A Syntactic Characterization of Compliance in Inquisitive Semantics

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written by

Irma Cornelisse, 5619750
(born March 16st, 1987 in Amsterdam, Netherlands)

under the supervision of **Floris Roelofsen** and **Theo Janssen**, submitted in partial fulfillment of the requirements for the degree of

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Abstract

In this paper the notion of compliance (Groenendijk [2008a,b]) in inquisitive semantics (Groenendijk [2008b], Mascarenhas [2008], Ciardelli [2008], Ciardelli and Roelofsen [2009a], Groenendijk and Roelofsen [2009]) is brought into practise. An algorithm for compting compliant responses is presented, based on an earlier draft (Ciardelli and Roelofsen [2009b]). The presented algorithm is proved to be sound and complete. An implementation can be found at www.illc.uva.nl/inquisitive-semantics/computing-compliance. Furthermore the complexity of the presented algorithm is analysed and (the essential part of the algorithm) is in $O(2^{2^{2^{2^n}}})$, with n the number of proposition letters in the input formula. The large computation time may in the future be reduced by computing not all compliant responses, but only the best candidates.

Contents

Acknowledgements

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1 Introduction

This paper investigates and presents a syntactic characterization of what kind of responses are desirable given the issues raised in a human conversation. Thus it is about the moves in a conversation that are related to earlier raised issues. It are exactly those moves that create coherence in a conversation.
Understanding and modeling human conversation is very important for many disciplines. Except its philosophical and psychological interest to model and understand human conversation, there is also a more practical interest which lies in human-computer interaction and AI. A lot of examples are already available of communication between human and computer through human language (most known are the Questioning-Answer systems). However, keeping the conversation coherent from the computers side still is a bottleneck (especially when the domain of discourse is widely oriented). The main purpose of this paper is to bring an already available formalized definition (with a philosophical prospect) of what kind of responses are good conversation moves called compliance (Groenendijk [2008a,b]), into a more practical setting. Compliance is the central notion in a new formal system, called inquisitive semantics initiated by Groenendijk [2008b] and Mascarenhas [2008].
The formalized definition of compliance tells us how we can be compliant to an earlier raised issue. There are two ways one can be compliant to an issue:

1. provide information that (partially) resolves the issue
2. raise an easier to answer sub-question

A combination of these two possibilities is also allowed. This paper presents an algorithm that outputs exactly all answers with these properties given a certain issue.

Motivation Inquisitive semantics Traditionally, the meaning of a sentence is interpreted with classical logic. However classical logic is originated from the need to judge the validity of argumentation. As a consequence interpreting the meaning of a sentence with classical semantics gives us only the informative content of the sentence. But people are interested in modeling also other purposes of natural language outside argumentation. In order to fulfill this need (and the lack of a better system available) we use traditional logics and semantics to formalize these other purposes of natural language. However, the purpose of language seems to be not only descriptive and language seems to be used for more than providing information.
For instance a typical purpose of language is to exchange information between the participants in a conversation. To exchange information is to raise issues (inquisitive content of a sentence) and to provide information related to one of the raised issues (informative content of a sentence). When an issue is raised it is not only relevant what the informative content is of this issue, but especially what its inquisitive content is (to find out what a right response to this issue would be).

Inquisitive semantics (Groenendijk [2008b], Mascarenhas [2008], Ciardelli [2008], Ciardelli and Roelofsen [2009a], Groenendijk and Roelofsen [2009]) is a new formal system, developed to handle not only the informative content of a sentence, but also its inquisitive content. Inquisitive semantics (other then traditional semantics) is originated from the need to model human language in a conversation. It judges utterances not on their truth value, but it treats them as proposals to the participants of a conversation to update the common ground. If these proposals provide information, the participants can choose to either accept or reject this information. If these proposals provide several options to change the common ground, they serve the purpose of a question in a conversation. The participants are then invited to respond to the question by either (partially) proposing some of the options or by proposing an easier to answer proposal.
The central notion in inquisitive semantics, compliance (Groenendijk [2008a,b]), tells what kind of responses are related to the initiative. Relatedness is one of the maxims in inquisitive pragmatics. Together with the other maxims, the maxim of quality and the maxim of quantity, relatedness is telling us what the most compliant response is given the raised issue and the responders information state.

Throughout the paper we use a definition of compliance (Groenendijk and Roelofsen [2009]) which is based on a generalized version (Groenendijk [2008a], Ciardelli [2008]) of inquisitive semantics. The generalized version was first discussed by Groenendijk [2008a] and Ciardelli [2008] and the associated logic was axiomatized by Ciardelli and Roelofsen [2009a]. The notion of compliance presented by Groenendijk [2008b] is presented differently than the notion presented by Groenendijk and Roelofsen [2009] because of their use of generalized semantics. The two notions are proven to be equivalent by Groenendijk and Roelofsen [2009].

The aim of this paper is to create a better practical insight in the notion of compliance and work out a sound and complete syntactic characterization of compliance. A first draft of an algorithm for compliance is already available (Ciardelli and Roelofsen [2009b]), but it is not jet proved to be sound and complete. In this paper some adjustments to the algorithm are made in order to reduce space- and time complexity, which makes it possible to run the algorithm for short formulas.

2 Inquisitive Semantics

In this section we introduce some definitions and facts of generalized inquisitive semantics, which we will need to understand the definition of compliance and proof soundness and completeness later on in the paper. All the facts and definitions stated here can be found in earlier work (Groenendijk [2008b], Ciardelli [2008], Ciardelli and Roelofsen [2009a]).

Definition 1 (Language) *P will denote a finite set of proposition letters. L_P will denote the formulas built up from letters in P and $\bot$ combined with the binary connectives $\wedge$, $\vee$ and $\rightarrow$.*

Definition 2 (Abbreviations) *For all $\varphi \in L_P$,*

1. *$\neg\varphi$ is an abbreviation for $\varphi \rightarrow \bot$*
2. *$?\varphi$ is an abbreviation for $\varphi \vee \neg\varphi$*
3. *$!\varphi$ is an abbreviation for $\neg\neg\varphi$*

Note that these abbreviations suggest that the law of double negation and the law of the excluded middle do not hold as laws in inquisitive semantics.

Definition 3 (Indices) *An index is a function from P to $\{0,1\}$. We will denote the set of all indices with Ω.*

Definition 4 (States) *A state is a set of indices. We will denote the set of all states with S.*

One could explain an index as a possible world and a state as a set of possible worlds. This set of possible worlds can stand for the information state of an agent.

Next we define when a state supports a formula.

Definition 5 (Support) *For all $\varphi, \psi \in L_P$ and $s \in S$,*

1. $s \models p$ *iff* $\forall w \in s : w(p) = 1$
2. $s \models \bot$ *iff* $s = \emptyset$
3. $s \models \varphi \wedge \psi$ *iff* $s \models \varphi$ *and* $s \models \psi$
4. $s \models \varphi \vee \psi$ *iff* $s \models \varphi$ *or* $s \models \psi$
5. $s \models \varphi \rightarrow \psi$ *iff* $\forall t \subseteq s :$ *if* $t \models \varphi$ *then* $t \models \psi$

Note that the support rule for disjunction (although it looks quite familiar) creates a world of difference between traditional semantics and inquisitive semantics. Traditionally some formula $p \vee q$ is true if and only if p is true or q is true. In inquisitive semantics however, an information state supports some formula $p \vee q$ if and only if every possible world in that information state supports p or every possible world in that information state supports q (which is of course not the same as every possible world in that information state supports p or supports q).

Next we proof a base fact, persistence, which is an immediate result of the definition of support.

Fact 1 (Persistence) *For all $\varphi \in L_P$ and $s \in S$,*
If $s \models \varphi$ then for every $t \subseteq s$: $t \models \varphi$.

Proof By induction on the length of φ
Take an arbitrary $s \in S$, and suppose $s \models \varphi$,

1. if $\varphi = p$
   Then we have for all indices $w \in s$, that $w(p) = 1$. Take an arbitrary $t \subseteq s$, then for all indices $w' \in t$, we have $w' \in s$ and therefore we have $w'(p) = 1$.

2. if $\varphi =\perp$
   Then we have that $s = \emptyset$ and thus there are none substates of s.

3. if $\varphi = \psi \wedge \chi$
   Take an arbitrary $t \subseteq s$, we know $s \models \psi$ and therefore by IH we have that $t \models \psi$. We also know that $s \models \chi$ and therefore by IH $t \models \chi$. Then also $t \models \psi \wedge \chi$.

4. if $\varphi = \psi \vee \chi$
   We know $s \models \psi$ or $s \models \chi$. Take an arbitrary $t \subseteq s$. If it is the case that $s \models \psi$ then by IH $t \models \psi$ and therefore $t \models \psi \vee \chi$. Otherwise it has to be the case that $s \models \chi$ and then by IH $t \models \chi$ and therefore $t \models \psi \vee \chi$.

5. if $\varphi = \psi \to \chi$
   We know for all $s' \subseteq s$, if $s' \models \psi$, then $s' \models \chi$. Take an arbitrary $t \subseteq s$. Then we have that for all $t' \subseteq t$ also $t' \subseteq s$. Therefore we know that if $t' \models \psi$, then $t' \models \chi$. Thus $t \models \psi \to \chi$.

Definition 6 (Possibilities, propositions, truth-sets) *For all $\varphi \in L_P$,*

1. *A **possibility** for φ is a maximal state supporting φ, that is a state that supports φ and is not properly included in any other state supporting φ.*

2. *The **proposition** expressed by φ is the set of possibilities for φ, and is denoted by $[\varphi]$.*

3. *The **truth set** of φ is the set of indices where φ is classically true, and is denoted by $|\varphi|$.*

That the set of possibilities is not always a singleton is strongly related to the support rule for disjunction. Furthermore we can intuitively note that the maximal set of all worlds in which a formula is classically true, is the set of all worlds in which that formula is classically true.

Next we proof a base fact which is an immediate result of the definition of possibilities, states and support.

Fact 2 *For all $\varphi \in L_p$ and $s \in S$,*
If $s \models \varphi$ then there is some possibility π for φ, such that $s \subseteq \pi$.

Proof Take arbitrary $\varphi \in L_p$ and $s \in S$, and suppose $s \models \varphi$. Then either s is a maximal state that supports φ and therefore a possibility for φ, and thus $s \subseteq \pi$ with $\pi = s$ and π a possibility for φ. Or s is not a maximal state that supports φ, but then there is an index $i \notin s$, such that $s \cup \{i\} \models \varphi$. Then $s \cup \{i\}$ is again either a maximal state that supports φ, or there is some extension of $s \cup \{i\}$ with some index $i' \notin s \cup \{i\}$ that support φ. Given that P is finite, we know that the possible sets of functions from P to $\{0, 1\}$ (states) are finite and therefore we know there is some point at which there is no index not already in the state that supports φ, and that state is a maximal state π that supports φ. Therefore there is some possibility π for φ, such that $s \subseteq \pi$.

If a formula is inquisitive it means it is a proposal which offer you certain possibilities to choose from. The speaker ask you to resolve (partially) his issue. If the proposal offers only one possibility, then the speaker leaves you nothing to resolve. Which means that your only opportunity then is to accept or reject his proposal.

Definition 7 (Inquisitiveness, Questions and assertions) *For all $\varphi \in L_p$,*

1. *φ is **inquisitive** if and only if $[\varphi]$ contains at least two possibilities.*
2. *φ is a **question** if and only if it is inquisitive*[1].
3. *φ is an **assertion** if and only if it is not inquisitive.*

Fact 3 (Assertion) *The following are equivalent:*

1. *φ is an assertion*
2. *if $s \models \varphi$ and $t \models \varphi$ then also $s \cup t \models \varphi$.*
3. $[\varphi] = \{|\varphi|\}$

Proof

$1 \Rightarrow 2$ Take an arbitrary $\varphi \in L_p$, and suppose φ is an assertion. Take arbitrary $s, t \in S$, and suppose $s \models \varphi$ and $t \models \varphi$. Since φ has only one possibility π and all states that support φ has to be contained in a possibility for φ, we have $t \cup s \subseteq \pi$. We know $\pi \models \varphi$ and therefore (by persistence) $t \cup s \models \varphi$.

$2 \Rightarrow 3$ Take arbitrary $\varphi \in L_p$ and suppose that for every $t, s \in S$, if $s \models \varphi$ and $t \models \varphi$, then $t \cup s \models \varphi$. Take arbitrary $\pi, \pi' \in S$ and suppose π is a possibility for φ and π' is a possibility for φ and $\pi \neq \pi'$ (thus suppose φ has more than one possibility). This is a contradiction because then by the first assumption $\pi \cup \pi' \models \varphi$ and hence we have that π and π' aren't

[1] In earlier work questions are defined differently and is what we call here a question defined as a hybrid formula or a question. In this paper we don't need to distinguish between hybrids and questions.

maximal states supporting φ. Therefore φ has at most one possibility. Take an arbitrary $s \in S$, and suppose s is the unique possibility for φ. Then we have that $s \models \varphi$ and also for every $t \subseteq s$ we have that $t \models \varphi$ (by fact 1). In particular all singleton states (indices) in s support φ. And there is no index $i \notin s$, such that $i \models \varphi$, because otherwise it would be contained in some possibility for φ (by fact 2). Since φ has only one possibility, this is not possible. Since exactly every index that supports φ is contained in s, we have that $[\varphi] = \{|\varphi|\}$.

$3 \Rightarrow 1$ Take an arbitrary $\varphi \in L_p$, and suppose $[\varphi] = \{|\varphi|\}$. Then the set of possibilities for φ contains one possibility, which means that φ is an assertion.

Fact 4 (Assertions) *For all $p \in P$ and all $\psi, \chi \in L_p$,*

1. *p is an assertion*
2. *$\bot$ is an assertion*
3. *$\neg\psi$ is an assertion*
4. *if ψ, χ are assertions, then $\psi \wedge \chi$ is an assertion*
5. *if χ is an assertion, then $\psi \to \chi$ is an assertion*

Proof $\bot$ is an assertion because there is only one maximal state supporting $\bot$: $s = \emptyset$. For the other items see [Ciardelli, 2008].

Note that from the fact that a negation is an assertion it follows that any formula with an exclamation mark as main-connective is also an assertion.

Later on to prove soundness and completeness of the algorithm presented in this paper, we will need to know when two formulas are equivalent.

Definition 8 (Equivalence) *Two formulas $\varphi, \psi \in L_p$ are equivalent, $\varphi \equiv \psi$, iff $[\varphi] = [\psi]$*

The following fact about equivalence immediately follows from the definition of equivalence.

Fact 5 (Equivalence) *Two formulas $\varphi, \psi \in L_p$ are equivalent iff for all $s \in S$, we have $s \models \varphi$ iff $s \models \psi$.*

Proof

$\Rightarrow$ Take arbitrary $\varphi, \psi \in L_P$, and suppose $\varphi \equiv \psi$. By the definition of equivalence we have that $[\varphi] = [\psi]$. Take an arbitrary $t \in S$, and suppose $t \models \varphi$. We have that $t \subseteq s$ for some maximal state s that supports φ (by fact 2). Then t is a subset for some maximal state that supports ψ, namely s (because $[\varphi] = [\psi]$). And by persistence we have that $t \models \psi$. In the same way we have that $\forall t \in S$, if $t \models \psi$ then $t \models \varphi$. Thus for all $s \in S$, we have $s \models \varphi$ *iff* $s \models \psi$.

$\Leftarrow$ Take arbitrary $\varphi, \psi \in L_P$, and suppose $\forall s \in S$ $s \models \varphi$ *iff* $s \models \psi$. Take an arbitrary $\pi \in S$, and suppose π is a possibility for φ. We want to show that π is also a possibility for ψ. We have that π is a state that supports φ and by the original assumption we have that π also supports ψ. Suppose that π is not a maximal state that supports ψ (i.e. not a possibility for ψ), then there would be some index $w \in \Omega$, such that $\pi \cup \{w\} \models \psi$. But then by the original assumption $\pi \cup \{w\} \models \varphi$, which would mean that π is not a maximal state that supports φ. This leads to a contradiction so π is also a maximal state that supports (possibility for) ψ. In the same way we have $\forall \pi \in S$, if π is a possibility for ψ then π is also a possibility for φ. Thus $[\varphi] = [\psi]$ and therefore (by the definition of equivalence) we have that $\varphi \equiv \psi$.

We also need the fact that if two formulas are equivalent in inquisitive semantics they are eqivalent in classical semantics. But to proof that, we first have to proof that the truth set of a formula is the union of the possibilities for that formula.

Fact 6 *For all $\varphi \in L_p$, $|\varphi|$ = the union of all possibilities in $[\varphi]$.*

Proof The right-to-left direction: Take an arbitrary $\varphi \in L_P$. By persistence we know that all single states (a set of one index) in a possibility for φ support φ. We also know that every state that supports φ is contained in a possibility for φ (by fact 2), then in particular all single states that support φ are contained in a possibility for φ. Therefore the union of all possibilities contain exactly those indices in which φ is classically true.
Now we can proof that if two formulas are equivalent in inquisitive semantics, they are also equivalent in classical semantics.

Fact 7 *For all $\varphi, \psi \in L_p$, if $\varphi \equiv \psi$ then $|\varphi| = |\psi|$.*

Proof Take two arbitrary formulas $\varphi, \psi \in L_p$, and suppose $\varphi \equiv \psi$. This means $[\varphi] = [\psi]$. Then also the union of all possibilities in $[\varphi]$ = the union of all possibilities in $[\psi]$. Therefore $|\varphi| = |\psi|$ (by fact 6).

3 Compliance

The notion of compliance in inquisitive semantics judges whether a certain conversation move is related in a coherent way to some raised issue in the conversation. In other words it judges if the participants response is compliant to the issue he responded to. Before stating the formal definition, we give a basic intuition of what kind of responses are desirable given an issue.

Basic intuitions Participants joining a conversation can typically make two possible conversation moves. One can either raise a new issue or try to resolve an issue that have been raised earlier in the conversation. Such moves that try to resolve an issue that have been raised earlier in the conversation are supposed to be coherent conversation moves, and are the kind of moves compliance

is concerned with. One way to contribute to resolve an issue is to provide information that (partially) resolve the issue. If this is not possible (because you're information state doesn't contain such kind of information, or you don't know it does) a conversational move can still make a significant contribution when you replace the issue by an easier to answer sub-question. What is important is that being compliant, and thus making a coherent conversation move, means to do nothing more than this: it should not provide information that is not related to the given issue, and it should not raise issues that are not raised by the given issue, or that are more difficult to answer.
So there are basically two ways to be compliant:

1. Provide information that (partially) resolves an issue.
2. Raise an easier to answer sub-question.

Combinations are also possible: one can partially resolve the issue and at the same time raise an easier to answer sub-question of the remaining unresolved part of the issue.

The following definition captures exactly those properties that decide if a certain response φ is compliant to some issue ψ.

Definition 9 (Compliance) *φ is compliant with ψ, in symbols $\varphi \propto \psi$, iff*

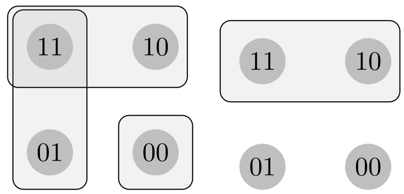
1. *every possibility in $[\varphi]$ is the union of a set of possibilities in $[\psi]$*
2. *every possibility in $[\psi]$ restricted to $|\varphi|$ is contained in a possibility in $[\varphi]$*

Desirable compliant responses are repsonses that (partially) resolve an issue, or responses that raise an easier to answer subquestion, or responses that partially resolve an issue and at the same time ask an easier to answer subquestion of the remaining issue. Now we will explore what kind of answers satisfy one of those properties and what kind of answers don't. Furthermore we will explain why the definition supports the kind of answers that do satisfy those properties and rules out the kind of answers that don't.

Provide information that (partially) resolves an issue The first thing we defined as being a compliant responses is to provide information that (partially) resolves an issue. To provide information is to rule out one or more indices. When you respond to an issue with an assertion, then this is the only thing you have to keep in mind if you want to give a compliant response. There are roughly three ways to be compliant by responding to an issue with an assertion.

Do: provide information that resolves an issue.
To totally resolve an issue is to respond by choosing one from the possibilities provided by the issue. The definition of compliance allows for this kind of responses since the possibility in the response is also an possibility in the issue. And the second condition is always true when one talks about assertions as responses.

Example 1 *Suppose we have an initiative 'Does John or Ben come?' ($?(p \vee q)$, figure 1(a)), one can resolve the issue by answering with John comes (p, figure 1(b)).*

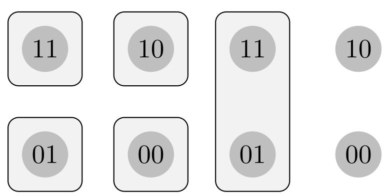


(a) $?(p \vee q)$ (b) p

Figure 1: totally resolve issue

Do: provide information that partially resolves an issue.
To partially resolve an issue is to respond with an answer that excludes one (or more) possibilities from the initiative by providing information. The definition of compliance allows for this kind of responses since the possibility in the response is the union of a set of (not all) possibilities in the issue. And the second condition is always true when one talks about assertions as responses.

Example 2 *Suppose we have an imitative 'Is John coming to the party and can I come?' ($?p \wedge ?q$, figure 2(a)), one can partially resolve the issue by answering with 'You can come' (q, figure 2(b)).*

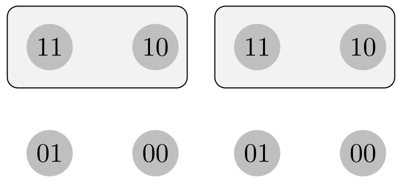


(a) $?p \wedge ?q$ (b) q

Figure 2: partially resolve issue

Do: confirme the initiative.
By confirming the initiative one doesn't really make a significant contribution in resolving the initiative, however it still is a compliant response because it is of great significance for the coherence of a conversation. The definition of compliance allows for this kind of responses because the possibility of the response is the union of a set of (all) possibilities in the issue. And the second condition is always true when one talks about assertions as responses.

Example 3 *Suppose we have an initiative 'It is beautiful day' (p, figure 3(a)), one can confirm the initiative by answering with 'Yes it is' (p, figure 3(b)).*



(a) p (b) p

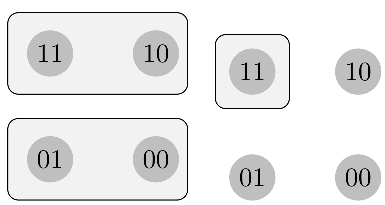
Figure 3: confirm the issue

The assertions as responses to a given issue that provide irrelevant information are not compliant responses. There are roughly two ways to provide irrelevant information.

Don't: be over-informative.

Being over-informative is saying something that is not asked. This is undesirable because its unnecessary risk of being rejected by the other participants of the conversation. It is not something that is asked to be resolved by the initiator of the issue. So it is not related to the domain of discussion [2]. The definition of compliance doesn't allow this kind of responses because the possibility of the response is a strict subset of some possibility in the issue and therefore not a union of a set of possibilities for the issue.

Example 4 *Suppose we have an initiative 'Does John comes to the party?' (?p, figure 4(a)), the reponse 'John and Ben come' ($p \wedge q$, figure 4(b)) would not be compliant to the initiative.*



(a) $?p$ (b) $p \wedge q$

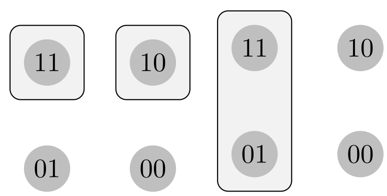
Figure 4: being over-informative

[2]In some case it is desirable to resolve an issue that is not asked directly, but then this issue that has to be resolved is an implicature. And then the desirable response is compliant to the implicature.

Don't: be less-informative.

Being less-informative then the issue is not a compliant response to that issue. The definition of compliance doesn't allow for this kind of responses because indices occur in the possibility for the response that don't occur in the issue and therefore it cannot be an union of a set of possibilities for the issue.

Example 5 *Suppose we have an initiative 'I'm hungry, would you also like to eat something' ($p \wedge ?q$, figure 5(a)),one is not answering compliant by saying 'Well if you're hungry or not, I like to eat something' (q, figure 5(b)).*



(a) $p \wedge ?q$ (b) q

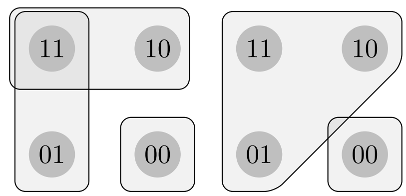
Figure 5: being less-informative

Raise an easier to answer sub-question The second property we defined as being a compliant response is to raise an easier to answer subquestion. Here we talk about a question as response that provides the same information as the issue at hand (no more and no less). An easier to answer subquestion of an issue means that enough information to answer the issue is certainly enough information to answer the question.

Do: ask an easier to answer subquestion.

Asking an easier to answer subquestion can make a significant contribution. If someone can't provide information that (partially) resolves the issue in the first place because he doesn't know he commands the right information, reformulating the issue can help him to make the relevant information available. The definition of compliance allows for this kind of questions as responses to an issue because you ask a question that unites possibilities for the issue (what makes the question easier to answer). This means that the possibilities for the response are all unions of a subset of possibilities for the issue. Furthermore, the information needed to answer the issue is always enough to answer the subquestion. This property is checked by the second condition of compliance, and is thus satisfied by this kind of responses.

Example 6 *Suppose we have an initiative 'Does John or Ben come?' ($?(p \vee q)$, figure 6(a)) then a compliant response would be 'Is one of them coming?' ($?!(p \vee q)$, figure 6(b)).*



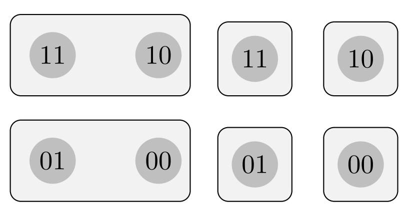
(a) $?(p \vee q)$ (b) $?!(p \vee q)$

Figure 6: ask an easier to answer sub-question

This means that asking a question that is not at least as easy to answer is not a compliant response. There are roughly two ways of asking a question that is not as least as easy to answer then the issue responded to.

Don't: ask for more specific information than the issue asks for.
Asking for more specific information than the issue asks for, means that enough information to answer the issue is not always enough information to answer the response. If the response ask for more specific information than the issue, the response is not totally relevant to the issue. The definition of compliance doesn't allow for this kind of answers because if you ask for more specific information, there is a possibility for the response that is a strict subset of some possibility for the issue. And therefore we have in that case that not all possibilities for the response are also unions of a subset of possibilities for the issue.

Example 7 *Suppose we have an initiative 'Does john come to the party?' ($?p$, figure 7(a)) then the following response would not be compliant to the issue 'Does John come?, and does Bea come?' ($?p \wedge ?q$, figure 7(b)).*



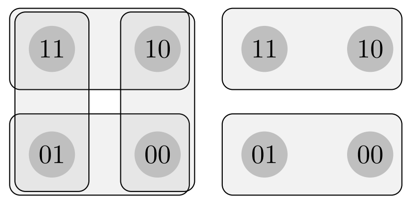
(a) $?p$ (b) $?p \wedge ?q$

Figure 7: ask for more specific information

Don't: rule out possible answers, without providing information.
Ruling out possible answers without providing information, means also that enough information to answer the issue is not always enough information to answer the response. The definition of compliance

doesn't allow for this kind of responses because it means that there is some possibility in the issue restricted to the truth set of the response that is not contained in a possibility for the response.

Example 8 *Suppose we have an initiative 'I want to know if John comes or I want to know if Bea comes!' ($?p \vee ?q$, figure 8(a)) then the response 'Does John come?' ($?p$, figure 8(a)) would not be compliant to the initiative* [3].



(a) $?p \vee ?q$ (b) $?p$

Figure 8: rule out possibilities without providing information

A combination of the previous two A combination of the previous two means a response that partially resolves the issue and simultaneously raises a subquestion that is at least as easy to answer as the remaining part of the issue.

Do: partially resolve the issue and raises a subquestion that is at least as easy to answer as the remaining part of the issue.

Don't: make a combination with a previous defined don't.
This means you can't give information that does not (partially) resolve the issue and you can't reformulate the unresolved part of the issue in a not as least as easy to answer sub-question, than the remaining part of the issue asks for.

4 Algorithm for Computing Compliant Responses

In this section we specify the algorithm which computes all compliant responses for a given initiative. A first draft for the algorithm was already available (Ciardelli and Roelofsen [2009b]). In this paper we made an addition to the algorithm which reduces the space and time complexity of the algorithm in order to make it possible for a computer to run an implementation of this algorithm in real time for initiatives with few proposition letters.
The algorithm takes as input some formula $\psi \in L_P$ and outputs all compliant reponses to ψ (i.e. every compliant response for ψ is equivalent with some formula in the output of the algorithm, and every formula in the output of the

[3]Since you can answer the initiative if you know if Bea comes, but you can't answer the response. Thus the response is not easier to answer.

algorithm is compliant to ψ). The algorithm its first steps are to compute the disjunctive normal form of ψ and from there on the clean disjunctive normal form of ψ. Therefore we first define how to calculate the disjunctive normal form and the clean disjunctive normal form of a formula ψ.

Definition 10 (DNF(ψ)) *For all $p \in P$ and all $\varphi, \psi, \chi \in L_p$,*

1. $\textsc{dnf}(p) = p$
2. $\textsc{dnf}(\bot) = \bot$
3. $\textsc{dnf}(\neg\varphi) = \neg\varphi$
4. $\textsc{dnf}(\varphi \vee \chi) = \textsc{dnf}(\varphi) \vee \textsc{dnf}(\chi)$
5. $$\textsc{dnf}(\varphi \wedge \chi) = \bigvee_{\substack{1 \leq i \leq n \\ 1 \leq j \leq m}} (\varphi_i \wedge \chi_j)$$
   where:
   - $\textsc{dnf}(\psi) = \varphi_1 \vee ... \vee \varphi_n$
   - $\textsc{dnf}(\chi) = \chi_1 \vee ... \vee \chi_m$
6. $$\textsc{dnf}(\varphi \rightarrow \chi) = \bigvee_{\substack{1 \leq k_1 \leq m \\ \vdots \\ 1 \leq k_n \leq m}} \left(\bigwedge_{1 \leq i \leq n} (\varphi_i \rightarrow \chi_{k_i}) \right)$$
   where:
   - $\textsc{dnf}(\varphi) = \psi_1 \vee ... \vee \varphi_n$
   - $\textsc{dnf}(\chi) = \chi_1 \vee ... \vee \chi_m$

To compute the clean disjunctive form we will need the notion of classical entailment.

Definition 11 (classical entailment) *For all $\varphi_1, ..., \varphi_n, \psi \in L_p$,*
We say that ψ is entailed [4] by $\varphi_1, ..., \varphi_n$, in symbols $\varphi_1, ..., \varphi_n \models \psi$, iff
$|\varphi_1| \cup ... \cup |\varphi_n| \subseteq \psi$

Definition 12 (CDNF(ψ)) *For all $\psi \in L_p$,*
$\textsc{cdnf}(\psi)$ *is obtained from* $\textsc{dnf}(\psi)$ *by removing any disjunct that assymetrically entails any other disjunct and remove all but one equivalent disjuncts.*

[4] We will refer to classical entailment with entailment. In this article we will not use inquisitive entailment and therefore we can drop 'classically' without confusion.

With this information we can specify an algorithm for computing the compliant responses for an input ψ.

Definition 13 (Algorithm)

Input $\psi \in L_P$

1. *Compute* DNF(ψ) *from* ψ *according to the definition.*
2. *Compute* CDNF(ψ)*,* $\psi_1 \vee ... \vee \psi_n$*, from* DNF$(\psi)$ *according to the definition.*
3. *Compute the set of potentially compliant assertions,* PCA(ψ)*, as follows:* PCA(ψ) $= \{!(\psi_{i_1} \vee ... \vee \psi_{i_m}) \mid i_1, ..., i_m \in \{1, ..., n\}\}$
4. *Compute the set of potentially compliant responses,* PCR(ψ)*, as follows:*

$$\text{PCR}(\psi) = \{\chi_1 \vee ... \vee \chi_n \mid 1 \leq n \leq |\text{PCA}(\psi)| \;\; \textit{and} \;\; \chi_1, ..., \chi_n \in \text{PCA}(\psi) \;\; \textit{and}$$
$$\textit{for no } i, j \in \{1, ..., n\} \chi_i \models \chi_j\}$$

5. *For all* $\varphi \in$ PCR(ψ) ***do:***
   If *for all disjuncts* ψ_j *in* CDNF(ψ)*,*
   there is a disjunct φ_g *in* φ*,*
   such that $\psi_j \wedge !\varphi \models \varphi_g$*,*
   then φ *is a compliant response,* $\varphi \in$ CR(ψ)*.*

Output CR(ψ)*, compliant responses (in clean disjunctive normal vorm) to* ψ [5]

Basic intuitions for soundness and completeness of the presented algorithm Given an input ψ the algorithm computes the disjunctive normal form of ψ (DNF(ψ)) because from there the clean disjunctive normal form of ψ (CDNF(ψ)) can be easily calculated. The definition of CDNF(ψ) leads towards a form where ψ is a disjunction of assertions. All assertions have only one possibility and therefore every possibility of ψ has to be contained in the possibility for some disjunct of CDNF(ψ). According to the definition CDNF(ψ) has only disjuncts left for which the possibility is not contained in the possibility for some other disjunct of CDNF(ψ). So the possibility for a disjunct of CDNF(ψ) is also a possibility for ψ. Thus computing the CDNF(ψ) results in getting all possibilities for ψ.
Then the Potentially Compliant Assertion of ψ (PCA(ψ)) are computed by taking the power set (without the empty set) of the disjuncts of CDNF(ψ), make a disjunction of every set and put it under the scope of an exclamation mark. What you get is a set of assertions (due to the exclamation mark), which have as only possibility the union of a set of possibilities in $[\psi]$. All possible unions have been created by the powerset. Thus the result is the set of *assertions* that satisfy the first condition of compliance and all the *assertions* that satisfy the

[5] In the algorithm described by Ciardelli and Roelofsen [2009b], there is a clean-up step before the formulas are putted out. This is redundant in this adapted version of the algorithm, because only responses that are in CDNF are generated by step 4.

first condition are equivalent with a formula in $\textsc{pca}(\psi)$.
Then the Potentially Compliant Responses ($\textsc{pcr}(\psi)$) are computed by taking the power set of $\textsc{pca}(\psi)$ make a disjunction of every set in the powerset and throw away all those formulas in which there is some disjunct that entails some other disjunct in that formula. By throwing away all formulas in which there is some disjunct that entails some other disjunct, there are only formulas left that are all in disjunctive normal form. This clean-up step in this phase is added to the algorithm described by Ciardelli and Roelofsen [2009b]. What you get is a set of responses which only possibilities are possibilities for the formulas in $\textsc{pca}(\psi)$ (which are unions of a set of possibilities in ψ). All possible responses are created, since all possible unions have been created by the power set and only those formulas that are equivalent with some other formula in $\textsc{cdnf}(\psi)$ are thrown away. Thus the result is the set of *responses* (questions and assertions) that satisfy the first condition of compliance and all the *responses* that satisfy the first condition are equivalent with a formula in $\textsc{pcr}(\psi)$.
In this stage we have all formulas that satisfy the first condition of compliance. We now only need to keep those formulas in $\textsc{pcr}(\psi)$ that also satisfy the second condition of compliance. That is every possibility in $[\psi]$ restricted to $|\varphi|$ must be contained in a possibility in $[\varphi]$. In the algorithm we check for every formula $\varphi \in \textsc{pcr}(\psi)$ if for each of the disjuncts ψ_j in ψ, we have $\psi_j \wedge !\varphi$ (every possibility in $[\psi]$ restricted to $|\varphi|$) entails some disjunct of φ (must be contained in a possibility in $[\varphi]$). All formulas that survive this check end up in the Compliant Responses of ψ, $\textsc{cr}(\psi)$. So all formulas in $\textsc{cr}(\psi)$ are compliant responses to ψ and all compliant responses to ψ are equivalent with a formula in $\textsc{cr}(\psi)$.

5 Soundness and Completeness of the Algorithm

In this section we want to proof that the presented algorithm is sound and complete. Thus with ψ as input all the compliant formulas that are found by the algorithm really are, according to the definition, compliant to ψ, and every formula that is, according to the definition, compliant to ψ is equivalent with some formula in the output of the algorithm.
To proof this we first have to proof that by computing $\textsc{dnf}(\psi)$ of ψ, we do not lose any properties of ψ that are of importance for compliance. The only properties playing a role in the definition of compliance concerning ψ, are the possibilities for ψ. Therefore we have to be sure that the possibilities for ψ are the same as the possibilities for $\textsc{dnf}(\psi)$. We earlier defined the property between two formulas of having exactly the same possibilities as being equivalent.
So if we don't want to lose any information concerning ψ that is of importance for compliance, we want to show that for any formula $\varphi \in L_P$, we have that $\textsc{dnf}(\varphi) \equiv \varphi$.
Since it will be easier to proof that $\textsc{dnf}(\varphi) \equiv \varphi$ trough terms of support, we will use fact 5 and proof that for every $s \in S$, we have that $s \models \varphi$ iff $s \models \textsc{dnf}(\varphi)$.
To proof this we first need to proof equivalence between steps that must be taken to compute $\textsc{dnf}(\varphi)$, namely when φ is of the form $\psi \wedge \chi$ and when φ is

of the form $\psi \rightarrow \chi$.
We begin with the case that $\varphi = \psi \wedge \chi$.
We have to show that

$$\text{DNF}(\psi) \wedge \text{DNF}(\chi) \quad \equiv \bigvee_{\substack{1 \leq i \leq n \\ 1 \leq j \leq m}} (\psi_i \wedge \chi_j) \tag{1}$$

where:

- $\text{DNF}(\psi) = \varphi_1 \vee ... \vee \varphi_n$
- $\text{DNF}(\chi) = \chi_1 \vee ... \vee \chi_m$

Therefore we will first need a proof of the fact that

$$s \models \bigvee_{1 \leq i \leq n} \psi_i \quad \equiv \quad s \models \psi_1 \textit{ or } ... \textit{ or } s \models \psi_n \tag{2}$$

This proof will become very important during the soundness and completeness proof of the presented algorithm.

Fact 8 *For all* $\varphi_1, ..., \varphi_n \in Lp$,
$s \models \varphi_1 \vee ... \vee \varphi_n$ *iff* $s \models \varphi_1$ *or ... or* $s \models \varphi_n$

Proof
For all $\varphi_1, ..., \varphi_n \in Lp$,
$s \models \varphi_1 \vee ... \vee \varphi_n$ iff (by definition 5.4)
$s \models \varphi_1$ or $s \models \varphi_2 \vee ... \vee \varphi_n$
By IH $s \models \varphi_2 \vee ... \vee \varphi_n$ iff
$s \models \varphi_2$ or ... or $s \models \varphi_n$

Now we can proof equation 1.

Fact 9 *For all* $\varphi_1, ..., \varphi_n, \psi_1, ..., \psi_m \in Lp$ *and* $s \in S$,
$s \models (\varphi_1 \vee ... \vee \varphi_n) \wedge (\psi_1 \vee ... \vee \psi_m)$ *iff*
$s \models (\varphi_1 \wedge \psi_1) \vee ... \vee (\varphi_1 \wedge \psi_m) \vee ... \vee (\varphi_n \wedge \psi_1) \vee ... \vee (\varphi_n \wedge \psi_m)$

Proof For all $\varphi_1, ..., \varphi_n, \psi_1, ..., \psi_m \in Lp$,
$s \models (\varphi_1 \vee ... \vee \varphi_n) \wedge (\psi_1 \vee ... \vee \psi_m)$ iff (by definition 5.3)
$s \models (\varphi_1 \vee ... \vee \varphi_n)$ and $s \models (\psi_1 \vee ... \vee \psi_m)$ iff (by fact 8)
($s \models \varphi_1$ or ... or $s \models \varphi_n$) and ($s \models \psi_1$ or ... or $s \models \psi_m$) iff (by convention)
($s \models \varphi_1$ and $s \models \psi_1$) or ... or ($s \models \varphi_1$ and $s \models \psi_m$) or ... or
($s \models \varphi_n$ and $s \models \psi_1$) or ... or ($s \models \varphi_n$ and $s \models \psi_m$) iff (by definition 5.3)
$s \models (\varphi_1 \wedge \psi_1)$ or ... or $s \models (\varphi_1 \wedge \psi_m)$ or ... or
$s \models (\varphi_n \wedge \psi_1)$ or ... or $s \models (\varphi_n \wedge \psi_m)$ iff (by fact 8)
$s \models (\varphi_1 \wedge \psi_1) \vee ... \vee (\varphi_1 \wedge \psi_m) \vee ... \vee (\varphi_n \wedge \psi_1) \vee ... \vee (\varphi_n \wedge \psi_m)$

Next we want to proof equivalence between φ and $\text{DNF}(\varphi)$ when φ is of the

form $\psi \to \chi$.
Therefore we have to proof that:

$$\text{DNF}(\psi) \to \text{DNF}(\chi) \equiv \bigvee_{\substack{1\leq k_1 \leq m \\ \dots \\ 1 \leq k_n \leq m}} \left(\bigwedge_{1 \leq i \leq n} (\psi_i \to \chi_{k_i}) \right) \tag{3}$$

where

- $\text{DNF}(\psi) = \psi_1 \vee ... \vee \psi_n$
- $\text{DNF}(\chi) = \chi_1 \vee ... \vee \chi_m$

We split this up in several parts.

$$\text{DNF}(\psi) \to \text{DNF}(\chi) \equiv (\psi_1 \to \text{DNF}(\chi)) \wedge ... \wedge (\psi_n \to \text{DNF}(\chi)) \tag{4}$$

$$\psi_1 \to \text{DNF}(\chi) \equiv \psi_1 \to \chi_1 \vee ... \vee \psi_1 \to \chi_m \tag{5}$$

$$\begin{aligned} ((\psi_1 \to \chi_1) \vee ... \vee (\psi_1 \to \chi_m)) \wedge ... \wedge ((\psi_n \to \chi_1) \vee ... \vee (\psi_1 \to \chi_m)) \equiv \\ ((\psi_1 \to \chi_1) \wedge ... \wedge (\psi_n \to \chi_1)) \vee \\ ... \vee \\ ((\psi_1 \to \chi_m) \wedge ... \wedge (\psi_n \to \chi_m)) \end{aligned} \tag{6}$$

With the right side of equation 6 we mean the formula that you get when you take one disjunct of every conjunct of the left side of the equation and bring those together in a new conjunction. Do this for all possible combinations and bring this new conjunction together in a new disjunction. You get the same formula as described by the right side of equation 3.

First we proof equation 4.

Fact 10 *For all $\varphi_1, ..., \varphi_n, \psi_1, ..., \psi_m \in Lp$ and $s \in S$,*
$s \models (\varphi_1 \vee ... \vee \varphi_n) \to (\psi_1 \vee ... \vee \psi_m)$ *iff*
$s \models (\varphi_1 \to \psi_1 \vee ... \vee \psi_m) \wedge ... \wedge (\varphi_n \to \psi_1 \vee ... \vee \psi_m)$

Proof $\varphi_1, ..., \varphi_n, \psi_1, ..., \psi_m \in Lp$,

$\Rightarrow$ Take an arbitrary $s \in S$, and suppose $s \models (\varphi_1 \vee ... \vee \varphi_n) \to (\psi_1 \vee ... \vee \psi_n)$. We want to show that $s \models (\varphi_1 \to \psi_1 \vee ... \vee \psi_m) \wedge ... \wedge (\varphi_n \to \psi_1 \vee ... \vee \psi_m)$. Take an arbitrary conjunct $\varphi_k \to \bigvee_{1 \leq i \leq m} \psi_i$, with $k \in \{1, ..., n\}$. We want to show that s supports that conjunct. Then we want to show that $\forall t \subseteq s$: if $t \models \varphi_k$ then also $t \models \bigvee_{1 \leq i \leq m} \psi_i$. Take an arbitrary $t \subseteq s$, and suppose $t \models \varphi_k$. Then $t \models \varphi_1 \vee ... \vee \varphi_n$. Then, by the original assumption, $t \models \bigvee_{1 \leq i \leq m} \psi_i$.

$\Leftarrow$ Take an arbitrary $s \in S$, and suppose $s \models (\varphi_1 \to \psi_1 \vee ... \vee \psi_m) \wedge ... \wedge (\varphi_n \to \psi_1 \vee ... \vee \psi_m)$. We want to show that $s \models (\varphi_1 \vee ... \vee \varphi_n) \to (\psi_1 \vee ... \vee \psi_m)$. Then we have to show that $\forall t \subseteq s$: if $t \models \bigvee_{1 \leq j \leq n} \varphi_j$ then also $t \models \bigvee_{1 \leq i \leq m} \psi_i$. Take an arbitrary $t \subseteq s$, and suppose $t \models \bigvee_{1 \leq j \leq n} \varphi_j$. Then there is a j, such that $t \models \varphi_j$. Then by the original assumption: $t \models \bigvee_{1 \leq i \leq m} \psi_i$.

Now we proof equation 5.

Fact 11 *For all $\varphi \in L_p$, all assertions $\psi_1, ..., \psi_m \in L_p$ and all $s \in S$,*
$s \models \varphi \to (\psi_1 \vee ... \vee \psi_m)$ iff
$s \models (\varphi \to \psi_1) \vee ... \vee (\varphi \to \psi_m)$

Proof

$\Rightarrow$ Take an arbitrary $s \in S$, and suppose $s \models \varphi \to (\psi_1 \vee ... \vee \psi_m)$. We want to show that $s \models (\varphi \to \psi_1) \vee ... \vee (\varphi \to \psi_m)$. Therefore we want to show there is some $k \in \{1, ..., m\}$, such that $s \models \varphi \to \psi_k$. Therefore we want to show that $\forall t \subseteq s$: if $t \models \varphi$ then there is some $k \in \{1, ..., m\}$, such that $t \models \psi_k$. Let $u \in S$ be the union of all $t \subseteq s$, such that $t \models \varphi$. Then $u \subseteq s$ (because it is the union of subsets from s) and $u \models \varphi$ (by fact 3 and because φ is an assertion). By the original assumption $u \models \psi_1 \vee ... \vee \psi_m$. Therefore there is some k, such that $u \models \psi_k$. Take an arbitrary t, and suppose $t \models \varphi$ and $t \subseteq s$, then $t \subseteq u$ and therefore (by fact 1) $t \models \psi_k$.

$\Leftarrow$ Take an arbitrary $s \in S$, and suppose $s \models (\varphi \to \psi_1) \vee ... \vee (\varphi_1 \to \psi_m)$. We want to show that $s \models \varphi \to (\psi_1 \vee ... \vee \psi_m)$. Thus we have to show that $\forall t \subseteq s$: if $t \models \varphi$ then also $t \models \psi_1 \vee ... \vee \psi_m$. Take an arbitrary $t \subseteq s$, and suppose $t \models \varphi$. Then by the original assumption there is some $k \in \{1, ..., m\}$, such that $t \models \psi_k$. Then also $t \models \psi_1 \vee ... \vee \psi_m$.

The essential part of the last step from equation 3, equation 6, is actually already proven by fact 9. Then finally we can proof equivalence between an arbitrary formula φ and its disjunctive normal form, $\textsc{dnf}(\varphi)$.

Fact 12 *For all $\varphi \in L_p$, we have that $\textsc{dnf}(\varphi) \equiv \varphi$*

Proof
For all $p \in P$ and all $\psi, \chi \in L_p$:

1. $\textsc{dnf}(p) = p$, thus
   $\textsc{dnf}(p) \equiv p$
2. $\textsc{dnf}(\bot) = \bot$, thus
   $\textsc{dnf}(\bot) \equiv \bot$
3. $\textsc{dnf}(\neg\psi) = \neg\psi$, thus
   $\textsc{dnf}(\neg\psi) \equiv \neg\psi$

4. $\text{DNF}(\psi \vee \chi) = \text{DNF}(\psi) \vee \text{DNF}(\chi)$
According to the IH:
$\text{DNF}(\psi) \equiv \psi$ and
$\text{DNF}(\chi) \equiv \chi$.
Therefore $\text{DNF}(\psi) \vee \text{DNF}(\chi) \equiv \psi \vee \chi$, thus
$\text{DNF}(\psi \vee \chi) \equiv \psi \vee \chi$.

5.
$$\text{DNF}(\psi \wedge \chi) = \bigvee_{\substack{1 \le i \le n \\ 1 \le j \le m}} (\psi_i \wedge \chi_j)$$
where:

- $\text{DNF}(\psi) = \psi_1 \vee ... \vee \psi_n$
- $\text{DNF}(\chi) = \chi_1 \vee ... \vee \chi_m$

According to the IH:
$\text{DNF}(\psi) \equiv \psi$ and
$\text{DNF}(\chi) \equiv \chi$.
Therefore $\psi \wedge \chi \equiv (\psi_1 \vee ... \vee \psi_n) \wedge (\chi_1 \vee ... \vee \chi_m)$
$\equiv$ (by fact 9 and by definition equivalence)
$(\psi_1 \wedge \chi_1) \vee ... \vee (\psi_1 \wedge \chi_m) \vee ... \vee (\psi_n \wedge \chi_1) \vee ... \vee (\psi_n \wedge \chi_m)$
$\equiv \text{DNF}(\psi \wedge \chi)$.
Thus $\text{DNF}(\psi \wedge \chi) \equiv \psi \wedge \chi$.

6.
$$\text{DNF}(\psi \rightarrow \chi) = \bigvee_{\substack{1 \le k_1 \le m \\ \dots \\ 1 \le k_n \le m}} \left(\bigwedge_{1 \le i \le n} (\psi_i \rightarrow \chi_{k_i}) \right)$$
where:

- $\text{DNF}(\psi) = \psi_1 \vee ... \vee \psi_n$
- $\text{DNF}(\chi) = \chi_1 \vee ... \vee \chi_m$

According to the IH:
$\text{DNF}(\psi) \equiv \psi$ and
$\text{DNF}(\chi) \equiv \chi$.
Therefore $\psi \rightarrow \chi \equiv (\psi_1 \vee ... \vee \psi_n) \rightarrow (\chi_1 \vee ... \vee \chi_m)$
$\equiv$ (by fact 10 and by definition equivalence)
$(\psi_1 \rightarrow (\chi_1 \vee ... \vee \chi_m)) \wedge ... \wedge (\psi_n \rightarrow (\chi_1 \vee ... \vee \chi_m))$
$\equiv$ (by fact 11 and by definition equivalence)
$((\psi_1 \rightarrow \chi_1) \vee ... \vee (\psi_1 \rightarrow \chi_m)) \wedge ... \wedge ((\psi_n \rightarrow \chi_1) \vee ... \vee ((\psi_n) \rightarrow \chi_m))$
$\equiv$ (by fact 9 and by definition equivalence)
$((\psi_1 \rightarrow \chi_1) \wedge ... \wedge (\psi_n \rightarrow \chi_1)) \vee ... \vee$
$((\psi_1 \rightarrow \chi_1) \wedge ... \wedge (\psi_n \rightarrow \chi_m)) \vee ... \vee$
$((\psi_1 \rightarrow \chi_m) \wedge ... \wedge (\psi_n \rightarrow \chi_1)) \vee ... \vee$

$((\psi_1 \to \chi_m) \wedge ... \wedge (\psi_n \to \chi_m))$
$\equiv$ DNF$(\psi \to \chi)$.
Thus DNF$(\psi \to \chi) \equiv \psi \to \chi$.

Now that we know that when we compute DNF(ψ) we don't lose any properties of importance of ψ to compute its compliant responses when we compute its disjunctive normal form, we can focus on the next step in the algorithm: compute the clean disjunctive normal form of ψ. Of course we also have to proof that computing the CDNF(ψ) maintain the properties of importance of ψ to compute its compliant responses. And therefore we proof equivalence between φ and CDNF(φ).

Fact 13 *For $\varphi \in L_p$, we have that* CDNF$(\varphi) \equiv \varphi$.

Proof We already know (by fact 12) that DNF$(\varphi) \equiv \varphi$, so we only need to show that CDNF$(\varphi) \equiv$ DNF(φ).

$\Rightarrow$ Take an arbitrary $s \in S$, and suppose $s \models$ CDNF(φ). We want to show that $s \models$ DNF(φ). We know that s supports some of the disjuncts in CDNF(φ) (by fact 8). All the disjuncts in CDNF(φ) also occur in DNF(φ) (by definition 12) and therefore s also supports some of the disjuncts in DNF(φ) (the same disjuncts as in CDNF(φ)). Therefore (by fact 8) $t \models$ DNF(φ).

$\Leftarrow$ Take an arbitrary $s \in S$, and suppose $s \models$ DNF(φ). We want to show that $s \models$ CDNF(φ). There is some disjunct d in DNF(φ) which is supported by s (by fact 8). Two possibilities: (1) d is also a disjunct in CDNF(φ), then also $s \models$ CDNF(φ). Or (2) d is not a disjunct in CDNF(φ), but then (by definition 12) d entails some other disjunct d' in CDNF(φ). Because $s \models d$ we have also that $s \models d'$. Therefore also $s \models$ CDNF(φ).

We compute CDNF(ψ) to get all possibilities of ψ. We want to proof that every disjunct of CDNF(ψ) has a one-on-one relation with the possibilities for ψ. Therefore we want to show that evey disjunct in CDNF(ψ) has only one possibility, i.e. that it is an assertion. To proof that, we first have to proof that DNF(ψ) is a disjunction of assertions.

Fact 14 *For all $\varphi \in L_p$, we have that* DNF(φ) *is a disjunction of assertions*

Proof

1. DNF$(p) = p$
   p is an assertion (fact 4.1) and therefore also a disjunction of assertions.

2. DNF$(\bot) = \bot$
   $\bot$ is an assertion (fact 4.2) and therefore also a disjunction of assertions.

3. DNF$(\neg\psi) = \neg\psi$
   $\neg\psi$ is an assertion (fact 4.3) and therefore also a disjunction of assertions.

4. $\textsc{dnf}(\psi \vee \chi) = \textsc{dnf}(\psi) \vee \textsc{dnf}(\chi)$
According to the IH $\textsc{dnf}(\psi), \textsc{dnf}(\chi)$ are disjunctions of assertions.
Therefore also $\textsc{dnf}(\psi) \vee \textsc{dnf}(\chi)$ is a disjunction of assertions.

5. $$\textsc{dnf}(\psi \wedge \chi) = \bigvee_{\substack{1 \leq i \leq n \\ 1 \leq j \leq m}} (\psi_i \wedge \chi_j)$$

   where:

   - $\textsc{dnf}(\psi) = \psi_1 \vee ... \vee \psi_n$
   - $\textsc{dnf}(\chi) = \chi_1 \vee ... \vee \chi_m$

   According to the IH $\textsc{dnf}(\psi), \textsc{dnf}(\chi)$ are disjunctions of assertions.
Therefore $\psi_1, ..., \psi_n, \chi_1, ..., \chi_m$ are assertions.
Therefore and by fact 4.4 $\psi_i \wedge \chi_j$ are assertions.
Therefore the disjunction of $\psi_i \wedge \chi_j$ is a disjunction of assertions.
Thus $\textsc{dnf}(\psi \wedge \chi)$ is an assertion.

6. $$\textsc{dnf}(\psi \rightarrow \chi) = \bigvee_{\substack{1 \leq k_1 \leq m \\ \cdots \\ 1 \leq k_n \leq m}} \left(\bigwedge_{1 \leq i \leq n} (\psi_i \rightarrow \chi_{k_i}) \right)$$

   where:

   - $\textsc{dnf}(\psi) = \psi_1 \vee ... \vee \psi_n$
   - $\textsc{dnf}(\chi) = \chi_1 \vee ... \vee \chi_m$

   According to the IH $\textsc{dnf}(\chi)$ is a disjunction of assertions.
Therefore $\chi_1, ..., \chi_m$ are assertions.
Therefore and by fact 4.5 $\psi_i \rightarrow \chi_{k_i}$ are assertions.
Therefore and by fact 4.4 the conjunction of $\psi_i \rightarrow \chi_{k_i}$ is an assertion.
Therefore the disjunction of conjunctions of $\psi_i \rightarrow \chi_{k_i}$ is a disjunction of assertions.
Thus $\textsc{dnf}(\psi \rightarrow \chi)$ is an assertion.

Now we can proof that $\textsc{cdnf}(\psi)$ is a disjunction of assertions.

Fact 15 *For all $\varphi \in L_p$,*
$\textsc{cdnf}(\varphi)$ is a disjunction of assertions.

Proof According to the definition $\textsc{cdnf}(\varphi)$ is obtained from $\textsc{dnf}(\varphi)$ by only removing some disjuncts. Nothing is added, and because all disjuncts in $\textsc{dnf}(\varphi)$ are assertions (by fact 14) also the disjuncts in $\textsc{cdnf}(\varphi)$ are assertions.

Next we can proof that the unique possibility for a disjunct of $\textsc{cdnf}(\psi)$ is a possibility for $\textsc{cdnf}(\psi)$ and that for every possibility for $\textsc{cdnf}(\psi)$ there is some disjunct of $\textsc{cdnf}(\psi)$ for which it is the possibility. Which means $\textsc{cdnf}(\psi)$ makes the possibilities for ψ come into sight.

Fact 16 *For $\varphi \in L_p$ and $\pi \in S$,*
π is a possibility for φ iff π is a possibility (the unique possibility) for some disjunct of $\textsc{cdnf}(\varphi)$.

Proof

$\Rightarrow$ Take an arbitrary $s \in S$, and suppose s is a possibility for $\textsc{cdnf}(\varphi)$, then s is a maximal state that supports $\textsc{cdnf}(\varphi)$. Then there is a disjunct d of $\textsc{cdnf}(\varphi)$, such that $s \models d$ (by fact8). Take an arbitrary $s' \in S$, and suppose $s \subset s'$ and $s' \models d$. Then $s' \models \textsc{cdnf}(\varphi)$ (by fact 8). But then $s \not\subset s'$ (because s is a maximal state supporting $\textsc{cdnf}(\varphi)$). This leads towards a contradiction. Thus s is a maximal state that supports d, and therefore a possibility for d. Moreover s is the unique possibility because d is an assertion (by fact 15).

$\Leftarrow$ Take some disjunct d of $\textsc{cdnf}(\varphi)$, then d is an assertion (by fact 15). Therefore d has only one possibility. This unique possibility is a state s, such that $s \models d$. Then also $s \models \textsc{cdnf}(\varphi)$ (by fact 8). Take an arbitrary other state $s' \in S$, and suppose $s \subset s'$. Then according to the left-to-right direction of this proof, there is some disjunct d' of $\textsc{cdnf}(\varphi)$, such that s' is the unique possibility for d', and so $s' \models d'$. Then $s \models d'$ (by fact 1). But this means that $d \models d'$. This is only possible if $d = d'$ (by definition 12). But then $s = s'$. This leads towards a contradiction. Thus s is a maximal state that supports $\textsc{cdnf}(\varphi)$, and therefore a possibility for $\textsc{cdnf}(\varphi)$.

The next step in the algorithm is to compute the potentially compliant assertion of ψ, $\textsc{pca}(\psi)$. What we want to show is that every $\varphi \in \textsc{pca}(\psi)$ satisfies the first condition of being compliant to ψ and that every assertion that satisfies the first condition of being compliant to ψ is equivalent with some $\varphi \in \textsc{pca}(\psi)$.

Fact 17 *For $\varphi, \psi, \chi \in L_P$,*
φ is logically equivalent with some $\chi \in \textsc{pca}(\psi)$ iff
$[\varphi] = \{|\varphi|\}$ and $|\varphi|$ is the union of a set of possibilities in $[\psi]$.

Proof

$\Rightarrow$ Take an arbitrary $\varphi \in L_p$, and suppose there is some $\chi \in \textsc{pca}(\psi)$, such that $\varphi \equiv \chi$. Since $\chi \in \textsc{pca}(\psi)$, we have that $\chi = !(\psi_{i_1} \vee ... \vee \psi_{i_m})$, with $i_1, ..., i_m \in \{1, ..., n\}$ and $\textsc{cdnf}(\psi) = \psi_1 \vee ... \vee \psi_n$. Since $\chi = !(...)$, we know that χ is an assertion (by definition of ! and fact 4.3) and therefore $[\chi] = \{|\chi|\}$ (by fact 3). By the original assumption (and by definition 8) $[\chi] = [\varphi]$, and therefore $[\varphi] = \{|\chi|\} = \{|\varphi|\}$. We want to show that $|\varphi|$, that is $|\chi|$, is the union of a set of possibilities in $[\psi]$. We know $|\chi| = |!(\psi_{i_1} \vee ... \vee \psi_{i_m})| =$ (by double negation law in classical logic)

$|\psi_{i_1} \vee ... \vee \psi_{i_m}|$ = (because the truthset of a disjunction is the union of truthsets of all disjuncts) $|\psi_{i_1}| \vee ... \vee |\psi_{i_m}|$. Take an arbitrary ψ_{i_k}, with $k \in \{1, ..., m\}$. Then $i_k \in \{1, ..., n\}$, so ψ_{i_k} is a disjunct of CDNF(ψ). Therefore $|\psi_{i_k}|$ is a possibility for ψ (by fact 16). And therefore $|\chi|$ is the union of a set of possibilities in $[\psi]$.

$\Leftarrow$ Take arbitrary $|\psi_{i_1}|, ..., |\psi_{i_m}| \in [\psi]$. Then $\psi_{i_1}, ..., \psi_{i_m}$ are disjuncts of CDNF(ψ) (by fact 16). Take an arbitrary φ, and suppose $[\varphi] = \{|\varphi|\}$ and $|\varphi|$ is the union of a set of possibilities in $[\psi]$. Then we know $|\varphi| = |\psi_{i_1}| \bigcup ... \bigcup |\psi_{i_m}|$ = (because the union of truthsets is the truthset of the disjunction) $|\psi_{i_1} \vee ... \vee \psi_{i_m}|$ = (by double negation law in classical logic) $|!(\psi_{i_1} \vee ... \vee \psi_{i_m})|$. Therefore $[\varphi] = \{|!(\psi_{i_1} \vee ... \vee \psi_{i_m})|\}$. Take $\chi = !(\psi_{i_1} \vee ... \vee \psi_{i_m})$. Then χ is in CDNF(ψ), and therefore $[\chi] = \{|!(\psi_{i_1} \vee ... \vee \psi_{i_m})|\}$ (by fact 16). Then $[\varphi] = [\chi]$ and therefore $\varphi \equiv \chi$. And by definition $\chi \in$ PCA(ψ).

Now that we have the potentially compliant assertion of ψ the algorithm its next step is to compute the potentially compliant responses of ψ (PCR(ψ)). What we want to show is that every $\varphi \in$ PCR(ψ) satisfies the first condition of being compliant to ψ and every response that satisfies the first condition of being compliant to ψ is equivalent with some $\varphi \in$ PCR(ψ).

Fact 18 *For $\varphi, \psi, \chi \in L_P$,*
φ is logically equivalent with some $\chi \in$ PCR(ψ) iff
every possibility in $[\varphi]$ is the union of a set of possibilities in $[\psi]$.

Proof

$\Rightarrow$ Take an arbitrary $\varphi \in L_p$, and suppose that there is some $\chi \in$ PCR(ψ), such that $\varphi \equiv \chi$. Then $[\varphi] = [\chi]$. Thus since we want to show that every possibility in $[\varphi]$ is the union of a set of possibilities in $[\psi]$, it is sufficient to show that every possibility in $[\chi]$ is the union of a set of possibilities in $[\psi]$. Because $\chi \in$ PCR(ψ), we have that $\chi = (\chi_1 \vee ... \vee \chi_n)$, with $1 \leq n \leq |\text{PCA}(\psi)|$ and $\chi_1, ..., \chi_n \in$ PCA(ψ) and for no $i, j \in \{1, ...n\}$ $\chi_i \models \chi_j$. Then χ is in DNF(χ) (because χ is a disjunction of assertions of the form !(..)) and χ is also in CDNF(χ) (because χ is in DNF(χ) and there is no disjunct in χ which entails some other disjunct in χ). Then every possibility in $[\chi]$ is the unique possibility for some disjunct of χ (by fact 16). Take an arbitrary disjunct d of χ, then $d \in$ PCA(ψ). Therefore the unique possibility for d is the union of a set of possibilities in $[\psi]$ (by fact 17). So every possibility in $[\chi]$ is the union of a set of possibilities in $[\psi]$.

$\Leftarrow$ Take an arbitrary $\varphi \in L_P$, and suppose every possibility in $[\varphi]$ is the union of a set of possibilities in $[\psi]$. We want to show that φ is logically equivalent with some χ such that $\chi \in$ PCR(ψ). We know $\varphi \equiv$ CDNF(φ) (by fact 13) and every possibility in [CDNF(φ)] is the unique possibility for some disjunct in CDNF(φ). Take an arbitrary disjunct d of CDNF(φ), then $[d] = \{|d|\}$ (because d is an assertion by fact 15) and by the original assumption d is the union of a set of possibilities in $[\psi]$. But then $d \in$ PCA(ψ) (by fact 17). So CDNF(φ) is a disjunction of formulas in PCA(ψ). Because by definition all possible disjunctions of formulas in PCA(ψ) are in PCR(ψ) in which no disjunct entails another disjunct, also CDNF(φ) $\in$ PCR(ψ). So φ is logically equivalent with χ ($\chi =$ CDNF(φ)) and $\chi \in$ PCR(ψ).

The last step in the soundness and completeness proof for the algorithm presented in the paper is that all the formulas in the output, that is in CR(ψ), really are compliant to ψ and that every formula that is compliant to ψ is equivalent with a formula in the output, that is CR(psi).

Theorem 1 (Soundness and Completeness of the Algorithm) *For $\varphi, \psi, \chi \in L_P$,*
φ is logically equivalent with some $\chi \in$ CR(ψ) iff
φ is compliant with ψ.

Proof

$\Rightarrow$ Take an arbitrary $\varphi \in L_p$, and suppose φ is logically equivalent with some $\chi \in$ CR(ψ). We want to show that φ is compliant with ψ, therefore we have to show that 1. every possibility in $[\varphi]$ is the union of a set of possibilities in $[\psi]$ and 2. every possibility in $[\psi]$ restricted to $|\varphi|$ is contained in a possibility in $[\varphi]$.
1. All formulas in CR(ψ) are also in PCR(ψ). Hence $\chi \in$ PCR(ψ), and therefore every possibility in $[\varphi]$ is the union of a set of possibilities in $[\psi]$ (by fact 18).
2. Since $\varphi \equiv \chi$, we know $|\varphi| = |\chi|$ (by fact 7) and also $[\varphi] = [\chi]$ (by definition 8). Therefore it is sufficient to prove that every possibility in $[\psi]$ restricted to $|\chi|$ is contained in a possibility in $[\chi]$. Take an arbitrary possibility π in $[\psi]$, then we know π is the unique possibility for some disjunct ψ_j of CDNF(ψ) (by fact 16). Since $\chi \in$ CR(ψ), we have $\psi_j \wedge !\chi \models \chi_g$ for some disjunct χ_g of χ. Therefore we have $|\psi_j| \cap |\chi| \subseteq |\chi_g|$. We know that $|\psi_j| \cap |\chi|$ is the restriction of a possibility in $[\psi]$ to $|\chi|$ and is contained in the unique possibility α for $|\chi_g|$. So it remains to show that α is contained in some possibility for χ. Since $\alpha \models \chi_g$, we have also $\alpha \models \chi$ (by fact 8). But then we know that α is contained in a possibility for χ (by fact 2).

$\Leftarrow$ Take arbitrary $\varphi, \psi \in L_p$, and suppose φ is compliant to ψ. We want to show there is some $\chi \in$ CR(ψ), such that $\varphi \equiv \chi$. We know every possibility in $[\varphi]$ is the union of a set of possibilities in $[\psi]$ and therefore there is some $\chi \in$ PCR(ψ), such that $\varphi \equiv \chi$ (by fact 18). Now we will show that χ is also in CR(ψ). We know that every possibility in $[\psi]$ restricted to $|\varphi|$ is contained in a possibility in $[\varphi]$. Since $\varphi \equiv \chi$, we have that $|\varphi| = |\chi|$ (by fact 7) and $[\varphi] = [\chi]$ (by definition 8). Therefore we have also that every possibility in $[\psi]$ restricted to $|\chi|$ is contained in a possibility in $[\chi]$.
Take an arbitrary disjunct ψ_j of CDNF(ψ). We want to show that $\psi_j \wedge !\chi \models \chi_g$ for some disjunct χ_g of χ. That is $|\psi_j| \bigcap |\chi| \subseteq |\chi_g|$ for some disjunct

χ_g of χ (since $\psi_j, !\chi$ and χ_g are assertions). We know that $|\psi_j| \cap |\chi|$ is the restriction of a possibility in $[\psi]$ to $|\chi|$. We also know that every possibility for ψ restricted to $|\chi|$ is contained in a possibility π for χ. Then we know that $|\psi_j| \cap |\chi| \subseteq \pi$. So it remains to show that $\pi \subseteq |\chi_g|$ for some disjunct χ_g of χ. We know that $\pi \models \chi$. So (by fact 8) $\pi \models \chi_g$ for some disjunct χ_g of χ. Therefore $\pi \subseteq |\chi_g|$.

6 Complexity

In this section we explore the time complexity of the algorithm presented in this paper. First we look at the complexity for computing the disjunctive normal form of some formula ψ with l occurrences of propositions.

Complexity Disjunctive Normal Form. The first 3 cases of the disjunctive normal form can be computed in polynomial time. The complexity of the 4th case of the disjunctive normal form, $O(\textsc{dnf}(\varphi \vee \chi)) = O(O(\textsc{dnf}(\varphi)) + O(\textsc{dnf}(\chi)))$. The complexity of the 5th case of the disjunctive normal form, $O(\textsc{dnf}(\varphi \wedge \chi)) = O(O(\textsc{dnf}(\varphi)) * O(\textsc{dnf}(\chi)))$. The complexity of the 6th and last case of the disjunctive normal form, $O(\textsc{dnf}(\varphi \to \chi)) = O(O(\textsc{dnf}(\varphi)) * O(\textsc{dnf}(\chi))^{O(\textsc{dnf}(\varphi))})$.
This means that worst case complexity of computing $\textsc{dnf}(\psi)$, with l occurrences of propositions in ψ is $O(l^l)$.

Next we look at the complexity for computing the clean disjunctive normal form of some formula ψ with l occurrences of propositions.

Complexity Clean Disjunctive Normal Form. For every disjunct in $\textsc{dnf}(\psi)$ we have to check if it entails any of the other disjunct. Entailment over (worst-case) l^l disjuncts is in $O(2^{l^l})$. In worst case one has to check it for every disjunct over all other disjuncts which is in $O(l^2)$. But the $O(O(l^2) * O(2^{l^l})) = O(2^{l^l})$.

We know that the clean disjunctive normal form of some formula ψ gives us just as much disjunctions as there are possibilities for ψ. Possibilities are better known is mathematics as 'antichains'. From antichains is known that the maximal possible antichains over m indices (according to the Sperner's theorem) is the middle of the binominal coefficient ([Sperner, 1928, 544-548]). Suppose we have n proposition letters in formula ψ, then there are 2^n indices, which means at most $\binom{2^n}{0.5*2^n}$ possibilities. Now we can compute the complexity of the next step, computing the potentially compliant assertions of ψ, on the basis of n which denotes the number of proposition letters in ψ.

Complexity Potentially Compliant Assertions. By computing the potentially compliant assertions we take the power set (without the empty set) of the possibilities for ψ. There are $O(2^{2^n})$ possibilities for n propositions in ψ, which means a complexity for computing potentially compliant assertions is in

$O(2^{2^{2^n}})$.

Next we explore the complexity of computing the potentially compliant responses.

Complexity Potentially Compliant Responses. By computing the potentially compliant responses we (roughly) take the power set of the potentially compliant assertions for ψ (without the empty set). There are $O(2^{2^{2^n}})$ potentially compliant assertions for n propositions in ψ, which means a complexity for computing potentially compliant responses is in $O(2^{2^{2^{2^n}}})$.

And then finally the last step of the algorithm is to compute the compliant responses.

Complexity Compliant Responses. By computing the compliant responses we look for every potentially compliant response $O(2^{2^{2^{2^n}}})$ if for every disjunct of ψ together with the truthset of the potentially compliant, there is some disjunct of the potentially compliant response which entails it. This is in $O(2^{2^{2^{2^{2^n}}}})$.

The complexity analyses tells us that the source of the complexity lies in computing the $\text{DNF}(\psi)$, but this is only the case when ψ is built up from (some) arrows. However, in pratice computing the disjunctive normal form of ψ doesn't seem to be the real comptational time bottleneck for this algorithm since in human conversation an utterance is rarely built up from more than three arrows. And when a sentence does reach an amount of more than three arrows humans get confused themselves. So this is a good reason not to worry so much about the complexity of computing $\text{DNF}(\psi)$, since computing the disjunctive normal form of formulas that have no implication is not very complex in time steps.
None the less is the time complexity for computing all compliant responses of a formula still in $O(2^{2^{2^{2^{2^n}}}})$, with n the number of proposition letters in the formula. Here we can bring in that the total amount of potentially compliant responses is certainly not as great as the amount of the power set of the power set. The exact amount of how many compliant responses are possible for n proposition letters is the same question as, how many antichains are there for the 2^n-set when the empty set is not considered a valid antichain. This question is unresolved and known as the Dedekind problem ([Dedekind, 1897, 103-148]). For small 2^n-sets the the number of antichains is known (see figre 9).
Despite the fact that the number of possible compliant responses for a formula is much less than the computation steps it takes the presented algorithm to compute all those responses, this number is still extremely large. Suppose we would expand the system inquisitive semantics to first order logic, for question like 'Who comes to the party', for n people in the domain of discourse, we have as much as compliant responses as there are antichains for the 2^n-set.

n	2^n	highest amont of compliant responses
0	0	1
1	2	4
2	4	165
3	8	56130437228687557907786

Figure 9: amount compliant answers, Sloane http://www.research.att.com/ njas/sequences/table?a=14466&fmt=4

7 Implementation

The algorithm presented in this paper is implemented and can be found at:

www.illc.uva.nl/inquisitive-semantics/computing-compliance

8 Conclusion, Discussion and Future Work

In this paper the notion of compliance in inquisitive semantics has been brought into practice. An algorithm for compliance is presented and investigated. The algorithm presented is proved to be sound and complete according to the definition of compliance.
The computation time has also been analyzed. The result is that the algorithm for computing all compliant responses for some formula ψ with length l is in $O(l^l)$. However this is worst case complexity and depends on the worst case assumption that the connectives in ψ all are implications. When we talk about human conversation this assumption seems to be very unlikely. Most people don't speak in sentences with many implications. Still, when we ignore the large computation time when many implications are involved, the computation time is in $O(2^{2^{2^{2^{2^n}}}})$.
The algorithm is implemented and can be found at www.illc.uva.nl/inquisitive-semantics/computing-compliance.

The impact of these results (especially of the analyzed complexity) for the philosophical and psychological interest is that it is somewhat unlikely that people consider all possible compliant answers to an issue when they choose how to respond. Question like 'Who comes to the party?' are for humans easy to answer when we talk about 30 possible candidates. But the amount of compliant answers for $n = 30$ is not even known by scientists, which makes it very unlikely that people consider al these answers.
The impact of these results (especially of the analyzed complexity) for the AI/HCI is that it is too bad that it take so much computational power to calculate the compliant answers for the input. On the other side it is good to know that it is possible to compute the compliant responses (at least for small formulas), but there is still room for improvement.

Fortunately inquisitive semantics provides us a framework on which we can base further improvements. It would be straight forward to go on with computing the best compliant response (or maybe the n-best compliant responses) and try to find a way in which it would not be necessary to first compute all compliant responses and then pick the best, but to only calculate the best responses. A step further would be to calculate the best compliant response relative to an information state. This means you have to consider except the maxim of relatedness (dependent on compliance) also the other maxims of inquisitive pragmatics that are the maxims of quantity and quality.

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