about the issues that other agents entertain, etcetera. This information, as we saw, is not represented in the models of DELQ. It follows that, if we want public issues to enter the picture in DELQ, they will have to be specified as an independent component of the models. But this would miss the fundamental relation existing between public issues and the individual states. Furthermore, in the dynamics we would be forced to postulate a special maintenance rule for the public issue state, independent of the maintenance rules for the individual states. This is not necessary in IDEL, where public issues automatically change *as a result* of changes in the agents' private states.

4.3.3 Different notions of issues

Issues play a central role in the models of both DELQ and IDEL. However, the systems are based on two different formal notions of issues. In DELQ, an issue is an equivalence relation $\approx$ on the set of worlds. As we saw, such an equivalence relation corresponds to a partition $\pi_\approx$ of the logical space, whose blocks correspond to the basic *answers* to the issue. In IDEL, on the other hand, an issue I is defined as a non-empty downward closed set of information states, to be thought of intuitively as those information states that contain enough information to resolve the issue. Basic answers may be taken to correspond to the minimal pieces of information that resolve the issue, that is, to the maximal elements of I with respect to the $\subseteq$-ordering.

The notion of issues adopted in IDEL is strictly more general than the one adopted in DELQ. Every issue in the sense of DELQ, modeled by an equivalence relation $\approx$, immediately translates to an issue in IDEL, namely:

$$I_{\approx} = \{t \mid w \approx w' \text{ for all } w, w' \in t\}$$

consisting of all information states that are included in a block of the partition induced by $\approx$. However, the converse does not hold: there are many issues in the sense of IDEL that do not correspond to any issue in the sense of DELQ, namely, all those issues whose maximal elements (corresponding to basic answers) do not form a partition of the logical space.

The question is: are there natural examples of issues whose basic answers do not correspond to the blocks of a partition of the logical space? The answer is *yes*. First, issues need not cover the entire logical space. As mentioned before, an issue may assume certain information. To see this, consider the so-called *alternative question* in (3), where $^{\uparrow}$ and $^{\downarrow}$ denote rising and falling intonation, respectively:

(3) Does Manu speak English$^{\uparrow}$ or French$^{\downarrow}$?

The corresponding sentence in our logical language is $?\{p \wedge \neg q, \neg p \wedge q\}$. The issue expressed by this sentence in IDEL is depicted in Figure 3(a), assuming, as before, a simple model with just four possible worlds. Clearly, the issue does not cover the whole logical space, since it assumes the information that exactly one of p and q is true. Indeed, the presupposition of this interrogative is the non-tautological declarative $(p \wedge \neg q) \vee (\neg p \wedge q)$.

Second, the basic answers to an issue need not be mutually exclusive. To see this, consider the conditional interrogative $p \to ?q$. A natural language counterpart of this interrogative was given on page 18. The proposition expressed by $p \to ?q$ in IDEL is depicted in Figure 3(b). This proposition captures the fact that the two basic answers to $p \to ?q$ are $p \to q$ and $p \to \neg q$. These basic answers clearly overlap, and therefore do not form a partition of the logical space.

Other types of natural language interrogatives expressing issues with non-mutually exclusive answers include *open disjunctive questions*, i.e., disjunctive questions with rising intonation on all disjuncts, such as (4):

(4) Does Manu speak English$^{\uparrow}$, or French$^{\uparrow}$?

whose meaning is depicted in Figure 3(c), and so-called *mention-some questions*, such as (5):

(5) Where can I buy an Italian newspaper?

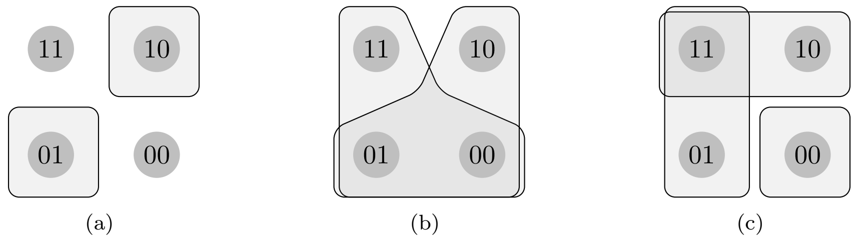

Figure 3: Three issues that do not correspond to a partition of the logical space.

Evidently, such questions admit of several non-mutually exclusive answers (for the latter question, e.g., "at the bookstore" and "at the train station").

We conclude that the notion of issues adopted in DELQ, while natural and formally well-behaved, is not rich enough to deal with several types of issues that play a significant role in information exchange.[25,26]

4.3.4 Questions as interrogative sentences or as speech acts

So far we identified two important differences between the *epistemic issue models* of DELQ and the *inquisitive epistemic models* of IDEL. One difference concerns the way issues are modeled, the other the way issues are embedded into the framework of epistemic logic. A third crucial difference concerns the treatment of questions.

In DELQ, the static language consists entirely of declarative sentences. No sentence is syntactically interrogative or semantically inquisitive. Questions only come into the picture in the dynamic component of the system, as a particular kind of speech act. As we saw, the effect of a question involving a sentence α is to raise the issue of whether α holds.

In IDEL, questions enter the picture already at the level of the static language, in the form of interrogative sentences. Just like declaratives, interrogative sentences have a semantic value, which captures their inquisitive content. This semantic value enters the compositional interpretation process, allowing us to

[25]Similar arguments, not addressing DELQ directly but rather the partition theory of questions that it is based on (Groenendijk and Stokhof, 1984), have been made by Mascarenhas (2009), Groenendijk (2011), and Ciardelli et al. (2013a).

[26]Readers may wonder—with one of the reviewers of this paper—whether the notion of issues adopted in DELQ may be generalized suitably by weakening the condition that $\approx$ be an *equivalence* relation. Early versions of inquisitive semantics did indeed seek to overcome the limitations of the partition theory of questions by modeling issues as reflexive and symmetric, but not necessarily transitive relations (Groenendijk, 2009; Mascarenhas, 2009; Sano, 2009). However, in more recent work (Ciardelli, 2009; Ciardelli and Roelofsen, 2011; Ciardelli et al., 2013a) it has been argued in detail that such a notion is still not general enough to model certain natural types of issues, such as the ones expressed by open disjunctive questions and mention-some questions. More generally, the arguments in Ciardelli et al. (2013a) can be phrased in such a way as to show that the relevant issues do not correspond to a binary relation on worlds *at all*, that is, they are not of the form $I_{\approx}$ for *any* binary relation $\approx$ on $\mathcal{W}$.

compositionally assign meanings to sentences where interrogatives are embedded under modal operators. It also allows us to keep the dynamic component of the system simpler: we only need a single action of announcing a sentence, be it declarative or interrogative, rather than two distinct actions for announcing and questioning. It is the content of the sentence that is being announced which determines whether the announcement brings about a change in information or in issues.

We will argue that there are good reasons to prefer the latter approach. In DELQ, all questions have the effect of raising a polar issue, namely the issue whether a certain declarative sentence α holds. The same effect is obtained in IDEL by an announcement of the polar interrogative $?\alpha$. Thus, the effect of a question action in DELQ may be simulated by the announcement of an interrogative in IDEL. However, the converse is problematic. Not all interrogatives that may be asked in IDEL express polar issues. Consider for instance the conditional interrogative $p \to ?q$, whose meaning was depicted in figure 3(b) above. The effect of such an interrogative cannot be modeled in DELQ since, as we saw, the notion of issues adopted in DELQ is not rich enough. But suppose this problem were amended. Then DELQ would still be in trouble, since asking $p \to ?q$ does not correspond in any way to asking whether a certain declarative sentence is true or not. In order to address this problem, DELQ may be extended with an additional, more complex action of *conditional questioning*, which would involve two declarative sentences, one serving as the antecedent and one serving as the consequent of the question. But one can of course easily imagine more and more complex question types, which would force DELQ to postulate a richer and richer repertoire of question actions.

Whether or not DELQ might eventually succeed in making its repertoire of actions rich enough, its treatment of questions as speech acts faces another difficulty as well. In IDEL, as mentioned above, an interrogative sentence is assigned a semantic value, which does not only determine the effect of announcing that interrogative, but also the meaning of more complex expressions in which the interrogative may be *embedded*. In particular, these more complex expressions may be declaratives, whose truth-conditions depend on the issue expressed by the embedded interrogative. Concretely, the basic way to construct a declarative from an interrogative μ is to embed μ under a modal operator, such as K_a, E_a, W_a, or their public counterparts K_*, E_*, W_*, all of which allow for an interrogative complement. In this way, we can construct declaratives such as $K_a\mu$, which expresses the fact that a can resolve μ; $W_a\mu$, which expresses the fact that a wonders about μ; $K_*\mu$, which expresses the fact that μ is publicly settled among the agents; and $W_*\mu$, which expresses the fact that μ is an open issue among the agents.

DELQ does not allow the construction of sentences that involve an interrogative embedded under a modal operator. The possibility of expressing the corresponding facts depends on the possibility of analyzing claims about interrogatives in terms of claims concerning declaratives. In some cases, such analyses are indeed possible. For instance, consider a polar interrogative $?\alpha$. In DELQ, "*a knows whether* α" may be analyzed as $K_a\alpha \vee K_a\neg\alpha$. In general, if an interrogative μ has a finite set of predetermined answers $\alpha_1, \ldots, \alpha_n$, then "*a knows* μ" may be analyzed in DELQ as $K_a\alpha_1 \vee \cdots \vee K_a\alpha_n$. However, this strategy faces three problems.

First, it is far from clear whether such a strategy is viable for "truly inquisitive" modalities such as *wonder*, which express a relation holding between an agent and an issue as a whole, a relation that cannot be reduced—as in the case of *know*—to a more basic relation holding between an agent and some piece of information. It is true that in DELQ we can analyze "*a wonders whether* α" as $U(Q_a\alpha \vee Q_a\neg\alpha)$. But this analysis only works if issues are treated as being global, world-independent, and we have argued above that this has serious drawbacks. Once issues are relativized to worlds, however, this account does not work anymore, and it seems no longer possible to express "*a wonders whether* α" with the tools available in DELQ (and more generally, as far as we can see, in terms of standard Kripke modalities).

Second, analyzing sentences involving an interrogative μ in this way requires knowledge of the set of answers to μ. Thus, in order to express facts about a question, DELQ needs to outsource the analysis of the question to some theory that predicts what its answers are. Our semantics, on the contrary, *includes* such a theory of questions. Equivalences such as:

$$K_a?\alpha \;\leftrightarrow\; (K_a\alpha \vee K_a\neg\alpha)$$

characterizing the knowledge that it takes to resolve a certain question, are obtained as *logical validities* of the theory, not merely assumed as definitions.[27]

Finally, even supposing the 'paraphrase' strategy would work for the propositional case—where questions have a finite, predetermined set of answers—the transition to a first-order case would be problematic. Many types of questions—such as *wh*-questions (*Who attended the party?*), *which*-questions (*Which students attended the party?*), and quantified questions (*Which party did every student attend?*)—have a set of answers that is neither predetermined (since it depends on the domain of interpretation) nor necessarily finite. The particular paraphrase strategy sketched above cannot be applied to questions of these kinds. Perhaps for any particular type of question some paraphrase in terms of declaratives may be found. However, it seems very unlikely that a uniform analysis of embedded questions in terms of declaratives exists. For any particular question, we will have to come up with a 'custom-made' translation.

Our compositional strategy, on the other hand, carries over straightforwardly to the first-order case. Drawing on ideas from first-order inquisitive semantics (Ciardelli, 2009; Roelofsen, 2013a), we could define a first-order language in which a broad range of issues can be expressed, including those corresponding to the question types above. For instance, our language would contain interrogatives of the form $\forall x?\alpha(x)$, corresponding to *wh*-questions such as *Who attended the party?*. Just like in the propositional case, such interrogatives may be embedded under modal operators to yield sentences such as $K_a\forall x?\alpha(x)$ and

[27] The same point is made by Aloni et al. (2009), who also propose a compositional interpretation of questions embedded under a knowledge operator.

$W_a \forall x?\alpha(x)$, expressing that *a knows who attended the party* and that *a wonders who attended the party*, respectively.[28]

5 Conclusion

We proposed an inquisitive dynamic epistemic logic in which the issues that the agents entertain are treated on a par with the information that they have, as an integral component of the state of affairs in each world, thus preserving the general philosophy of standard epistemic logic. We imported from inquisitive semantics a notion of issues which is more general than the traditional partition-notion. Moreover, we enriched the logical language with interrogative sentences, and generalized the semantics in order to treat declaratives and interrogatives in a uniform way, moving from single worlds to information states as points of evaluation, while still being able to derive the natural, truth-conditional interpretation of declaratives relative to single worlds. We specified a natural public state construction, analogous to the familiar common knowledge construction, which allows us to derive the public information and issues from the description of the private ones. Finally, we provided a basic dynamics for actions of public announcement. This results in a system in which a rich spectrum of facts concerning public and private knowledge as well as public and private issues can be modeled and reasoned about.

Several directions for future work naturally suggest themselves. Perhaps most urgently, the logic that the system gives rise to needs to be investigated. As we saw, our uniform semantics gives rise to a unified notion of entailment, where premisses and conclusions can belong to either syntactic category. As discussed in Ciardelli et al. (2013b), this notion is especially interesting since it subsumes the four central notions of logics dealing with both information and issues: restricted to declaratives, entailment is simply standard declarative entailment: $\alpha \models \beta$ amounts to α being more informative, or having more stringent truth-conditions, than β. An entailment $\alpha \models \mu$ from a declarative to an interrogative holds iff α provides enough information to settle μ; thus, declarative-to-interrogative entailment captures *sufficient answerhood*. An entailment $\mu \models \alpha$ from an interrogative to a declarative holds iff μ can only be resolved in worlds where α is true; thus, interrogative-to-declarative entailment captures the notion of *presupposition* of an interrogative. Finally, an entailment $\mu \models \nu$ among interrogatives holds just in case ν is resolved whenever μ is, or in other words, just in case answering ν may be reduced to answering μ. Evidently, we would like to characterize this notion axiomatically. Previous work on inquisitive logic (Ciardelli, 2009; Ciardelli and Roelofsen, 2011; Ciardelli et al.,

[28] In the liguistic literature, the point that a proper treatment of questions, especially embedded questions, requires inquisitiveness to enter the picture at the semantic level, and not just at the speech act level, has been made in much detail by Groenendijk and Stokhof (1997). At the time, it was directed mostly at the speech act treatment of questions proposed by Searle (1969) and Vanderveeken (1990), and at the imperative-epistemic treatment of questions proposed by Åqvist (1965) and Hintikka (1976, 1983). The argument we just gave is similar, but now directed specifically at the speech act treatment of questions in DELQ.

2013b) provides a useful starting point for such a characterization.[29]

The system proposed in the present paper is based on a specific incarnation of inquisitive semantics, namely, the system InqD_π developed in Ciardelli et al. (2013b). This system assumes a dichotomous syntax, distinguishing declarative and interrogative sentences, and lets the declarative fragment behave entirely classically. In this respect, it differs from the standard inquisitive semantics system InqB (Ciardelli, 2009; Groenendijk and Roelofsen, 2009; Ciardelli and Roelofsen, 2011; Ciardelli et al., 2012), which assumes a simple propositional language where disjunction has issue-raising potential, and whose logic is intermediate rather than classical. The ideas and notions propounded here could easily be adapted to construct a dynamic epistemic version of InqB. Investigating such a system, as well as the logic it would give rise to, constitutes a second direction for further research.

In this paper, our goal has been to show that information and issues are amenable to a uniform treatment in a natural extension of the basic DEL framework, and that inquisitive semantics provides the right tools for this enterprise. We illustrated this by building a system which is minimal in many respects. This basic system may be extended in several directions, incorporating insights from the existing literature on both dynamic epistemic logic and inquisitive semantics. For instance, on the dynamic epistemic logic side, besides the very strong notion of knowledge that we assumed here, which is characterized by factivity and full introspection, we may also consider the dynamics of weaker notions of knowledge and belief (see, e.g., van Ditmarsch, 2005; van Benthem, 2007). As already mentioned above, the basic dynamic component of the framework may be extended to deal with a richer repertoire of actions (e.g., Baltag et al., 1998; Benthem et al., 2006). On the inquisitive semantics side, besides the notion of issues adopted here, which captures inquisitive content, we may also import semantic structures that capture *attentive* content (Ciardelli et al., 2009; Roelofsen, 2011, 2013b; Westera, 2013). Finally, it may be interesting to see how our framework can be applied in the analysis of *question-answer games*, as investigated previously based on DELQ by Ågotnes et al. (2011).

For now, we hope to have shown that the basic machinery of dynamic epistemic logic can be extended in a natural and principled way so as to allow for a more inclusive logical analysis of information exchange, encompassing both informative and inquisitive aspects.

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