An Inquisitive Formalization of Interrogative Inquiry: Logical and Computational Aspects

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Abstract

Interrogative inquiry refers to the process of knowledge-seeking by questioning. In this paper, we investigate the process of interrogative inquiry in the context of conversations. To this end, we develop a formalization of interrogative inquiry based on inquisitive semantics and pragmatics. This is motivated by the capacity of inquisitive semantics to provide a semantic account of questions and answers in natural language, and the capacity of inquisitive pragmatics to provide a pragmatic account of the behavior of questions and answers in conversations.

The paper begins with a presentation of the modelling of questions and answers in the inquisitive framework. Then, after a brief comparison of the inquisitive account of questions with Hintikka's treatment of questions in the Interrogative Model of Inquiry (IMI), we discuss and define the notion of *interrogative rule* which aims to characterize the question-answer steps that one can make in an interrogative inquiry. We then put the interrogative rule into a temporal perspective, by introducing the notion of *interrogative protocol*, which aims to govern interrogative inquiry as a temporal process. The notion of interrogative protocol enables us to reach formal definitions of the notion of *interrogative inquiry* and the associated logical notion of *interrogative consequence*, that we illustrate with some concrete examples. Our framework thus defined allows then for a formal logical and computational study of the process of interrogative inquiry. On the logical side, we relate the notion of interrogative consequence with the ones of distributed information and yes-no question. On the computational side, we shape the bases of a computational investigation of interrogative inquiry in our framework, and we present first computational results. From this computational perspective, we propose to revisit the so-called strategic aspects of inquiry, one of the main themes of Hintikka's IMI, from an algorithmic point of view. We end this paper with some concluding remarks and suggestions for further works.

1 Introduction and motivation

The notion of *interrogative inquiry* refers to the process of *knowledge-seeking by questioning*. Surprisingly, one encounters this process as a topic of interest across several fields. In philosophy, interrogative inquiry was at the heart of the first systematic method of philosophizing known as the socratic method, or *elenchus*, constitutive of the Platonic dialogues, in which progress in the philosophical investigations was based on the information obtained as answers to Socrates' questions. In computer science, interrogative inquiries take the form of sequences of queries into databases or into internet webpages, while one of the tasks of artificial intelligence is precisely to design artificial inquirers capable of collecting information in order to answer specific questions with an optimal use of available resources. Last but not least, interrogative inquiry can be seen as a language-game in which we find ourselves engaged in everyday in order to be brought in specific informational states, making the mechanisms

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that underlie interrogative inquiry a suitable topic of investigation from the point of view of the pragmatics of natural language.

One of the prominent contemporary philosophers and logicians to acknowledge the importance of questioning for the study of reasoning and to develop a systematic philosophical and logical investigation of interrogative inquiry is Jaakko Hintikka. His approach crystalized around the development of the so-called *Interrogative Model of Inquiry (IMI)* [10, 13] which represents interrogative inquiry as a *game* between two players called the *Inquirer* and *Nature*. The game is played on a fix model M and the role of the Inquirer is to establish a given conclusion C from a background theory T. To this end, the Inquirer can make *interrogative moves*, which consist in putting questions to Nature and registering the answers as additional premises, or *deductive moves*, which consist in drawing logical inferences from the information already obtained by the Inquirer. Hintikka and colleagues proposed a formalization of the IMI through the development of *Interrogative Logic* [15], which is based on Hintikka's theory of questions for representing interrogative moves, and on the tableau method for representing deductive moves. Recently, Hintikka turned his account of interrogative inquiry into a general approach to epistemology, called *Socratic Epistemology* [14], which proposes to shift the attention of current epistemology from a focus on the justification and evaluation of already acquired knowledge towards a focus on *how* new knowledge is obtained. Socratic epistemology proposes precisely to represent knowledge acquisition as a questioning process, echoing the socratic method of Platonic dialogues.

By making reference to the Platonic dialogues, Hintikka is implicitly making the important observation that conversation constitutes one of the main stages of interrogative inquiry. However, little attention has been given so far to the specificities of the process of interrogative inquiry in conversational contexts, even though, as we noticed at the beginning, everyone is conducting interrogative inquiries in conversations on a daily basis. In this paper, we will refer to the investigation of interrogative inquiry in conversations as the study of the *language-game of interrogative inquiry*. From a logical point of view on interrogative inquiry, such a perspective will require to attach a particular attention (i) to the formal representation of conversational contexts and (ii) to the logical modelling of questions and answers.

Recently, significant advances have been made in the field of formal semantics and pragmatics on the representation of questions in discourse. One of the most promising lines of research in this direction is the development of *inquisitive semantics* and *pragmatics* [4, 7]. The main idea of inquisitive semantics is to provide a notion of meaning which incorporates both *informative* and *inquisitive contents*, where the informative content of a sentence refers to its capacity to bring in new information, while its inquisitive content refers to its capacity to raise new issues. The inquisitive notion of meaning leads to a new perspective on pragmatics allowing a fine-grained analysis of the regulative principles of information exchange in conversations, along with a precise modelling of the meaning and the role of questions in conversational information flow. Thus, inquisitive semantics and pragmatics appears as a very suitable framework for investigating the language-game of interrogative inquiry.

The aim of this paper is precisely to develop a formalization of interrogative inquiry based on the inquisitive framework. Indeed, besides its adequacy to represent questions in a conversational context, several features of inquisitive semantics and pragmatics motivate such a project. More specifically, the inquisitive framework offers: (i) a sophisticated modelling of questions which allows to represent *embedded questions*, such as conditional and alternative questions; (ii) a semantic categorization of *questions* and *assertions*; (iii) a precise notion of *answerhood*; (iv) an account of *complete* and *partial answers*. In this paper, we will see that each one of these features turns out to be directly relevant to a formal investigation of interrogative inquiry.

Hintikka's IMI is obviously one of the main sources of inspiration of this paper. We will discuss important themes developed in the IMI, and we will try to relate and compare our work with the one of Hintikka. However, it is important to notice that our approach will deviate significantly from the game-theoretic approach of the IMI. More precisely, we will ground our investigation on a different approach to the pragmatics of question-answer steps in interrogative inquiry: we will consider that the pragmatic rules governing the production of answers to questions are *deterministic*, while in the framework of the IMI these rules are *non-deterministic*. To better understand this point, we introduce the distinction between the notions of *interrogative game* and *interrogative protocol*: in the former, the answerer has a choice between different answers that he can provide to a given question, whereas in the latter, the answer to a question is uniquely determined with respect to the informational state of the answerer. The IMI proposes to study interrogative inquiries in the context of *interrogative games*. This paper investigates interrogative inquiries in the context of *interrogative protocols*.

The paper is organized as follows. In section 2, we provide the elements of inquisitive semantics necessary to the logical modelling of questions and answers in the inquisitive framework. In section 3, we show how one can recover, in this context, the key notions constitutive of Hintikka's treatment of questions in the IMI, i.e., the notions of *propositional question*, *presupposition* and *desideratum*. In section 4, we discuss and define the notion of *interrogative rule* which aims to characterize completely the question-answer steps that one can make in an interrogative inquiry. In section 5, we put the interrogative rule into a temporal perspective by introducing the notion of *interrogative protocol* which aims to govern interrogative inquiry as a temporal process. This allows us to define the two key notions of *interrogative inquiry* and *interrogative consequence*, which we illustrate by some concrete examples. One of the main arguments in favor of the framework thus defined lies in its capacity to allow a precise investigation of the logical and computational aspects of interrogative inquiry, as illustrated respectively in section 6 and 7. On the logical side, we relate the logical notion of interrogative consequence to the ones of distributed information and yes-no question. On the computational side, we discuss and shape the bases of a computational investigation of interrogative inquiry in our framework, and we use it to obtain first computational results. Such a computational approach suggests an interesting way to revisit one of the key themes investigated by Hintikka within the IMI, namely the *strategic aspects* of interrogative inquiry. We call this approach the *algorithmic view* on the strategic aspects of interrogative inquiry, and we present it in section 8. Section 9 ends this paper with some concluding remarks and proposals for further works.

2 Modelling questions and answers in inquisitive semantics

In this section, we present the basic elements of inquisitive semantics necessary to define (i) the semantic categories of *questions* and *assertions* and (ii) the notion of *answerhood*.[1] We also introduce the notions of *settledness*, *characteristic proposition* and *characteristic question* that will be useful in the following sections. First of all, we consider a propositional language $\mathcal{L}$:

Definition 2.1 (Language $\mathcal{L}$)**.** *Let $\mathcal{P}$ be a finite set of propositional variables. The language $\mathcal{L}$ is given by:*

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \varphi \to \psi$$

where $p \in \mathcal{P}$.

[1] We will adopt the version of inquisitive semantics developed in [2], [4], [7] and [21]. An earlier version of inquisitive semantics was presented in [6] and [17]. In [4], the authors discuss the relation between the earlier and the new presentation of inquisitive semantics, and argue in favor of the new system. In this paper, our presentation of inquisitive semantics follows the ones of [4] and [7].

We also introduce two non-standard operators, ! and ?, where $!\varphi$ is defined as $\neg\neg\varphi$ and $\varphi?$ is defined as $\varphi \vee \neg\varphi$. We will make explicit the meaning of these two operators shortly.

On the semantic side, the first step is to define the notion of *support* which in turn will be used to define the notion of *proposition*. To this end, we need to define the notions of *index* and *state*:

Definition 2.2 (index and state). *The notions of index and state are defined as follows:*

- *An index v is a binary valuation $v : \mathcal{P} \to \{0, 1\}$ for the set of propositional variables $\mathcal{P}$,*
- *A state is a non-empty set of indices.*

We will use the following notations: v as a variable ranging over indices, s, t as variables ranging over states, ω to denote the set of all indices and $\mathcal{S}$ to denote the set of all states. We can now define the notion of *support*:

Definition 2.3 (Support). *The notion of support is defined recursively as follows:*

1. $s \models p$ *iff* $\forall v \in s : v(p) = 1$
2. $s \models \neg\varphi$ *iff* $\forall t \subseteq s :$ *not* $t \models \varphi$
3. $s \models \varphi \vee \psi$ *iff* $s \models \varphi$ *or* $s \models \psi$
4. $s \models \varphi \wedge \psi$ *iff* $s \models \varphi$ *and* $s \models \psi$
5. $s \models \varphi \to \psi$ *iff* $\forall t \subseteq s$: *if* $t \models \varphi$ *then* $t \models \psi$

In the above definition, we read $s \models \varphi$ as *state s supports φ*.[2] At this point, we have the choice of defining the main notions of inquisitive semantics either in an *absolute* way, i.e., with respect to the set of all states, or *relativized* to particular states. The first perspective is adapted for studying the logical properties of inquisitive semantics, while the second perspective is more suitable for studying specific conversational situations, as exemplified in the study of inquisitive pragmatics [7]. In the case of interrogative inquiry, we are interested in representing questions and answers in particular epistemic situations, and we adopt thereby the same perspective as inquisitive pragmatics, i.e., we define the main inquisitive notions relativized to particular states.

We now turn to the inquisitive notion of *proposition* which is defined via the notions of *support* and *possibility*:

Definition 2.4 (Possibility, proposition and truth set). *Let $\varphi \in \mathcal{L}$ and $s \in \mathcal{S}$.*

1. *A possibility for φ in s is a maximal substate of s supporting φ.*
2. *The proposition expressed by φ in s, denoted by $s[\varphi]$, is the set of possibilities for φ in s.*
3. *The truth set of φ in s, denoted by $s|\varphi|$, is the set of indices in s where φ is classically[3] true.*[4]

[2] For a discussion of the conceptual interpretation of the notion of support, we refer the reader to the section 9 of [4].

[3] In this paper, when we say that φ is classically true in index v, we mean that φ is true under the valuation v in the sense of propositional logic. Then, when we say that φ is classically true in a state s, we mean that φ is classically true in each index $v \in s$.

[4] Notice that the notion of *truth-set* recovers the classical definition of proposition as the set of all indices in s that make φ true.

In inquisitive pragmatics, the intended interpretation of the state s to which we relativize our definitions is to represent the *common ground* of the conversation. Thus, the intuitive idea behind inquisitive semantics is to conceive propositions as *proposals* to change the common ground, where the different *possibilities* constitutive of a proposition precisely encode the proposed ways to do so. We can then say that a proposition is *inquisitive* when it consists of more than two possibilities, and *informative* when the union of its possibilities excludes some indices of the common ground:

Definition 2.5 (Informativeness and inquisitiveness). *Let $\varphi \in \mathcal{L}$ and $s \in \mathcal{S}$.*

1. *φ is inquisitive in s iff $s[\varphi]$ contains at least two possibilities.*
2. *φ is informative in s iff $s[\varphi]$ contains at least one possibility and $\bigcup s[\varphi] \subset s$.*

The notions of *informativeness* and *inquisitiveness* allow us to define the semantic categories of *questions* and *assertions*:

Definition 2.6 (Question and assertion). *Let $\varphi \in \mathcal{L}$ and $s \in \mathcal{S}$.*

1. *φ is a question in s iff φ is inquisitive and not informative in s.*
2. *φ is an assertion in s iff φ is not inquisitive and informative in s.*[5]

We now have all the ingredients to define the notion of *answerhood*:

Definition 2.7 (Answerhood). *Let $\varphi, \psi \in \mathcal{L}$ and $s \in \mathcal{S}$ such that ψ is a question and φ an assertion in s.*

1. *φ is an answer to ψ in s iff $s|\varphi|$ coincides with the union of a set of possibilities for ψ in s and φ is informative in s.*
2. *φ is a complete answer to ψ in s iff $s|\varphi|$ coincides with one of the possibilities for ψ in s.*
3. *φ is a partial answer to ψ in s iff φ is an answer but not a complete answer to ψ in s.*

The above notion of answerhood is a particular case of a more general notion of *compliance* proposed by Groenendijk and Roelofsen in [7] which "judges whether a certain conversational move is *related* to the foregoing discourse" [7, p. 19]. More precisely, the notion of answerhood corresponds to the notion of compliance between a *question* and an *assertion.* However, notice that, in our definition, we add an additional requirement to the notion of compliance of assertions to questions presented in [7] by requiring the answer to a question to be *informative.*

In the context of interrogative inquiry, we want to have a precise definition of what it means for a question or an issue to be *settled.* In the inquisitive framework, the notion of *settledness* can formally be defined as follows:

Definition 2.8 (Settledness). *Let $\varphi \in \mathcal{L}$ and $s \in \mathcal{S}$. We say that φ is settled in s iff $s[\varphi] = \{s\}$.*

[5] We can now explicit the meaning of the two operators ! and ? introduced at the beginning: the operator ! transforms any formula φ into an assertion $!\varphi$ of which the proposition is composed of one possibility corresponding to the union of the possibilities of φ; the operator ? transforms, in general, any formula φ into a question $\varphi?$ which adds the possibility that $\neg\varphi$ to the possibilities of φ.

The intuitive reading of this definition is the following: an issue φ is settled in a state s when (i) φ is classically true in s and (ii) φ is not inquisitive in s, i.e., φ does not raise any more issues. In the context of inquisitive pragmatics [7], such sentences are called *insignificant* due to the fact that they do not contribute to the conversation by being informative or inquisitive with respect to the common ground of the conversation.

Finally, we shall define two notions that were not introduced in the existing literature on inquisitive semantics, but which will turn out to be very useful in our investigation of interrogative inquiry. These notions are the ones of *characteristic proposition* and *characteristic question* of an *index* v and a *state* s:

Definition 2.9 (Characteristic proposition and characteristic question of an index)**.** *Let $v \in \omega$.*

1. *The characteristic assertion χ_v of v is given by:*

$$\chi_v := \bigwedge \{\{p \mid p \in \mathcal{P} \text{ and } v(p) = 1\} \cup \{\neg p \mid p \in \mathcal{P} \text{ and } v(p) = 0\}\},$$

2. *The characteristic question $\chi_v?$ of v is given by: $\chi_v? := \chi_v \vee \neg\chi_v$.*

The intuitive meaning of the characteristic assertion χ_v of v is that χ_v is classically true only for the index $v \in \omega$. Consequently, the characteristic question $\chi_v?$ of v can be interpreted as saying "Is v the index corresponding to the actual world?". These properties can be stated formally:

Proposition 1. *Let $s \in \mathcal{S}$ and $v \in s$. We have:*

1. *The characteristic assertion χ_v of v is an assertion in s,*
2. *$s|\chi_v| = \{v\}$,*
3. *$s[\chi_v?] = \{\{v\}, s - \{v\}\}$.*

Proof. The proof is straightforward from the previous definitions. □

From the notions of characteristic proposition and characteristic question for indices, we can easily define the corresponding notions for *states*:

Definition 2.10 (Characteristic proposition and characteristic question of a state)**.** *Let $s \in \mathcal{S}$.*

1. *The characteristic assertion χ_s of s is given by:*

$$\chi_s :=!\chi'_s \quad \text{with} \quad \chi'_s := \bigvee\{\chi_v \mid v \in s\},$$

2. *The characteristic question $\chi_s?$ of s is given by: $\chi_s? := \chi_s \vee \neg\chi_s$.*

The intuitive meaning of the characteristic assertion χ_s of s is that χ_s is classically true only for the indices in s. Then, the characteristic question $\chi_s?$ of s can be interpreted as saying "Does the state s contain the actual world?". This reading is confirmed by the following proposition:

Proposition 2. *Let $s, t \in \mathcal{S}$ such that $t \subseteq s$. We have:*

1. *The characteristic assertion χ_t of t is an assertion in s,*
2. *$s|\chi_t| = t$,*
3. *$s[\chi_t?] = \{t, s - t\}$.*

Proof. The proof is straightforward from the previous definitions. □

The inquisitive framework offers us a powerful theory to represent questions and answers. Before turning to the definition of the notion of interrogative rule, it would be interesting to relate the inquisitive framework to Hintikka's theory of questions and answers, since Hintikka's developments and discussions of the IMI are intimately related to his own theory of questions and answers. This will then enable a cross-discussion between our framework and the IMI. We approach this task in the next section and we argue that the key notions of Hintikka's theory of questions find natural correspondents in the inquisitive framework.

3 Propositional questions, presuppositions and desiderata

The treatment of questions which underlines Hintikka's approach to interrogative inquiry is based on what we will call *Hintikka's theory of questions*, which originated in [9] and has been developed further in [13] and [14]. In the propositional case, the key notions of this theory are the ones of *propositional question*, *presupposition* and *desideratum.* In this section, we argue that these notions find natural correspondents in the framework of inquisitive semantics and pragmatics.[6]

According to Hintikka's theory of questions, a question is identified with its set of possible answers. Thus, in the propositional case, a *propositional question* Q is identified by a finite set of propositions that we denote by $Q = (\gamma_1, \ldots, \gamma_k)$ where $\gamma_1, \ldots, \gamma_k$ are propositional formulas. Such a question Q is read as *"Is it the case that γ_1, or is it the case that $\gamma_2, \ldots,$ or is it the case that γ_k?"* and is conceived as a request of information from the questioner to the answerer, where the answerer is supposed to choose between these k *alternatives* the one(s) that he knows is (are) the case. In inquisitive semantics, questions are also conceived as proposals inviting for a choice between several alternatives called *possibilities.* However, the notion of *alternativehood* of inquisitive semantics differs sharply from the one of Hintikka's theory of questions: whereas Hintikka conciders any set of propositional formulas as a set of alternatives, in inquisitive semantics, the notion of alternative is defined semantically for a propositional formula via the notion of support, and is constitutive of the inquisitive notion of meaning as we have seen in section 2. Interestingly, the corollary 3.15 of [4], saying that "any formulas is equivalent to a disjunction of negations", tells us that any question ψ in inquisitive semantics can be seen as a disjunction of formulas, echoing Hintikka's reading of questions. Of course, the important distinction lies in the fact that in the former the disjunction is interpreted *inquisitively*, whereas in the latter the disjunction is interpreted *classically.*

The notion of propositional question comes with the important notion of *presupposition.* According to Hintikka, a question can be *meaningfully* asked only if its presupposition has been established by the questioner. In the case of propositional questions, the presupposition of a question $Q = (\gamma_1, \ldots, \gamma_k)$ is formalized in the language of epistemic logic as follows:

$$K(\gamma_1 \vee \ldots \vee \gamma_k)$$

where K refers to the knowledge operator associated to the questioner. Thus, the notion of presupposition belongs to the pragmatics of asking questions: the questioner can only ask the question Q if he knows that at least one of the disjuncts γ_i is the case. In the

[6] We consider here the 'basic' version of Hintikka's theory of questions, i.e., the one based on the classical version of epistemic logic. In [14], Hintikka presents a 'Second-Generation Epistemic Logic' based on game-theoretic semantics which allows to deal with the notion of *informational independence.* This new version of epistemic logic provides an epistemic slash operator $/K$ which is used to define the notions of presupposition and desideratum. An in-depth comparison between this framework and the inquisitive account of questions and answers is out of the scope of this paper.

inquisitive framework, this condition is directly integrated into the definition of the semantic category of questions, which is consistent with the idea of inquisitive semantics to represent the interrogative aspect of sentences directly at the level of *meaning*, while Hintikka represents it at the level of *speech acts*. Thus, for ψ to be a question, and *a fortiori* for the questioner to be able to ask the question, ψ should satisfy the clause of *non-informativity* in the definition of the semantic category of questions: if ψ is a question in the common ground σ, then the questioner can only ask ψ if ψ is not eliminative in the common ground, which means that ψ has been classically established in the common ground of the conversation. If we refer to the corollary 3.15 of [4] and we consider ψ as a disjunction of formulas, then we can see the close parallel between the requirement of Hintikka that a questioner should have established the presupposition of a question in order to meaningfully address it, and the clause of non-informativity in the inquisitive definition of questions. Notice nevertheless that the inquisitive notion of question adds another condition to non-informativity which says that, in order for ψ to be a question, the proposition $\sigma[\psi]$ should be *inquisitive*, i.e., should actually correspond to a set of *alternatives*.

Finally, the notion of *desideratum* of a question is defined by Hintikka as follows: "The desideratum of a question specifies the epistemic state that a conclusive answer to it is calculated to bring about (on the assumption that the presupposition of the question is true) in a typical information-acquiring use of questions" [15, p. 71]. For a propositional question $Q = (\gamma_1, \ldots, \gamma_k)$, the desideratum of Q is formalized in the language of epistemic logic as follows:

$$K\gamma_1 \vee \ldots \vee K\gamma_k$$

where K refers to the knowledge operator associated to the questioner. In inquisitive semantics, the proposition $\sigma[\psi]$ of a question ψ specifies the different possibilities (alternatives) that ψ proposes to change the common ground σ. Thus, the desideratum of the conversational participant asking ψ is to transform the common ground of the conversation by restricting it to one of the possibilities of ψ. It is important to notice here that the notion of desideratum is defined by Hintikka relatively to the informational state of the questioner, whereas in the inquisitive framework the notion of desideratum is defined relatively to the common ground of the conversation.

Thus, the key notions of Hintikka's treatment of questions in his approach to interrogative inquiry have natural correspondents in the inquisitive framework. This speaks for a certain continuity between the inquisitive approach to interrogative inquiry that we will develop in the next sections and Hintikka's logical and philosophical works on the subject, and will allow a cross-discussion between our approach and the one of the IMI.

4 Interrogative rule

The notion of *interrogative rule* should be thought by analogy with the notion of *inference rule*: where the inference rules are governing the logical inferences that one is allowed to draw, the interrogative rule is governing the admissible question-answer steps that one is allowed to make. In Hintikka's own words:

> When we are recording the successive steps of interrogative inquiry on paper (or on a computer disk), logical inference steps and interrogative steps look rather similar. The former are steps from a premise (or number of premises) to a conclusion; the latter are steps from the presupposition(s) of a question to its answer.[7] [14, p. 71]

[7]From the discussion of the previous section, we can see how to translate Hintikka's description of interrogative steps in the inquisitive framework: in this case, an interrogative step is a step from a formula ψ belonging to the semantic category of questions in the current common ground of the conversation, to the

In this paper, the aim of the interrogative rule is to characterize completely the question-answer steps that one can make in an interrogative inquiry. For this purpose, we will 'split' the notion of interrogative rule into two components:

- A pragmatic rule for *answering:* which governs the production of answers to questions given (i) the informational state of the answerer and (ii) the common ground of the conversation.

- A pragmatic rule for *updating:* which governs the way the conversation, i.e., the common ground and the informational states of the participants, is updated after the reception of an answer to a question.

This section is organized as follows. Firstly, we stipulate basic rules for the language-game of interrogative inquiry, and we capture formally the idea of conversational context by defining the notion of *conversational state.* Secondly, we successively discuss and define the notions of *pragmatic rules for answering* and *updating*, and we use them to define the notion of *interrogative rule.* Then, we illustrate our definitions with two characteristic examples in which conversational participants produce maximally and minimally informative answers. Finally, we define the notion of *yes-no question*, and we show that, in our framework, responses to yes-no questions are independent of the pragmatic rule for answering adopted by the conversational participants.

4.1 A conversational setting for interrogative inquiry

In order to make precise the bases of our framework, we first need to stipulate some basic rules of the language-game of interrogative inquiry. To this end, we will set the following hypotheses on the type of conversation which serves as the setting of our investigation of interrogative inquiry:

- We designate one of the participants as the *inquirer* and the other participants as the *oracles*,[8]

- Each interrogative step takes the form of a question asked by the inquirer and (eventually) answered by one of the oracles or by the inquirer himself,[9]

- Each question asked by the inquirer is directed towards a particular conversational participant.[10]

In order to formalize the general informational and conversational context which corresponds to a given stage of a conversation of this type, we introduce the notion of *conversational state*:

Definition 4.1 (Conversational state)**.** *A conversational state C is defined as a $\mathcal{S}$-tuple $C = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n})$ where:*

(eventual) production of an answer by the conversational participant to whom the question is addressed.

[8]We follow Hintikka's terminology here and we designate the sources of information, in our case the participants of the conversation, by the technical term of *oracle*.

[9]This possibility for the inquirer to ask questions to himself might seem odd at first sight. However, since we investigate interrogative inquiry in the context of conversations, it is important that the inquirer can share his own information with the other participants. Nevertheless, we would like to consider every step of an interrogative inquiry as a question-answer step and to avoid the possibility of pure information announcement. This is why any information provided by the inquirer will have to take the form of an answer to a question that he addressed to himself.

[10]In the following, we will use the term *directed question* to designate the pair composed of a question and the conversational participant to whom the question is addressed.

- *σ denotes the common ground of the conversation,*
- *τ_I denotes the informational state of the inquirer,*
- *$\tau_{O_1}, \ldots, \tau_{O_n}$ denote the informational states of the oracles,*

and such that:

1. $\tau_I, \tau_{O_1}, \ldots, \tau_{O_n} \subseteq \sigma$,
2. $\left(\bigcap_{1 \leq i \leq n} \tau_{O_i}\right) \cap \tau_I \neq \emptyset$.

The set of all conversational states is denoted by $\mathcal{C}$, the set of all conversational states with n oracles is denoted by $\mathcal{C}^n$.

In the above definition, clause 1. says that the informational states of the participants are contained in the common ground of the conversation,[11] clause 2. says that the participants of the conversation have at least one index in common in their respective informational states, which in particular means that they all have the index corresponding to the actual world in their informational states. To illustrate our definition, we propose a graphical representation of the notion of conversational state in figure 4.1.

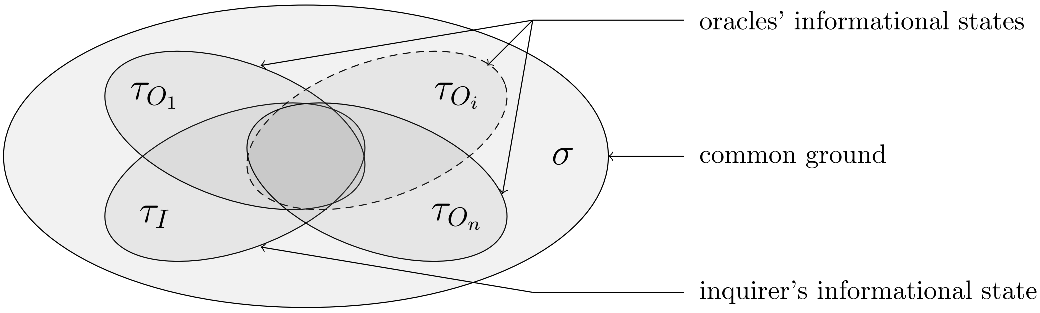


Figure 4.1: A conversational state $C = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n})$.

4.2 Defining the notion of interrogative rule

We now define the notion of *interrogative rule* through the notions of *pragmatic rules for answering* and *updating.*

Pragmatic rule(s) for answering. By a *pragmatic rule for answering*, we mean a rule that governs the production of answers to questions. In section 2, we provided a general notion of *answerhood* which says when a proposition φ should be considered as an answer to a question ψ in a given state s. However, if we are interested in the production of answers by a given conversational participant, we need to relativize this definition to the informational state of the answerer. To this end, we introduce the notion of *answer* which says when a proposition φ is an answer to a question ψ for an answerer with informational state τ in a conversational state with common ground σ. This is done by requiring the answer to a question to be non-eliminative in the informational state of the answerer:[12]

[11] See [7, p. 15] for a discussion on how the informational states of the conversational participants and the (external and internal) common ground of a conversation are related.

[12] In other words, this simply means that the informational state of the answerer supports the answer to the question, i.e., the answerer actually has the information corresponding to the answer. Notice that this condition coincides with the 'informative sincerity' clause of the sincerity maxim in the inquisitive pragmatics developed in [7].

Definition 4.2 (Answer). *Let $\varphi, \psi \in \mathcal{L}$ and $\sigma, \tau \in \mathcal{S}$ such that $\tau \subseteq \sigma$, ψ is a question and φ an assertion in σ. We say that φ is an answer to ψ for τ in σ if:*

1. *φ is an answer to ψ in σ,*
2. *$\tau|\varphi| = \tau$.*

We denote by $\mathsf{Answers}(\psi, \tau, \sigma)$ the set of all $\varphi \in \mathcal{L}$ such that φ is an answer to ψ for τ in σ.

The following proposition provides a necessary and sufficient condition for the existence of answers:

Proposition 3 (Answer existence). *Let $\psi \in \mathcal{L}$ and $\sigma, \tau \in \mathcal{S}$ such that $\tau \subseteq \sigma$ and ψ is a question in σ. Then:*

There exists an answer φ to ψ for τ in σ iff there exists $\alpha_1, \ldots, \alpha_k \in \sigma[\psi]$ such that

$$\tau \subseteq \bigcup_{1 \leq i \leq k} \alpha_i \subset \sigma.$$

Proof. Let $\psi \in \mathcal{L}$ and $\sigma, \tau \in \mathcal{S}$ such that $\tau \subseteq \sigma$. Suppose that there exists an answer φ to ψ for τ in σ. By definition of the notion of answer, this means that (i) there exists $\alpha_1, \ldots, \alpha_k \in \sigma[\psi]$ such that $\sigma|\varphi| = \bigcup_{1 \leq i \leq k} \alpha_i$ and (ii) $\sigma|\varphi| \subset \sigma$. Since φ is an answer to ψ for τ in σ, we also have that $\tau|\varphi| = \tau$. Thus, since $\tau \subseteq \sigma$, we have $\tau|\varphi| \subseteq \sigma|\varphi|$ and then $\tau \subseteq \sigma|\varphi|$. We finally have:

$$\tau \subseteq \bigcup_{1 \leq i \leq k} \alpha_i \subset \sigma.$$

For the other direction, suppose that there exists $\alpha_1, \ldots, \alpha_k \in \sigma[\psi]$ such that $\tau \subseteq \bigcup_{1 \leq i \leq k} \alpha_i \subset \sigma$. Let $s := \bigcup_{1 \leq i \leq k} \alpha_i$. We consider the characteristic proposition χ_s of s. Then, by proposition 2, χ_s is such that $\sigma|\chi_s| = s \subset \sigma$, and thereby χ_s is an answer to ψ in σ. Besides, since $\tau \subseteq \sigma|\chi_s|$, we also have $\tau|\chi_s| = \tau$. We conclude that χ_s is an answer to ψ for τ in σ. □

From the notion of answer, we can now define the notion of *answering rule* as a partial function which, for every triple (ψ, τ, σ) such that ψ is a question in σ, picks a formula $\varphi \in \mathcal{L}$ such that φ is an answer to ψ for τ in σ. Formally, this leads to the following definition:

Definition 4.3 (Answering rule). *An answering rule is a partial function*

$$\begin{array}{cccccc} A & : & \psi, \tau, \sigma & \longmapsto & A(\psi, \tau, \sigma) \\ & & \mathcal{L} \times \mathcal{S} \times \mathcal{S} & \longrightarrow & \mathcal{L} \end{array}$$

where $A(\psi, \tau, \sigma)$ is defined for all (ψ, τ, σ) such that ψ is a question in σ by[13]

$$A(\psi, \tau, \sigma) = \begin{cases} \varphi \in \mathsf{Answers}(\psi, \tau, \sigma) & \textit{if } \mathsf{Answers}(\psi, \tau, \sigma) \neq \emptyset, \\ \top & \textit{otherwise.} \end{cases}$$

[13] Here, $\top$ denotes the formula $!(p \vee \neg p)$, which is neither informative, nor inquisitive, in any state. Thus, we take the convention that the answerer utters the (always) insignificant formula $\top$ when he does not have the answer to a question.

Pragmatic rule for updating. In the previous paragraph, we defined a pragmatic rule for answering which governs the production of answers to questions. We then need to state how a produced answer *modifies* the informational states of the conversational participants along with the common ground of the conversation, i.e., how it modifies the current *conversational state*. To this end, we define an *updating rule* which maps a conversational state and a sentence to the conversational state updated after the utterance of this sentence:

Definition 4.4 (Updating rule)**.** *The updating rule is a partial function*

$$\begin{array}{cccc} U & : & C, \varphi & \longmapsto & C|\varphi \\ & & \mathcal{C} \times \mathcal{L} & \longrightarrow & \mathcal{C} \end{array}$$

where $C|\varphi$ *is defined for all* (C, φ) *such that* $s|\varphi| \neq \emptyset$*, with* $s := \left(\bigcap_{1 \leq i \leq n} \tau_{O_i}\right) \cap \tau_I$ *and* $C = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n})$*, by*

$$C|\varphi = (\sigma|\varphi, \tau_I|\varphi, \tau_{O_1}|\varphi, \ldots, \tau_{O_n}|\varphi), \text{ with } t|\varphi = t|\varphi| \text{ for } t \in \mathcal{S}.$$

In this definition, we adopt the most straightforward solution for updating conversational states which consists in ruling out the indices in the informational states of the conversational participants, along with the indices in the common ground of the conversation, which are incompatible with the uttered sentence. Notice that the updating rule preserves conversational states, which means that, if (C, φ) is such that $s|\varphi| \neq \emptyset$, where $s := \left(\bigcap_{1 \leq i \leq n} \tau_{O_i}\right) \cap \tau_I$, then $U(C, \varphi)$ is a conversational state according to definition 4.1.

Interrogative rule. Having defined the pragmatic rules for answering and updating, we can now straightforwardly define from these two components the notion of *interrogative rule*:

Definition 4.5 (Interrogative rule)**.** *Let* $n \in \mathbb{N}$ *and let* A *be an answering rule. The interrogative rule associated to* A *and* n *is a partial function*[14]

$$\begin{array}{cccc} I_n & : & C, \psi, i & \longmapsto & C|_i^? \psi \\ & & \mathcal{C}^n \times \mathcal{L} \times [\![0, n]\!] & \longrightarrow & \mathcal{C} \end{array}$$

where $C|_i^? \psi$ *is defined for all* (C, ψ, i) *such that* ψ *is a question in* σ *by*

$$C|_i^? \psi = C|A(\psi, \tau_i, \sigma) = U(C, A(\psi, \tau_i, \sigma)).$$

This definition characterizes completely the question-answer steps that an inquirer can make in an interrogative inquiry. However, interrogative inquiries are temporal processes composed of *sequences* of question-answer steps. The aim of the next section will precisely be to put the interrogative rule into a temporal perspective by introducing the notion of *interrogative protocol*. Before that, we first discuss two important types of answering rules, and we define and show some specific properties of the notion of *yes-no question*.

4.3 Characteristic examples: maximally and minimally informative answers

Our definition of pragmatic rules for answering only requires that the assertion uttered in response to a question is an answer to this question, and does not impose any more restrictions on the way answers are chosen. However, one might be interested to study particular

[14] In a conversational state $C = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n})$, we will take as a convention to refer to the informational state of the oracle τ_{O_i} as the informational state indexed by i, and to refer to the informational state of the inquirer τ_I as the informational state indexed by 0.

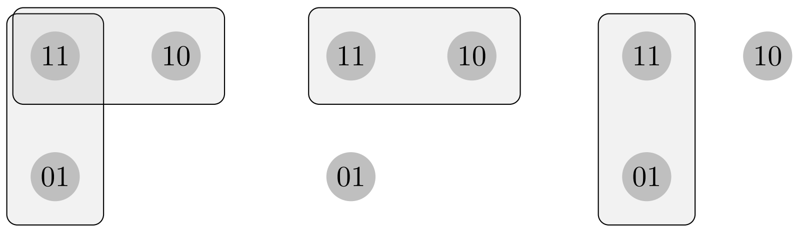
pragmatic rules for answering by precisely adding specific conditions on the way answers are chosen. One important aspect of the production of answers that one might want to represent in the context of a conversation is the willingness of the participants to be informative or, in other words, their cooperativeness in sharing their own information. We will show here how one can represent in our framework two canonical examples along this line in which answerers are respectively *maximally* and *minimally informative*. To this end, we introduce the notion of a *maximally informative answer*:

Definition 4.6 (Maximally informative answer)**.** *Let $\varphi, \psi \in \mathcal{L}$ and $\sigma, \tau \in \mathcal{S}$ such that $\tau \subseteq \sigma$, ψ is a question and φ an assertion in σ. We say that φ is a maximally informative answer (MaxIA) to ψ for τ in σ if:*

1. *φ is an answer to ψ for τ in σ,*

2. *there is no $\chi \in \mathcal{L}$ such that χ is an answer to ψ for τ in σ and $\sigma|\chi| \subset \sigma|\varphi|$.*

It is important to notice that, according to this definition, there can be several different maximally informative answers to a question for a participant in a conversation, as illustrated in figure 4.2.



(a) The question $p \vee q$ (b) The answer p (c) The answer q

Figure 4.2: In the state $\sigma = \{11, 10, 01\}$, there exist two maximally informative answers to $p \vee q$ for $\tau = \{11\}$ in σ, namely p and q.

Another property of MaxIA is that whenever there exists an answer to a question, there exists a maximally informative answer to it. This is the content of the following proposition:

Proposition 4 (MaxIA existence)**.** *Let $\psi \in \mathcal{L}$ and $\sigma, \tau \in \mathcal{S}$ such that $\tau \subseteq \sigma$ and ψ is a question in σ. We have:*

If there exists an answer to ψ for τ in σ, then there exists a maximally informative answer to ψ for τ in σ.

Proof. Suppose that there exists an answer φ to ψ for τ in σ. This means that we have $\tau \subseteq \sigma|\varphi| \subset \sigma$. If φ is not a maximally informative answer to ψ, then there exists an answer χ to ψ for τ in σ such that $\tau \subseteq \sigma|\chi| \subset \sigma|\varphi| \subset \sigma$. However, since σ and τ are finite, we cannot construct in this way an infinite descending chain of more informative answers. Thus, there must be a maximally informative answer to ψ for τ in σ. □

Given the definition of maximally informative answers, one can define the corresponding pragmatic rules for answering as follows:

Definition 4.7 (MaxIA answering rules)**.** *Let A be an answering rule. We say that A is a MaxIA answering rule if, for all (ψ, τ, σ) such that ψ is a question in σ, $A(\psi, \tau, \sigma)$ is a maximally informative answer to ψ for τ in σ.*

Then, one can use this definition to straightforwardly obtain a notion of MaxIA interrogative rule. In section 7, we will see that the choice of specific pragmatic rules for answering can play a crucial role from a computational perspective on the process of interrogative inquiry. Finally, in exactly the same way, one can obtain a notion of MinIA interrogative rule based on the following definition of *minimally informative answer*:

Definition 4.8 (Minimally informative answer)**.** *Let $\varphi, \psi \in \mathcal{L}$ and $\sigma, \tau \in \mathcal{S}$ such that ψ is a question and φ an assertion in σ. We say that φ is a minimally informative answer (MinIA) to ψ for τ in σ if:*

1. *φ is an answer to ψ for τ in σ,*
2. *there is no $\chi \in \mathcal{L}$ such that χ is an answer to ψ for τ in σ and $\sigma|\chi| \supset \sigma|\varphi|$.*

MaxIA and MinIA are two extremes on the scale of the *informativity* of answers. Thus, one could want to go further here and to represent different *degrees of cooperativeness* of the answerer by introducing pragmatic rules for answering with correspondent *degrees of informativity* of the produced answers. On possibility to do so is to adopt the approach of [26] and [27], which consists in introducing probabilities into the framework, and to define informativity of answers in terms of the information-theoretic notion of *entropy*. Although our framework could support such an approach, we will not pursue this line further here.

4.4 Yes-no questions

Yes-no questions constitute an important type of questions, which has been extensively discussed by Hintikka in his writings on interrogative inquiry. In the context of inquisitive semantics, we propose to associate a yes-no question to any formula of the language as follows:

Definition 4.9 (Yes-no question)**.** *Let $\varphi \in \mathcal{L}$. We define the yes-no question associated to φ by*

$$!\varphi \vee \neg !\varphi.$$

The idea of this definition is, for a given formula $\varphi \in \mathcal{L}$, to first transform φ in an assertion $!\varphi$ using the operator !, and then construct the yes-no question $!\varphi \vee \neg !\varphi$. This definition can be rendered by successively applying the operators ! and ? to the formula φ, as illustrated by the following fact:

Fact 1. *Let $\varphi \in \mathcal{L}$. The yes-no question associated to φ is $(!\varphi)?$.*

Then, the following proposition (i) tells us that our definition of yes-no questions behaves in the intended way at the level of proposition and (ii) provides a yes-no question associated to any bi-partition of the common ground:

Proposition 5. *Let $s \in \mathcal{S}$.*

1. *If $\varphi \in \mathcal{L}$ with $\emptyset \subset s|\varphi| \subset s$, then $s[!\varphi \vee \neg !\varphi] = \{s|\varphi|, s - s|\varphi|\}$,*
2. *If $t \in \mathcal{S}$ with $\emptyset \subset t \subset s$, then there is a yes-no question corresponding to $\{t, s - t\}$, namely $!\chi_t \vee \neg !\chi_t$.*

Proof. The proof is straightforward from the definition of the notion of proposition. □

Yes-no questions will play a crucial role in this paper. This is due to the fact that, at the level of proposition, responses to yes-no questions turn out to be independent of the answering rule adopted by the participants. We will show this property formally in several steps. First of all, we prove the following lemma which provides the truth-set of the answer to a yes-no question given the informational state of the answerer:

Lemma 1. *Let A be an answering rule, let $\tau, \sigma \in \mathcal{S}$ such that $\tau \subseteq \sigma$ and let $\varphi \in \mathcal{L}$. If $\psi := !\varphi \vee \neg !\varphi$ is a question in σ, then*

$$\sigma|A(\psi, \tau, \sigma)| = \begin{cases} \sigma|\varphi| & \textit{if } \tau|\varphi| = \tau, \\ \sigma - \sigma|\varphi| & \textit{if } \tau|\neg\varphi| = \tau, \\ \top & \textit{otherwise.} \end{cases}$$

Proof. If $\psi := !\varphi \vee \neg !\varphi$ is a question in σ, then by proposition 5 we have $\sigma[\psi] = \sigma[!\varphi \vee \neg !\varphi] = \{\sigma|\varphi|, \sigma - \sigma|\varphi|\}$. Thus, if χ is an answer to ψ in σ, this means that either $\sigma|\chi| = \sigma|\varphi|$ or $\sigma|\chi| = \sigma - \sigma|\varphi|$. Then, if χ is an answer to ψ for τ in σ, we necessarily have that $\sigma|\chi| = \sigma|\varphi|$ if $\tau|\varphi| = \tau$, and $\sigma|\chi| = \sigma - \sigma|\varphi|$ if $\tau|\neg\varphi| = \tau$. From this it follows that

$$\sigma|A(\psi, \tau, \sigma)| = \begin{cases} \sigma|\varphi| & \text{if } \tau|\varphi| = \tau, \\ \sigma - \sigma|\varphi| & \text{if } \tau|\neg\varphi| = \tau, \\ \top & \text{otherwise.} \end{cases}$$

□

This lemma allows us to show the following corollary:

Corollary 1. *Let A and A' be two answering rules, let $\tau, \sigma \in \mathcal{S}$ such that $\tau \subseteq \sigma$ and let $\varphi \in \mathcal{L}$. If $\psi := !\varphi \vee \neg !\varphi$ is a question in σ, then*

$$\sigma|A(\psi, \tau, \sigma)| = \sigma|A'(\psi, \tau, \sigma)|.$$

Proof. The theorem follows directly from the previous lemma. □

We can now show that the update of the current conversational state resulting from a yes-no question is independent of the adopted interrogative rule:

Theorem 1. *Let I_n and I'_n be two interrogative rules respectively based on the answering rules A and A'. Let $C \in \mathcal{C}^n$ and $\varphi \in \mathcal{L}$. If $\psi := !\varphi \vee \neg !\varphi$ is a question in C, then*

$$I_n(C, \psi, i) = I'_n(C, \psi, i).$$

Proof. If $\psi := !\varphi \vee \neg !\varphi$ is a question in $C = (\sigma, \tau_I, \tau_{O_1}, \dots, \tau_{O_n})$, then the previous corollary says that

$$\sigma|A(\psi, \tau_i, \sigma)| = \sigma|A'(\psi, \tau_i, \sigma)|.$$

It follows that

$$I_n(C, \psi, i) = U(C, A(\psi, \tau_i, \sigma)) = U(C, A'(\psi, \tau_i, \sigma)) = I'_n(C, \psi, i).$$

□

What this theorem says is that the transformation of a conversational state after a yes-no question is independent of the answering rule adopted by the answerer. We will see in sections 6 and 7 that this property of yes-no questions allows us to establish general logical and computational properties of interrogative inquiry. However, for this purpose, we first need to provide a framework for formally representing interrogative inquiry. This is precisely the topic of the next section.

5 Interrogative protocol, interrogative inquiry and interrogative consequence

The notion of interrogative rule defined in the previous section describes completely the epistemic effect of asking a question in a given conversational context. However, an interrogative inquiry is a *sequence* of question-answer steps, and is thereby a *temporal process*. We shall then account for this temporal dimension, and put the interrogative rule into a temporal perspective. To this end, our approach will consist in representing, in a temporal framework, the two main aspects of the language-game of interrogative inquiry that we defined so far: (i) the basic rules of the language-game of interrogative inquiry and (ii) the pragmatic rules for answering and updating. In logic and computer science, such temporal processes are represented using the notion of *protocol*, which refers to a set of rules governing a *temporal process*. In our context, we are interested in defining *interrogative protocols* describing all the possible paths that an interrogative inquiry can take. We will then be able to formally define the notion of *interrogative inquiry* as finite paths within such protocols, and the notion of *interrogative consequence* as the information that can be reached through this process. In this section, we will develop this approach by providing formal definitions for the notions of *interrogative protocol*, *interrogative inquiry* and *interrogative consequence*, and we will then illustrate our formal framework with an example.

5.1 Defining the main notions

Interrogative protocol. The notion of *protocol* has previously been used in logical contexts to capture the dynamics of informational flow in conversations such as in [20] and [24]. It has even been used to represent questioning procedures in the framework of dynamic epistemic logic of questions [25]. In this paper, we are interested in defining *interrogative protocols* governing the language-game of interrogative inquiry. Our interrogative protocols are built from two parameters: (i) a conversational state constituting the starting point of the conversation, and (ii) an interrogative rule which integrates the adopted pragmatic rules for answering and updating. Then, our notion of interrogative protocol encodes the basic rules of the language-game of interrogative inquiry by stipulating that the only admissible moves are questions addressed by the inquirer to the conversational participants. This leads to the following formal definition:

Definition 5.1 (Interrogative protocol). *Let $n \in \mathbb{N}$, $C \in \mathcal{C}^n$ and I_n be an interrogative rule. The interrogative protocol $P_?(C, I_n)$ based on C and I_n is defined as a tree built as follows:*

Root: *the root of the tree is C,*

Expanding rule: *if $C' = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n})$ is a node of the tree, then for each formula φ such that φ is a question in σ and for each $i \in [\![0, n]\!]$, C' has a successor $C'|_i^? \varphi = I_n(C', \varphi, i)$.*

In order to better understand this definition, we propose a graphical representation of the notion of interrogative protocol in figure 5.1. In this figure, C_0 corresponds to the initial state of the conversation and constitutes the root of $P_?(C_0, I_n)$. In C_0, the inquirer can address a directed question (φ_k, i_k) insofar as φ_k is a question in C_0 and i_k is the index of a conversational participant ($i_k \leq n$). Then, to each possible directed question (φ_k, i_k) corresponds a branch leading to the conversational state resulting from the question-answer step consisting in (i) addressing the question φ_k to the conversation participant i_k and (ii) updating the conversational state with the answer. In other words, each possible directed question (φ_k, i_k) corresponds to a branch leading to the node $I_n(C_0, \varphi_k, i_k)$. This construction

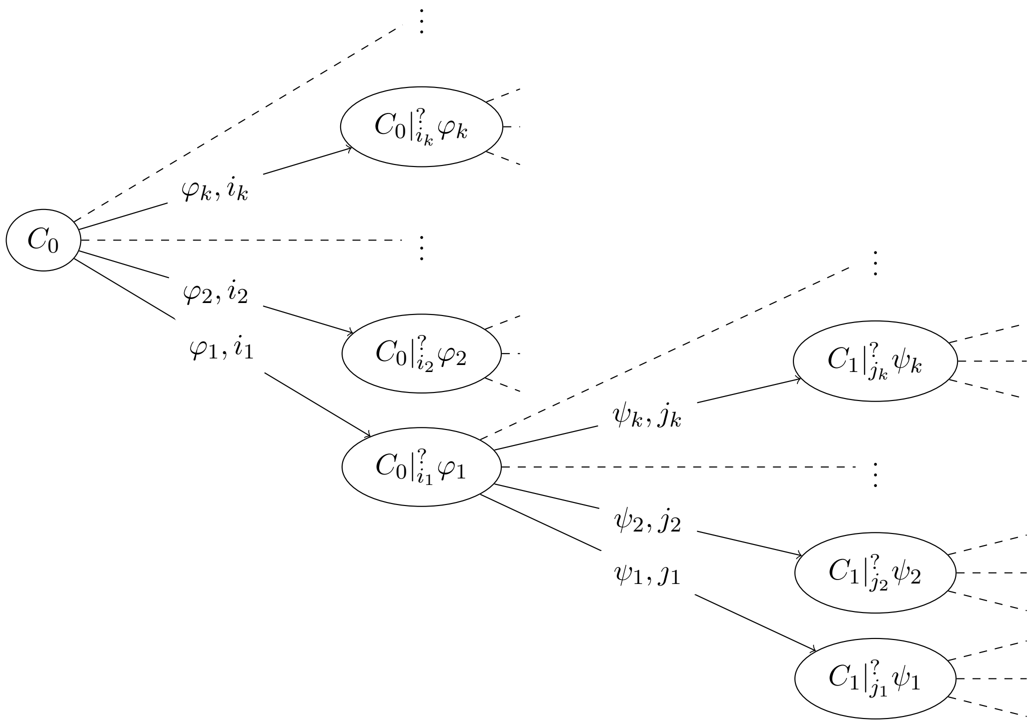


Figure 5.1: A graphical representation of the notion of interrogative protocol.

can then be repeated to the different conversational states resulting from this operation, as illustrated in figure 5.1 with the node $C_1 = C_0|^?_{i_1}\varphi_1$. This precisely describes the intuitive meaning of the expanding rule in the definition of interrogative protocols, and illustrates the fact that an interrogative protocol encodes all the possible lines of inquiry admissible in the language-game of interrogative inquiry given (i) a starting conversational state and (ii) an interrogative rule encoding the adopted pragmatic rules for answering and updating.

Interrogative inquiry. From the definition of interrogative protocols, we can then reach a formal definition of the notion of *interrogative inquiry*. To this end, we first notice that, for each node C of an interrogative protocol, each edge starting from C is identified by a directed question, which constitutes the *label* of this edge from C. This means that, in an interrogative protocol, any finite branch from the root can be identified by a finite sequence of labels, i.e., by a finite sequence of directed questions. This gives us our formal definition of the notion of *interrogative inquiry*:

Definition 5.2 (Interrogative inquiry)**.** *Let $P_?(C, I_n)$ be an interrogative protocol. An interrogative inquiry in $P_?(C, I_n)$ is a finite sequence $\langle(\varphi_1, i_1), \ldots, (\varphi_k, i_k)\rangle_k$ of elements in $\mathcal{L} \times [\![0, n]\!]$ such that $\langle(\varphi_1, i_1), \ldots, (\varphi_k, i_k)\rangle_k$ corresponds to the labels of a finite branch in $P_?(C, I_n)$ from the root C.*

This definition fits our intuitive representation of interrogative inquiries as sequences of questions addressed to specific conversational participants, i.e., as sequences of directed questions. Moreover, this definition also fits the idea that an interrogative inquiry takes place in a particular temporal process governs by certain rules. In our case, these rules are represented into an interrogative protocol which encodes (i) the particular type of conversation within which the interrogative inquiry takes place, (ii) the pragmatic rule for answering governing the production of answers and (iii) the pragmatic rule for updating governing the modifications of conversational states. In section 5.2, we will illustrate the modelling power of this definition through concrete examples. Then, in sections 6 and 7, we will see that this definition enables subtle logical and computational investigations of the process of interrogative inquiry.

Interrogative consequence. In the framework of interrogative logic [15], Hintikka and colleagues introduced the notion of *interrogative derivability*: C is interrogatively derivable from the initial set of premisses T in the model M if there exists a sequence of interrogative and deductive steps, made according to the rules of interrogative logic, leading to the conclusion C. In our framework, we will introduce an analogous notion of *interrogative consequence*: φ is an *interrogative consequence* in the interrogative protocol $P_?(C, I_n)$ if there exists an interrogative inquiry in $P_?(C, I_n)$ leading to a conversational state in which φ has been *established* in the common ground. However, in the inquisitive framework, there are two different possible ways to think of the term *established* here, corresponding to the *classical* and the *inquisitive views* on meaning:

- *Classically*, we consider that a proposition φ has been established in the common ground σ when φ is true in all the indices of σ, i.e., $\sigma|\varphi| = \sigma$,
- *Inquisitively*, we consider that a proposition φ has been established in the common ground σ when φ is composed of only one possibility covering σ, i.e., $\sigma[\varphi] = \{\sigma\}$.

In the classical view, we consider that a state encodes information φ as soon as φ is classically true in all the indices of the considered state, which represents then the current range of epistemic possibilities. However, due to the capacity of inquisitive semantics to encode both informative and inquisitive contents, the classical view is not enough in our case: φ can be classically true in all the indices composing the common ground while still being *inquisitive*, i.e., while still raising some issues. Thus, this observation speaks for a stronger[15] notion in which we will consider that a formula φ has been established in the common ground if not only φ is classically true in all the indices composing the common ground, i.e., φ is not informative in the common ground, but also φ does not raise any more issues. This precisely amounts to say that φ has been *settled* in the common ground. Consequently, we will adopt the following definition of the notion of *interrogative consequence*:

Definition 5.3 (Interrogative consequence)**.** *Let $P_?(C, I_n)$ be an interrogative protocol and $\varphi \in \mathcal{L}$. We say that φ is an interrogative consequence in $P_?(C, I_n)$ iff there exists an interrogative inquiry in $P_?(C, I_n)$ leading to a conversational state C' in which φ is settled, i.e., $\sigma'[\varphi] = \{\sigma'\}$.*

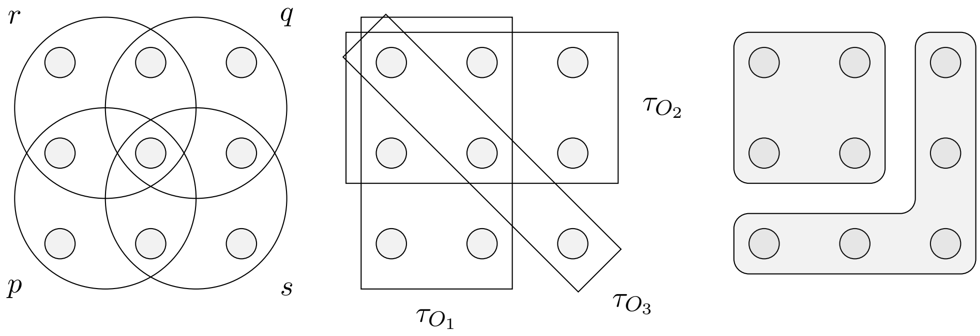
In section 6, we will investigate logical properties of the notion of interrogative consequence.

5.2 An illustrative example

We now illustrate the notions that we have just introduced with an example. To this end, we consider a language $\mathcal{L}$ based on the finite set of atomic propositions $\mathcal{P} = \{p, q, r, s\}$. We then consider a conversational state $C = (\sigma, \tau_I, \tau_{O_1}, \tau_{O_2}, \tau_{O_3})$ in which we assume that $\tau_I = \sigma$, i.e., that the inquirer has already shared all his information. The composition of the common ground σ is depicted in figure 5.2(a), and the informational states of the oracles O_1, O_2 and

[15]This stronger property is a formal feature of the framework: if $\sigma \in \mathcal{S}$, $\varphi \in \mathcal{L}$ and $\sigma[\varphi] = \{\sigma\}$, then $\sigma|\varphi| = \sigma$. Notice that the converse is not true (a counterexample is provided by figure 4.2 where $p \vee q$ is classically established in σ, yet not inquisitively established).

O_3 are depicted in figure 5.2(b). As can be observed from these two figures, none of the three oracles has knowledge about the atomic propositions. This feature of the example aims to already points out the importance of the distributed aspect of information, in the context of conversations, for the process of interrogative inquiry. In this example, we will assume that the goal of the inquiry is to settle the question $r?$, which is depicted in figure 5.2(c).



(a) Common ground σ. (b) Informational states. (c) Inquiry goal $r?$.

Figure 5.2: Conversation state $C = (\sigma, \tau_I, \tau_{O_1}, \tau_{O_2}, \tau_{O_3})$ and inquiry goal $r?$.

Then, we consider an interrogative rule I_3 based on a MaxIA answering rule A and the updating rule U. In this example, we will only consider interrogative inquiries with questions for which there exists a unique MaxIA. For this reason, we do not have to give more precisions on the answering rule A. Finally, from C and I_3, we can consider the *interrogative protocol* $P_?(C, I_3)$. In order to illustrate the notions of interrogative protocol and interrogative inquiry, we will consider three interrogative inquiries in $P_?(C, I_3)$, which are listed in table 1.

Inquiry 1	**Inquiry 2**	**Inquiry 3**
$O_3 :\ r?$	$O_2 :\ s \to q?$	$O_2 :\ p \wedge \neg q \to r?$
$O_1 :\ p \vee q \vee r \vee s$	$O_3 :\ p \to s?$	$O_1 :\ p?$
$O_2 :\ p \vee r$	$O_1 :\ q \to r?$	$O_3 :\ \neg q?$

Table 1: Three examples of interrogative inquiries in $P_?(C, I_n)$.

We start by considering inquiry 1, which is depicted in figure 5.3. In this figure, we first represent the initial common ground of C along with the inquiry goal $r?$. Then, each '$\Rightarrow$' represents a question-answer step, where the top figure depicts the question which is asked, the middle down-arrow mentions the oracle to which the question is addressed, and the down picture depicts the conversational state resulting from the question-answer step. Finally, the last picture represents the final common ground and depicts the inquiry goal $r?$ at the end of the inquiry. All the following figures representing interrogative inquiries follow the same construction pattern.

If we look closer to inquiry 1, we can see that the initial, legitimate, attempt of the inquirer to address directly the question $r?$ to oracle O_3 is unsuccessful. Actually, as we mentioned at the beginning, none of the three oracles has information about r, and it is only by combining information obtained from several oracles that the inquirer can hope to settle his inquiry goal. The second question-answer step consists in addressing an alternative question $p \vee q \vee r \vee s$ to oracle O_1. This step is successful, and the proposition of the received answer is the one corresponding to the sentence $!(p \vee r)$, which is supported by the informational state of O_1. At this point, we can see from our external point of view that O_2 and O_3 can answer the inquiry goal $r?$. However, the inquirer, having no access to the informational states of the oracles, chooses to ask the alternative question $p \vee r$, which is successfully answered by O_2. Notice that these two question-answer steps use *alternative questions*, and illustrate thereby the interest of using the inquisitive framework for its capacities to represent so-called *embedded questions*. Finally, we can see that inquiry 1 was able to settle the initial inquiry goal $r?$. Thus, $r?$ is, according to definition 5.3, an *interrogative consequence* in $P_?(C, I_3)$.

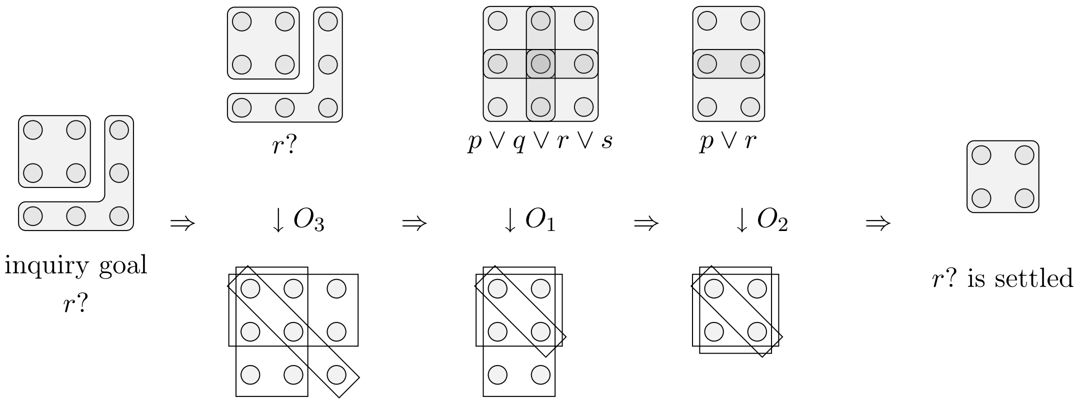


Figure 5.3: Interrogative inquiry 1.

In inquiry 2, depicted in figure 5.4, we aim to give an example of an interrogative inquiry composed of *conditional questions*. Here again, we can see that one of the main interests of inquisitive semantics, namely its capacity to represent *conditional questions*, directly transposes to the study of interrogative inquiry.[16] As it can be observed from figure 5.4, inquiry 2 is also a successful inquiry for settling $r?$. However, one of the important differences between inquiry 1 and 2 is that they do not result into the same conversational state. Indeed, in the conversational state C, it is sufficient to eliminate the 5 indices in which r is not supported for settling $r?$, as inquiry 1 did. Inquiry 2 actually leads to the elimination of one more index than necessary, and so obtains more information than inquiry 1. This illustrates the fact that several different inquiries might settle $r?$ with different informational 'paths', and might result into different conversational states.

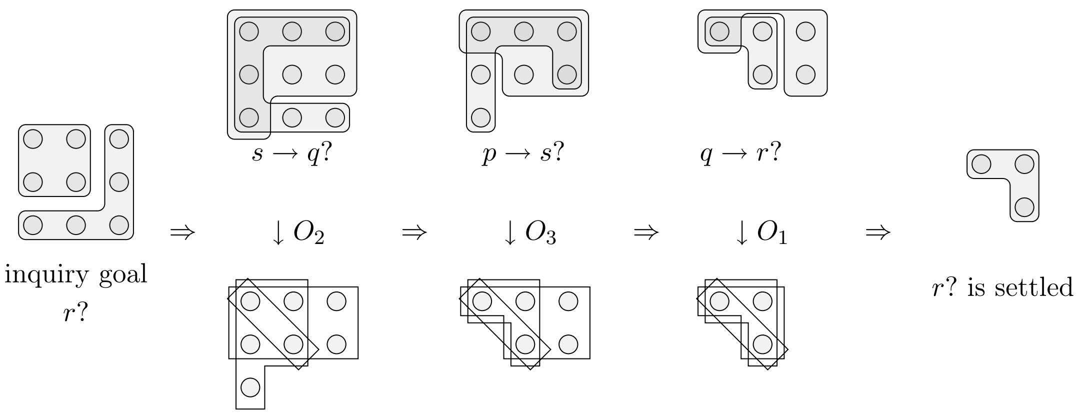


Figure 5.4: Interrogative inquiry 2.

[16]Thus, the examples of inquiries 1 and 2 show that the inquisitive framework allows to represent different kinds of questions and to investigate their behaviors in the process of interrogative inquiry. This illustrates one of the main motivations for adopting the inquisitive framework in the study of interrogative inquiry, namely its modelling power for representing questions.

Finally, with inquiry 3, depicted in figure 5.5, we give an example of an unsuccessful, yet plausible, inquiry. Inquiry 3 starts by addressing the question $p \wedge \neg q \rightarrow r?$ to oracle O_2. This step is successful and leads to an update of the conversational state with an answer corresponding to $p \wedge \neg q \rightarrow r$. Since the inquirer wants to settle $r?$, and he now has the information that p and $\neg q$ implies r, he might want to establish that p and $\neg q$ are the case. To this end, he addresses the questions $p?$ to O_1 and $\neg q?$ to O_3. However, we can see from figure 5.5 that these two questions are unsuccessful. Consequently, even though inquiry 3 was an intuitively legitimate line of inquiry, inquiry 3 results in a conversational state in which $r?$ is not settled.

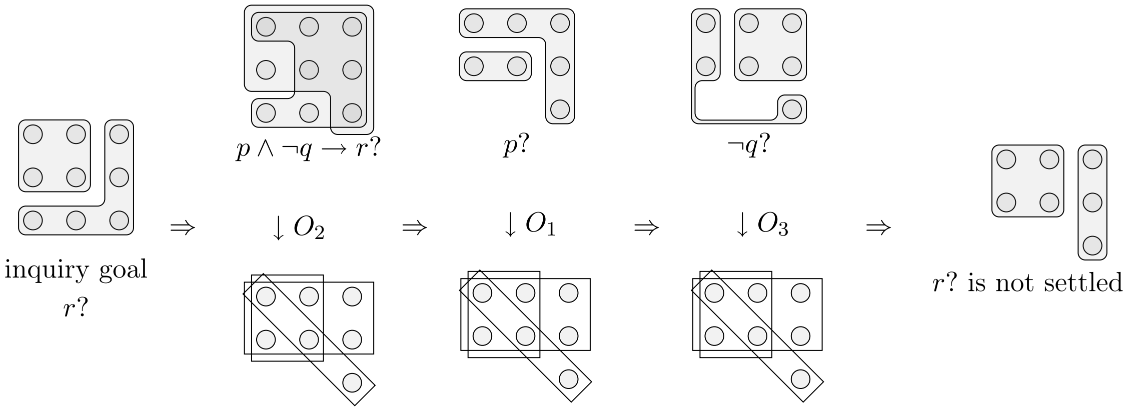


Figure 5.5: Interrogative inquiry 3.

These examples already show the modelling power of our framework for representing interrogative inquiries in conversations, and confirm the important interest of using the inquisitive framework for its fine-grained representation of questions and answers in conversational contexts. In the next two sections, we will show that our framework also allows for subtle logical and computational investigations of the process of interrogative inquiry.

6 Logical aspects

In this section, we investigate two issues related to the logical notion of interrogative consequence. The first one is concerned with a characterization of the information reachable by the process of interrogative inquiry in terms of the information possessed by the conversational participants, and shows how the notions of interrogative consequence and distributed information are formally related. The second one is concerned with the information reachable by interrogative inquiries exclusively composed of yes-no questions, and shows that any interrogative consequence is reachable by only asking yes-no questions.

6.1 Interrogative consequence and distributed information

In section 5.2, we saw through an example the importance of the distributed aspect of information among the participants of a conversation for the process of interrogative inquiry. We would like here to formally investigate this aspect within our framework. To this end, we first need to precisely define the notion of *distributed information*.

In the context of epistemic logic, the notion of distributed information intuitively refers to the information that a group of epistemic agents would have if they would put all the information they individually have together. This notion is generally semantically defined as follows [5, 18]: φ is distributed information among a group of agents G if and only if φ is true in all the worlds that every agent in G considers epistemically possible. In our framework, we are interested in defining a notion of distributed information among the participants of a conversation. To this end, we propose the following definitions:

Definition 6.1 (Distributed information)**.** *Let $C = (\sigma, \tau_I, \tau_{O_1}, \dots, \tau_{O_n}) \in \mathcal{C}^n$ and $\varphi \in \mathcal{L}$.*

- *We define the distributed information state $D(C)$ of C as $D(C) := \bigcap_{1 \leq i \leq n} \tau_{O_i} \cap \tau_I$,*
- *We say that φ is distributed information in C iff φ is settled in $D(C)$, i.e., $D(C)[\varphi] = \{D(C)\}$,*
- *We define the saturated conversational state C_D of C as $C_D := (D(C), \dots, D(C))$.*

Given a conversational state C, the *distributed information state* $D(C)$ refers to the intersection of the informational states of all the conversational participants in C, as depicted in figure 6.1. In the epistemic logic terminology, this corresponds to all worlds that every conversational agent in C considers epistemically possible. From this notion of distributed information state, we can define the notion of *distributed information* in C as the formulas in $\mathcal{L}$ which would be settled if the common ground of C would be reduced to the intersection of the informational states of all the conversational participants in C. We then refer to the conversational state resulting from C through each agent sharing entirely his own information as the *saturated conversational state* C_D of C.

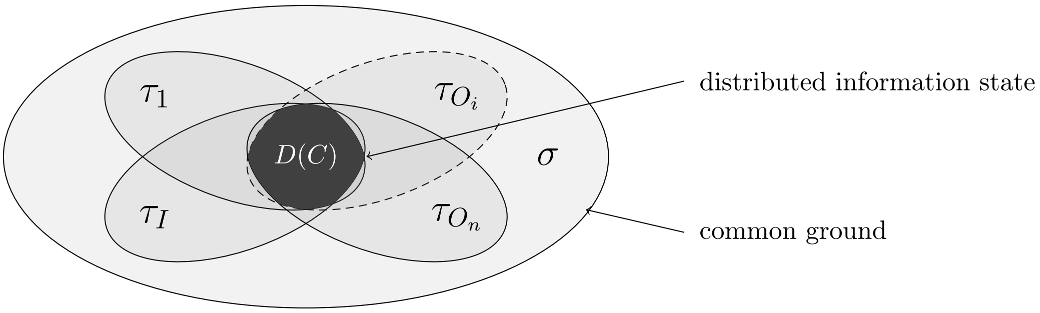


Figure 6.1: Common ground and distributed information state.

Intuitively, we would expect that whenever φ is distributed information among the participants of a conversation, φ can be reached by the inquirer through the process of interrogative inquiry. In other words, we would expect that the notions of distributed information and interrogative consequence *coincide*. This intuitive relation between distributed information and interrogative consequence can formally be established in our framework:

Theorem 2 (Interrogative Consequence and Distributed Information)**.** *Let $P_?(C, I_n)$ be an interrogative protocol and $\varphi \in \mathcal{L}$.*

φ is an interrogative consequence in $P_?(C, I_n)$ iff φ is distributed information in C.

Proof. Let $P_?(C, I_n)$ be an interrogative protocol, with $C = (\sigma, \tau_I, \tau_{O_1}, \dots, \tau_{O_n}) \in \mathcal{C}^n$, and let $\varphi \in \mathcal{L}$. Assume that φ is an interrogative consequence in $P_?(C, I_n)$. By definition, this means that there exists an interrogative inquiry $\langle(\varphi_1, i_1), \dots, (\varphi_k, i_k)\rangle_k$ in $P_?(C, I_n)$ leading to a node $C' = (\sigma', \tau'_I, \tau'_{O_1}, \dots, \tau'_{O_n})$ such that $\sigma'[\varphi] = \{\sigma'\}$. We will show that $D(C) \subseteq \sigma'$.

Let $v \in D(C)$. Suppose towards a contradiction that $v \notin \sigma'$. Since $v \in \sigma$, this would mean that the announcement of the answer χ_p to φ_p for some $p \in [\![1, k]\!]$ has led to the elimination of

v. However, since $v \in D(C)$, this means that v is a member of $\tau_I, \tau_{O_1}, \ldots, \tau_{O_n}$, i.e., a member of the informational state of each conversational participant. Thus, the announcement of χ_p must have been eliminative in the informational state τ_{i_p} of the participant with index i_p, which is not possible due to the definition of the notion of answer.[17] By contradiction, we get that $v \in \sigma$ and finally that $D(C) \subseteq \sigma'$. Then, it follows from $D(C) \subseteq \sigma'$ and $\sigma'[\varphi] = \{\sigma'\}$ that $D_C[\varphi] = \{\varphi\}$, i.e., that φ is distributed information in C.

Now assume that φ is distributed information in C. By definition, this means that $D(C)[\varphi] = \{D(C)\}$. We will show that there exists an interrogative inquiry in $P_?(C, I_n)$ which leads to the saturated conversational state C_D of C. Consider the following interrogative inquiry:

$$\langle(\chi_{\tau_I}?, 0), (\chi_{\tau_{O_1}}?, 1), \ldots, (\chi_{\tau_{O_n}}?, n)\rangle_{n+1}.$$

We claim that this interrogative inquiry leads to to the saturated conversational state C_D. To see this, consider the sequence $\langle C_1, \ldots, C_{n+1}\rangle_{n+1}$ of conversational states associated to $\langle(\chi_{\tau_I}?, 0), (\chi_{\tau_{O_1}}?, 1), \ldots, (\chi_{\tau_{O_n}}?, n)\rangle_{n+1}$. We then get that:

$$\begin{aligned}
\sigma_1 &= \sigma|\chi_{\tau_I} = \tau_I, \\
\sigma_2 &= \sigma|\chi_{\tau_I}|\chi_{\tau_{O_1}} = \tau_I \cap \tau_{O_1}, \\
&\vdots \\
\sigma_{n+1} &= \sigma|\chi_{\tau_I}|\chi_{\tau_{O_1}}|\ldots|\chi_{\tau_{O_n}} = \tau_I \cap \bigcap_{1 \leq i \leq n} \tau_{O_i},
\end{aligned}$$

where σ_q denotes the common ground of the conversational state C_q. Thus, we get that $C_{n+1} = C_D$, which means that the interrogative inquiry $\langle(\chi_{\tau_I}?, 0), (\chi_{\tau_{O_1}}?, 1), \ldots, (\chi_{\tau_{O_n}}?, n)\rangle_{n+1}$ leads to the saturated conversational state C_D of C. From that and the initial assumption that φ is distributed information in C, i.e., $D(C)[\varphi] = \{D(C)\}$, we conclude that φ is an interrogative consequence in $P_?(C, I_n)$. $\square$

Interestingly, one might see this result as stating a *completeness* property of interrogative inquiry: all the available information to the inquirer from the conversational participants, i.e., all the *distributed information*, can be obtained via the process of interrogative inquiry. The theorem also provides a *soundness* property of interrogative inquiry: all the information that can be obtained via the process of interrogative inquiry is necessarily *distributed information.* Thus, this result might be seen as a soundness and completeness theorem of interrogative inquiry with respect to the notion of distributed information.

Illustration. The proof of the above theorem provides a reason for the correspondence between interrogative consequence and distributed information in our framework: this correspondence is based on the existence of *characteristic questions* for the informational states of the conversational participants. Thus, such a characteristic question addressed to a conversational participant obliges him to give all the information he has. In order to illustrate this particular use of characteristic questions in the proof of theorem 2, we picture in figure 6.2 the *characteristic inquiry*[18] associated to the conversational state C from the example of section 5.2.

[17] In other words, this would mean that the answer χ_p produced by the participant indexed i_p does not respect the 'informative sincerity' clause of the sincerity maxim.

[18] By a characteristic inquiry of a conversational state C we mean the interrogative inquiry composed of the characteristic questions associated to the informational states of the conversational participants in C, abstracted of the order in which the questions are asked.

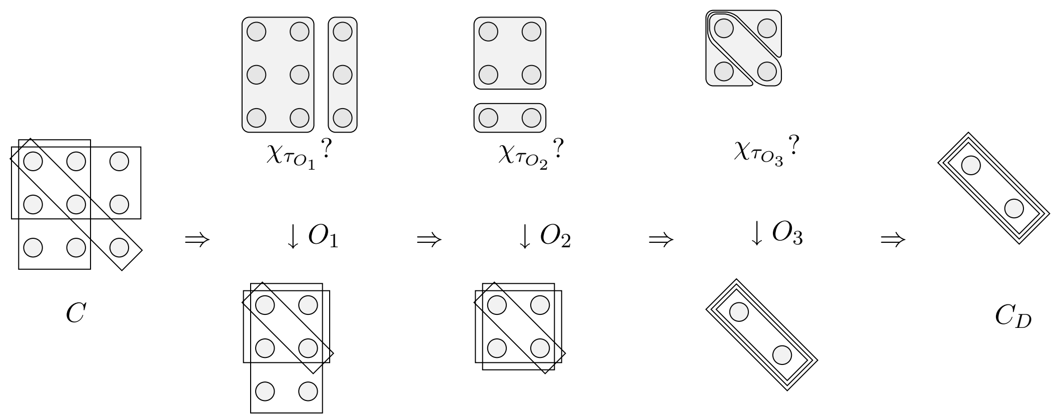


Figure 6.2: Characteristic inquiry associated to C from the example of section 5.2.

Corollary. The idea of *characteristic inquiry* provides us with an upper bound to the length of the shortest successful interrogative inquiry settling any interrogative consequence in a given interrogative protocol:

Proposition 6. *Let $P_?(C, I_n)$ be an interrogative protocol and $\varphi \in \mathcal{L}$ such that φ is an interrogative consequence in $P_?(C, I_n)$. The length of the shortest successful interrogative inquiry for φ in $P_?(C, I_n)$ is of at most $n+1$ steps.*

Proof. The proof of the previous theorem tells us that, in order to settle φ, it is sufficient to ask to each participant the characteristic question corresponding to its informational state. In this way, each participant will have to give all the information he has, and this interrogative inquiry will bring to the saturated conversational state D_C of C. Since this interrogative inquiry is of length $n+1$, and since, from the previous theorem, we know that this interrogative inquiry is sufficient to settle any φ such that φ is an interrogative consequence in $P_?(C, I_n)$, we get that the length of the shortest successful interrogative inquiry settling φ in $P_?(C, I_n)$ is of at most $n+1$ steps. □

6.2 Interrogative consequence and yes-no questions

In the framework of interrogative logic [15], Hintikka and colleagues proved the so-called *yes-no theorem.* This theorem says that, whenever a conclusion C can be established through an interrogative inquiry, C can be established by only asking *yes-no questions.* In section 5, we provided a definition of yes-no questions which now allows us to show that the yes-no theorem also holds in our framework:

Theorem 3 (Yes-no theorem)**.** *Let $P_?(C, I_n)$ be an interrogative protocol and $\varphi \in \mathcal{L}$. If φ is an interrogative consequence in $P_?(C, I_n)$, then there exists an interrogative inquiry composed exclusively of yes-no questions which settles φ, i.e., which leads to a conversational state C' such that $\sigma'[\varphi] = \{\sigma'\}$.*

Proof. Let $P_?(C, I_n)$ be an interrogative protocol and $\varphi \in \mathcal{L}$. Assume that φ is an interrogative consequence in $P_?(C, I_n)$. By definition, this means that there exists an interrogative inquiry $\langle(\varphi_1, i_1), \ldots, (\varphi_k, i_k)\rangle_k$ in $P_?(C, I_n)$ leading to a conversational state C' such that $\sigma'[\varphi] = \{\sigma'\}$. Now, let $\chi_1, \ldots, \chi_k$ be the answers respectively obtained as responses to the directed questions of the sequence $\langle(\varphi_1, i_1), \ldots, (\varphi_k, i_k)\rangle_k$. Then, we claim that $\langle(\chi_1?, i_1), \ldots, (\chi_k?, i_k)\rangle_k$ is an interrogative inquiry in $P_?(C, I_n)$ leading to the same node

C'.[19] To see this, notice that for any $q \in [\![1, k]\!]$, if χ_q is the answer to φ_q for i_q in C_q, where C_q denotes the node corresponding to the end-point of the branch $\langle(\varphi_1, i_1), \ldots, (\varphi_q, i_q)\rangle_q$, the answer to χ_q? for i_q in C_q is necessarily χ_q. The sequence of conversational states $\langle C_1, \ldots, C_k\rangle_k$ resulting from $\langle(\varphi_1, i_1), \ldots, (\varphi_k, i_k)\rangle_k$ in $P_?(C, I_n)$ is then identical to the sequence of conversational states associated to $\langle(\chi_1?, i_1), \ldots, (\chi_k?, i_k)\rangle_k$. Consequently, the two interrogative inquiries $\langle(\varphi_1, i_1), \ldots, (\varphi_k, i_k)\rangle_k$ and $\langle(\chi_1?, i_1), \ldots, (\chi_k?, i_k)\rangle_k$ lead to the same conversational state C', i.e., to a conversational state in which φ is settled. Hence, we have found an interrogative inquiry composed exclusively of yes-no questions which settles φ, namely $\langle(\chi_1?, i_1), \ldots, (\chi_k?, i_k)\rangle_k$. □

What the yes-no theorem says is that, whenever φ is an interrogative consequence in an interrogative protocol $P_?(C, I_n)$, φ can be reached in $P_?(C, I_n)$ through an interrogative inquiry composed exclusively of yes-no questions.

Illustration. The proof of the yes-no theorem is based on the idea that we can construct an interrogative inquiry composed of yes-no questions from any given interrogative inquiry, by asking for each question the yes-no question associated to the obtained answer. In order to illustrate this idea, we represent in figure 6.3 the interrogative inquiry composed of yes-no questions associated to inquiry 1 from the example of section 5.2.

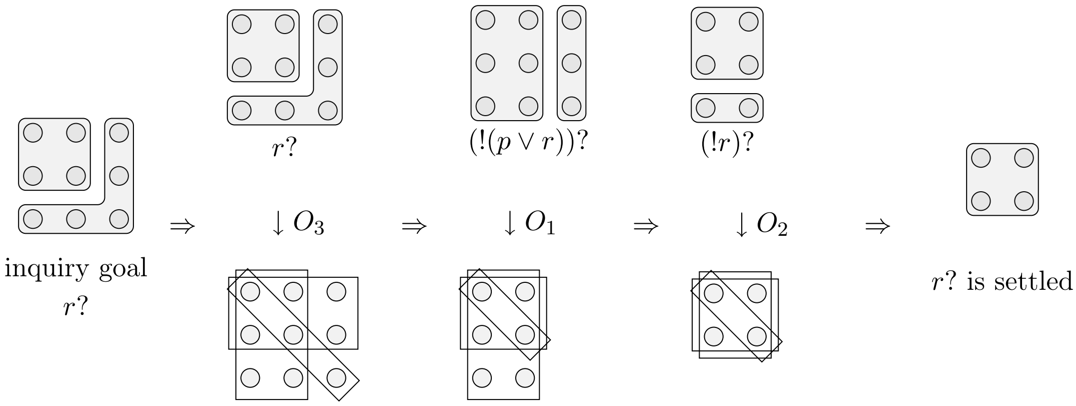


Figure 6.3: Yes-no inquiry associated to interrogative inquiry 1 from section 5.2.

7 Computational aspects

Computational aspects of the process of interrogative inquiry play a crucial role in practice: one is often interested to know how difficult it is to settle a given a question, how much time it will take, what are the best mechanical ways to choose the questions to ask etc. In this section, our goal is to shape the bases of a computational investigation of the process of interrogative inquiry in our framework, and to provide first computational results. Of course, a complete computational study of interrogative inquiry would deserve a dedicated paper. Our hope here is rather to convince the reader that our inquisitive formalization of interrogative inquiry is a very suitable framework for carrying out such a study. In the next section, we will argue that this computational approach can shed light on an important theme discussed by Hintikka in the IMI, namely the *strategic aspects of inquiry*.

[19]Since $\chi_1, \ldots, \chi_k$ are assertions, as answers to some questions, we write χ_i? instead of $(!\chi_i)$? for the yes-no question associated to χ_i.

7.1 Computational model

In section 4, we introduced the notion of interrogative rule through an analogy with the notion of inference rule. Here again, an analogy between the processes of *proving* and *questioning* can help in formulating the bases of our computational model. For the process of proving, we are interested in how much time one needs to find a proof of a given statement. For the process of questioning, we are interested in how much time one needs to find an interrogative inquiry leading to the settlement of a given question. In the case of proving, the computational unit is the application of an *inference rule*, whereas in the case of questioning, the computational unit is the application of an *interrogative rule*. In the process of proving, the prover has at his disposal a set of initial axioms along with the different statements that he has established. In the process of questioning, the inquirer has at his disposal his own information along with the information obtained through previous questions recorded into the common ground of the conversation.

Thus, we consider in our computational model that (i) the computational unit is a *question-answer step*[20] and (ii) the inquirer has at his disposal at each step his own informational state and the composition of the common ground. We will focus here on *time* complexity. Of course, this does not mean that space complexity is not important in the case of interrogative inquiry.

One of the main difficulties of a computational investigation of the process of interrogative inquiry lies in the many parameters that are susceptible to play a role in the computation. This appears clearly in our framework. If we consider an interrogative protocol $P_?(C, I_n)$, where $C = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n})$, we can already list the following parameters that will intuitively play a role in a computational study of interrogative inquiry:

- The number of oracles n,
- The number of indices composing the inquirer's informational state: $|\tau_I|$,
- The number of indices composing the common ground: $|\sigma|$,
- The complexity of the inquiry goal ψ in σ to be settled,
- The pragmatic rules for answering and updating adopted in I_n,
- The cardinality of the set of atomic variables $\mathcal{P}$.

One may then be interested to investigate what are the different impacts of each one of these parameters taken *individually*, but also the impacts of different *combined* variations of these parameters. Thus, this list already shows how complex and subtle an exhaustive computational study based on our framework could be.

7.2 An upper bound to the process of interrogative inquiry

It turns out that, in our framework, there exists an algorithmic method that can be used to settle any question that can be settled in a given interrogative protocol, i.e., that can be used to settle any *interrogative consequence*. We will call this method the *scan algorithm*. The main idea of this algorithm is, for each conversational participant, to successively ask the

[20] We consider in our model that the inquirer does not need to compute the proposition associated to a given formula, for instance for deciding whether a given formula corresponds to a question in the current common ground, but that it can have direct access to such information. In computational terminology, one would say that the inquirer can consult an *oracle*, avoiding the computational cost necessary to reach such information. Of course, the term oracle in the previous sentence should not be confused with the term oracle we use in our framework.

characteristic question associated to each one of the indices composing the common ground. In other words, this amounts to ask for each participant and for each index composing the common ground, if the participant's informational state allows to eliminate the considered index. Formally, the scan algorithm can be stated in the following way:

Definition 7.1 (Scan algorithm)**.** *The scan algorithm is defined as follows:*

1. *The algorithm takes as its input an interrogative protocol $P_?(C, I_n)$,*
2. *For each $v \in \sigma$, where σ refers to the current common ground, the inquirer successively asks the characteristic question χ_v? to each conversational participant,*
3. *The algorithm outputs the resulting conversational state.*

We now want to show that the scan algorithm can actually be used to settle any question that can be settled in a given interrogative protocol. To this end, we first show that the use of the scan algorithm in a given interrogative protocol leads to the saturated conversational state:

Proposition 7. *Let $P_?(C, I_n)$ be an interrogative protocol. The output of the scan algorithm is the saturated conversational state C_D.*

Proof. Let $P_?(C, I_n)$ be an interrogative protocol with $C = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n})$. Assume towards a contradiction that the scan algorithm does not output the saturated conversational state C_D. This means that there exists an index $v \in \sigma$ which is not in $D(C) := \bigcap_{1 \leq i \leq n} \tau_{O_i} \cap \tau_I$ and which has not been eliminated by the scan algorithm. Since $v \notin D(C)$, this means that there is a conversational participant whom the informational state does not contain v. However, the scan algorithm requires that the characteristic question χ_v? is addressed to this participant at some point. By appealing to lemma 1, we have that necessarily the answer to this participant should have led to the elimination of v from the common ground of the conversation, which contradicts our initial assumption. We conclude that the scan algorithm has for effect to eliminate all the indices in the common ground which are not in $D(C)$, and thereby output the saturated conversational state $C_D = (D(C), \ldots, D(C))$. □

This proposition allows us to show that the scan algorithm settles any question that can be settled in a given interrogative protocol:

Corollary 2. *Let $P_?(C, I_n)$ be an interrogative protocol. If φ is an interrogative consequence in $P_?(C, I_n)$, then the scan algorithm leads to a conversational state in which φ has been settled.*

Proof. Let $P_?(C, I_n)$ be an interrogative protocol. Assume that φ is an interrogative consequence in $P_?(C, I_n)$. By theorem 2, φ is distributed information in C. By definition, this means that φ is settled in $D(C)$. Since the output of the scan algorithm is the saturated conversational state $C_D = (D(C), \ldots, D(C))$, as proved by the previous proposition, we get that the scan algorithm leads to a conversational state in which φ has been settled. □

Since the scan algorithm can be used to settle any question that can be settled in a given interrogative protocol, by obtaining an upper bound to this algorithm, we will automatically obtain an upper bound to the process of interrogative inquiry in our framework. We let $\mathsf{T}_{scan}(C)$ be the number of question-answer steps necessary for the scan algorithm to reach the saturated conversational state C_D. We then have the following result:

Theorem 4 (Upper bound)**.** *Let $P_?(C, I_n)$ be an interrogative protocol with $C = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n})$. We have:*

$$\mathsf{T}_{scan}(C) \leq n \cdot |\tau_I| + 1 \leq n \cdot |\sigma| + 1 \leq n \cdot 2^{|\mathcal{P}|} + 1.$$

Proof. The first inequality is obtained as follows. First, the inquirer addresses the characteristic question $\chi_{\tau_I}?$ to himself. This forces him to give all the information he has, and leads to a reduction of the common ground to τ_I. Then, the inquirer uses the scan algorithm. This algorithm consists in asking at most $|\tau_I|$ questions, corresponding to the number of indices in the common ground after the first inquiry step, to the n oracles. This corresponds to at most $n \cdot |\tau_I|$ question-answer steps, and we have proved in proposition 7 that this leads to the saturated conversational state. Consequently, the inquirer was able to reach the saturated conversational state in at most $n \cdot |\tau_I| + 1$, hence the first inequality. The second inequality follows from the fact that $\tau_I \subseteq \sigma$. The third inequality follows from the fact that, given a finite set of atomic propositions $\mathcal{P}$, there are $2^{|\mathcal{P}|}$ possible indices in ω, and thereby there are at most $2^{|\mathcal{P}|}$ indices in a common ground based on $\mathcal{P}$. $\square$

What this theorem says is that the inquirer can settle any question that can be settled in a given interrogative protocol $P_?(C, I_n)$ in at most n times the number of indices composing his informational state. One interesting point of this result is that it shows the computational impact of the distributed nature of information among the conversational participants through the parameter n appearing in the upper bound. This fits the intuitive idea that, in practice, the more people we have to consult, the more complicated it is to obtain the desired information. It is also important to notice that, in the worst case in which (i) the inquirer has no information at all, i.e., $|\tau_I| = |\sigma| = 2^{|\mathcal{P}|}$, and (ii) the scan algorithm reaches the saturated conversational state in its last step, all the inequalities in theorem 4 turn into equalities.

7.3 The special case of MaxIA

It seems intuitive to think that the answering rule adopted by the conversational participants will play a role from a computational perspective. For instance, we would expect to be easier in some sense to settle an issue if the conversational participants are maximally informative. We will show a result in our framework which goes in this direction. To this end, we first define what we call the *all-in-all algorithm*:

Definition 7.2 (The all-in-all algorithm)**.** *The all-in-all algorithm is defined as follows:*

1. *The algorithm takes as its input an interrogative protocol $P_?(C, I_n)$,*
2. *For successively each conversational participant, the inquirer addresses to the considered participant the question*
$$\bigvee_{v \in \sigma} \chi_v$$
where σ denotes the current common ground and χ_v denotes the characteristic proposition of the index v,
3. *The algorithm outputs the resulting conversational state.*

The idea of the all-in-all algorithm is to address to each participant a specific question for which the possibilities are singletons corresponding exactly to the different indices composing the current common ground. In other words, the algorithm consists in proposing to all the conversational participants, all the possibilities corresponding to the different indices composing the common ground. The functioning of the all-in-all algorithm will be illustrated below with figure 7.1.

It turns out that, in the case where the conversational participants adopt a MaxIA answering rule, the all-in-all algorithm can reach the saturated conversational state in at most as much steps as the number of participants, as formally established by the following proposition:

Proposition 8. *Let $P_?(C, I_n)$ be an interrogative protocol such that I_n is based on a MaxIA answering rule. The all-in-all algorithm outputs the saturated conversational state C_D of C in at most $n + 1$ question-answer steps, i.e.,*

$$\mathsf{T}_{all}(C) \leq n + 1.^{21}$$

Proof. Let $P_?(C, I_n)$ be an interrogative protocol with $C = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n})$, and such that I_n is based on a MaxIA answering rule. Suppose that the first question is addressed to the inquirer. Then, it is easy to see that there is a unique proposition corresponding to a MaxIA to $\bigvee_{v \in \sigma'} \chi_v$, namely the proposition composed only of the possibility $\{v \mid v \in \sigma \text{ and } v \in \tau_I\}$. Updating the conversational state with this proposition leads to an elimination of all the indices in the common ground, and in the informational states of the participants, which are not in τ_I. By repeating this operation for all the remaining conversational participants, we end up in a conversational state in which the common ground and the informational states of the participants contain exactly the indices that are present in all the initial informational states of the participants. In other words, this means that the all-in-all algorithm outputs the conversational state $(D(C), \ldots, D(C))$, namely the saturated conversational state of C. Finally, since the all-in-all algorithm consists in asking one question for each participant, the algorithm runs in $n + 1$ steps in $P_?(C, I_n)$. It follows from this that $\mathsf{T}_{all}(C) \leq n + 1$. □

What this proposition says is, in the case of maximally informative participants, the all-in-all algorithm can settle any question that can be settled by asking at most one question to each participant. It is easy to see that this result relies very much on the fact that participants are adopting a MaxIA answering rule, and that the proposition does not hold, for instance, if the participants are adopting a MinIA answering rule. This result suggests that our framework, and our computational model, can be used to formally relate given classes of answering rules, corresponding to specific properties, with different degrees of difficulty of settling issues by the process of interrogative inquiry.

Illustration. We now illustrate in figure 7.1 the functioning of the all-in-all algorithm in the case of the example from section 5.2.

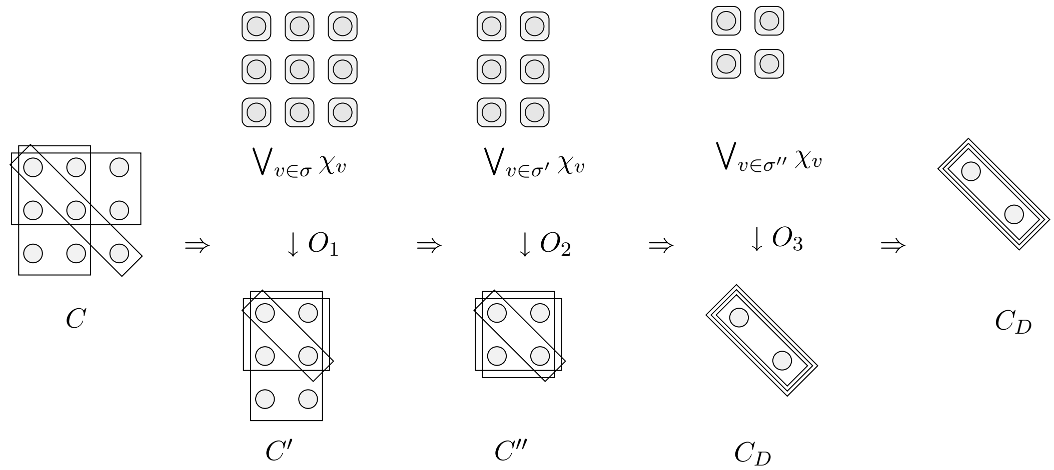


Figure 7.1: The all-in-all algorithm applied to the example of section 5.2.

[21]We let $\mathsf{T}_{all}(C)$ be the number of question-answer steps necessary for the all-in-all algorithm to reach the saturated conversational state C_D.

7.4 Interrogative inquiry and computational complexity

One of the most promising lines of research from the computational perspective on interrogative inquiry concerns the study of interrogative inquiry from the point of view of *computational complexity*.[22] However, as we said at the beginning, treating this aspect fully would require a dedicated paper. In this section, we will restrict ourselves to state the *interrogative inquiry decision problem*, which indicates a possible way to study the complexity of interrogative inquiry, and we will show that this problem belongs to the class NP in our computational model. We will also state the associated *interrogative inquiry optimization problem*, and spell out its epistemological significance.

Interrogative inquiry decision problem. Here again, the analogy between proving and questioning can help shape ideas. In the case of proving, one is often interested into the complexity of deciding whether or not there exists a 'manageable' proof of a given statement, i.e., a proof of length less than a given $k \in \mathbb{N}$. In the case of questioning, the analogous problem is to decide whether or not there exists a 'manageable' inquiry settling a given issue, i.e., an inquiry composed of less than k question-answer steps. This is what we will call the *interrogative inquiry decision problem*, which we define as follows:

Definition 7.3 (Interrogative inquiry decision problem)**.** *The interrogative inquiry decision problem is a decision problem defined by*

INPUT: *An interrogative protocol $P_?(C, I_n)$, a formula $\varphi \in \mathcal{L}$ s.t. φ is a question in C and a natural number $k \in \mathbb{N}$.*

QUESTION: *Can φ by settled by the process of interrogative inquiry in less than k steps?*

Trying to classify the interrogative inquiry decision problem into some of the main complexity classes would already provide a first way to study the complexity of interrogative inquiry. Indeed, we can already easily show that this decision problem is in NP with respect to the parameter k:

Theorem 5. *The Interrogative inquiry decision problem is in* NP*.*

Proof. Let $P_?(C, I_n)$ be an interrogative protocol, $\varphi \in \mathcal{L}$ such that φ is a question in C and $k \in \mathbb{N}$. To see that the interrogative decision problem is in NP, non-deterministically guess an interrogative inquiry $\langle(\varphi_1, i_1), \ldots, (\varphi_k, i_k)\rangle_k$ in $P_?(C, I_n)$. According to our computational model, the inquirer can check if this interrogative inquiry settles φ by simply successively addressing the directed questions in $\langle(\varphi_1, i_1), \ldots, (\varphi_k, i_k)\rangle_k$, and by checking if this procedure results into a conversational state in which φ is settled. This verification procedure is done in k-steps, and so in polynomial-time with respect to the parameter k. $\square$

Of course, having determined that the interrogative inquiry decision problem is in NP, the next question would be to determine whether it is NP-*complete*. In this paper, we will leave this issue as an open problem for further research.

Interrogative inquiry optimization problem. In the previous paragraph, we state a *decision* problem, which was concerned with determining whether or not there exists a manageable interrogative inquiry settling a given issue. Although this perspective is useful to get first information on the complexity of the process of interrogative inquiry, we are in practice more interested in the *search* problem of *finding* an interrogative inquiry settling a given issue. Indeed, we are often interested in even more, namely finding an *optimal* interrogative

[22] We refer the reader to [1] for an introduction to the field of computational complexity.

inquiry which will minimize the number of questions to ask. This kind of problems are called *optimization* problems, and we can state the optimization problem relative to interrogative inquiry as follows:

Definition 7.4 (Interrogative inquiry optimization problem). *The interrogative inquiry optimization problem is an optimization problem defined by*

INPUT: *An interrogative protocol* $P_?(C, I_n)$, *a formula* $\varphi \in \mathcal{L}$ *s.t.* φ *is a question in* C *and a natural number* $k \in \mathbb{N}$.

TASK: *Find an interrogative inquiry settling* φ *which minimizes the number of inquiry steps.*

Although we leave an exhaustive investigation of the complexity of the interrogative inquiry optimization problem for further research, it is important to stress here the epistemological significance of the interrogative inquiry optimization problem. This optimization problem is directly related to the concepts of *research strategies* and *research agendas*. This is due to the fact that this problem can be interpreted as a search problem for determining the *best* questions to ask in order to progress as quickly as possible in a given inquiry. In other words, the interrogative inquiry optimization problem can be seen as a search problem for *optimal* research strategies, and for determining *optimal* research agendas in the prospect of settling a given issue. This aspect is very much connected with what Hintikka calls the *strategic aspects* of inquiry. In the following section, we will show how the computational perspective developed in this section leads to an interesting view on this issue.

8 Strategic aspects of inquiry: the algorithmic view

According to Hintikka, one of the main advantages of the game-theoretic formulation of the IMI is to offer a precise way to investigate the so-called *strategic aspects of inquiry*. In this section, we begin by presenting Hintikka's view on the strategic aspects of inquiry. Then, we present an alternative view emerging from the computational approach of the previous section which we call the *algorithmic view* on the strategic aspects of inquiry.[23] In several papers, Hintikka draws some implications of his game-theoretic view for the study of research strategies in philosophy of science and for the investigation of the logic of conversation in pragmatics. We will review Hintikka's works in these directions, and show how to reformulate them from the algorithmic view.

8.1 Hintikka on the strategic aspects of inquiry

In the following quote, Hintikka explains what he means by the strategic aspects of inquiry:

> Another main requirement that can be addressed to the interrogative approach—and indeed to the theory of any goal-directed activity—is that it must do justice to the strategic aspects of inquiry. Among other things, it ought to be possible to distinguish the definitory rules of the activity in question from its strategic rules. The former spell out what is possible at each stage of the process. The latter express what actions are better and worse for the purpose of reaching the goals of the activity. This requirement can be handled most naturally by doing what Plato already did to the Socratic *elenchus* and by construing knowledge-seeking by questioning as a game that pits the questioner against the answerer. Then

[23] By contrast, we sometimes refer to Hintikka's view as the *game-theoretic view* on the strategic aspects of inquiry. Notice that the term 'strategic', in the expression 'strategic aspects of inquiry', is used here in a broad sense and does not carry out the technical meaning that it has in game theory.

the study of the strategies of knowledge acquisition becomes another application of the mathematical theory of games [...]. The distinction between the definitory rules—usually called simply the rules of the game—and strategic principles is built right into the structure of such games. [14, p. 19]

Thus, Hintikka introduces the idea of the strategic aspects of inquiry by drawing a distinction between the *definitory* and the *strategic rules* of the game of inquiry: the definitory rules say what are the questions that the inquirer is *allowed* to ask, whereas the strategic rules say what are the *best* questions to ask in order for the inquirer to reach his inquiry goal. The investigation of the strategic aspects of inquiry refers thereby to the study of the *strategic rules* of the game of inquiry, and the formulation of the IMI as a *game* allows to use the mathematical framework of game theory for studying them.

In the framework developed in this paper, the definitory rules delimiting the questions that the inquirer is allowed to ask are encoded into the specific *interrogative protocol* within which the interrogative inquiry is conducted. What would then be the notion corresponding to the strategic rules in our case? Of course, since our framework is not stipulated in game-theoretic terms, we cannot refer to the game-theoretic notion of *strategy* here. However, the computational approach developed in the previous section suggests an alternative notion for studying the strategic aspects: the one of *algorithmic method* for computing the questions to ask. This is the intuitive content of the *algorithmic view* on the strategic aspects of inquiry that we now present.

8.2 The algorithmic view

One of the main arguments of Hintikka for adopting the game-theoretic view is that the strategic aspects cannot be studied on a move-by-move basis, but only at the level of complete strategies: "From the general theory of games, we know that such rules cannot be formulated in move-by-move terms—for instance, in terms of the relationship of premises to a conclusion—but only in terms of complete strategies." [14, p. 45]. As we have seen through the two examples of algorithmic methods for computing questions introduced in the previous section, i.e., the *scan algorithm* and the *all-in-all algorithm*, algorithmic methods are also not formulated in move-by-move terms insofar as they make sense only if the inquirer sticks to the adopted algorithmic method during all his inquiry. Then, in the same way as strategies can be compared in a game-theoretic framework, we can compare algorithmic methods with respect the their *efficiency* for settling issues. What makes this approach interesting is that there are many different ways to compare algorithmic methods. For one thing, the efficiency of algorithmic methods can be measured in terms of different specific parameters of the framework, as we have seen in the previous section. Besides, one might also want to compare algorithmic methods in specific contexts, by formally restricting the comparison with respect to a given class of interrogative protocols. This has been illustrated in section 7.3, when we exhibit a very efficient algorithmic method for settling issues when the participants are maximally informative.

Thus, we can now formulate the algorithmic view as follows: the *algorithmic view* on the strategic aspects of inquiry consists in investigating the strategic aspects of inquiry by studying the average efficiency of algorithmic methods for computing the question to ask at each step of an interrogative inquiry. It is important to notice that the game-theoretic view and the algorithmic view are not mutually exclusive views on the strategic aspects of inquiry. Rather, we see them as complementary and as illuminating different aspects. Indeed, one might even imagine studying these two views jointly by developing a framework allowing to represent both computational and game-theoretic aspects of interrogative inquiry.[24]

[24]There are a lots of concrete cases in which computational and interactive aspects of inquiry are both

We now review implications of the strategic aspects of inquiry for the philosophy of science and for pragmatics drawn by Hintikka from the game-theoretic view, and we discuss how to approach them from the algorithmic view.

8.3 Implications for the philosophy of science

In his first works on the IMI, Hintikka's motivations were to study *scientific inquiry* by conceiving it as an interrogative procedure, where experiments and observations in science would be thought as *questions put to Nature*. In this context, Hintikka argues that the game-theoretic view on the strategic aspects of interrogative inquiry can shed light on the issue of *research strategies* studied in the field of philosophy of science:

> One of the most attractive features of my model of scientific inquiry is that it enables us to use game-theoretical concepts and methods. For the model can be cast in the form of a game against nature in the sense of the mathematical theory of games. Game theory is the best available general tool for considerations of strategy. This should make my model especially attractive for the purpose of studying such dynamics of science as are manifested in sequences of choices by a scientist, as distinguished from, e.g., a one-shot choice of a hypothesis on the basis of evidence. Along the same lines, we can also hope to cash in on one of the favorite metaphors of recent theorists of science, the idea of research strategy. The study of research strategies can now in principle be subsumed under the study of strategies in general. [10, p. 81]

It is clear that the framework developed in this paper was oriented towards studying interrogative inquiry in the context of conversations, and thereby was not designed to study scientific inquiry. Nevertheless, the algorithmic view on the strategic aspects of inquiry can be transposed to the context of scientific inquiry. In this case, the algorithmic view would suggest to represent research strategies as algorithmic methods, and to compare different research strategies with respect to their computational efficiency. The algorithmic view might then shed light on the notion of *heuristics* which has been extensively discussed by philosophers of science. Of course, this would require to develop a plausible interrogative model of scientific inquiry which would in addition be able to support a computational study. This raises an interesting question for further research: could the inquisitive framework be used for representing *questions to Nature* and for developing an interrogative model of scientific inquiry?

8.4 Implications for pragmatics

In [11] and [12], Hintikka applies the IMI to the study of dialogues and the logic of conversation. In this context, the strategic aspects are used to construct a critics of the speach-act theories and of Gricean pragmatics:

> In past studies of various kinds of dialogues, philosophers and linguists have typically formulated their concepts and theses in a way that, in terms of my model, apply to individual moves in the interrogative "game", corresponding to particular utterances in a dialogue or discourse. Examples are provided by speech-act

important. Think of situations in which research communities are interacting and competing for settling a given research problem, or situations in which lawyers are competing for obtaining specific information etc., where in all these cases the involved agents have limited computational capacities. On the technical side, the emerging field of *algorithmic game theory* [19] might provide the necessary formal tools to develop a framework for representing jointly game-theoretic and computational aspects.

theories, whose very name betrays their conceptual focus; and Grice's conversational maxims. There is a sense in which no such theory focusing on particular "moves" can be fully satisfactory, for from game theory we know that no values ("utilities") can in the last analysis be assigned to individual moves in the game, only to (complete) strategies. In other words, there is no theoretically satisfactory way of relating particular moves to the general ends of the dialogue in question, in the case of my model, to the ends of inquiry. Thus theories of the kind just mentioned are bound to remain unsatisfactory in the last analysis. [11, p. 137]

The point that Hintikka is making here is that there might be *more* to the logic of conversation than simply rules for governing utterances on a move-by-move basis. In the case of inquiry, the inquirer might want to adopt specific strategies for settling his inquiry goal, and such strategic rules should, according to Hintikka, be part of the logic of conversation. According to the algorithmic view, such strategic rules correspond to the algorithmic methods that the inquirer is using to settle his inquiry goal. Thus, our framework allows to reformulate the point that Hintikka is making as follows: in the context of interrogative inquiry, definitory rules, encoded into interrogative protocols, and strategic rules, represented as algorithmic methods for choosing questions, participate *both* of the rules governing the language-game of interrogative inquiry.

9 Conclusion

In this paper, we have developed a formal framework for investigating the process of interrogative inquiry in conversational contexts. Our approach was built on recent developments on the modelling of questions and answers in conversations, and took as its foundations the framework of inquisitive semantics and pragmatics. We have proposed a way to define the key notions of *interrogative rule*, *interrogative protocol*, *interrogative inquiry* and *interrogative consequence*, and we hope to have convinced the reader that the inquisitive framework was very suitable for doing so. We have then shown how our formal framework enables subtle logical and computational investigations of the process of interrogative inquiry. Finally, the computational perspective on interrogative inquiry led us to a new view on the strategic aspects of inquiry that we called the *algorithmic view*.

This paper is a very first step towards connecting the inquisitive framework with the study of interrogative inquiry. Our hope is that this work is paving the way for future developments in this direction. Indeed, several lines of research already come to mind. We now sketch what are for us the most promising directions for further research.

One of the most direct improvements of our framework would consist in considering a richer language than the propositional language that we adopted at the beginning. Two recent developments in inquisitive semantics are directly relevant for this purpose. The first one is the first-order inquisitive semantics developed in [3] and [8], which could be a starting point for investigating interrogative inquiry in the *first-order case*. The second one is the Inquisitive Dynamic Epistemic Logic (IDEL) presented in [22], which could allow an investigation of interrogative inquiry about *higher-order information*. Indeed, the framework of IDEL could benefit from several works in the DEL tradition for representing interrogative inquiry as a process over time.[25] For instance, the already mentioned extension of dynamic epistemic logic of questions with protocols [25] could be a source of inspiration for developing an analogous extension of IDEL with interrogative protocols.

[25]We refer the reader to the PhD thesis [16], the paper [24] and the chapter 11 of [23], for an overview of the logical treatment of long-term processes in the DEL tradition.

Another direct improvement of our framework would consist in introducing *deduction* into the picture. Hintikka has heavily emphasized on the importance of the interaction between questions and inferences in the process of interrogative inquiry. The introduction of deduction is relevant for a study of interrogative inquiry based on the three languages just discussed, namely the propositional, first-order and epistemic languages. In the propositional case, on-going work by Wiśniewski [30] on merging Inferential Erotetic Logic [29] and inquisitive semantics constitutes a very promising way for jointly dealing with questions and inferences in the inquisitive framework. In the first-order case, inspiration can be taken from the *Interrogative Logic* developed by Hintikka and colleagues in [15] which accounts for both questions and inferences by providing a proof system with rules for deduction and a rule for questioning. In this case, it would then be interesting to see how to develop a logic of questions and inferences based on the first-order inquisitive semantics. In the epistemic case, one of the main issues would consist in dealing with the problem of *logical omniscience*. Here again, recent works in the DEL tradition point out possible directions for dealing with this problem and for representing inferences and questions in IDEL.[26]

We conclude with two last research directions that appear very important to us: the study of interrogative inquiry from a computational point of view and the study of the social dimension of interrogative inquiry. In this paper, we have shaped the bases of a computational approach to interrogative inquiry. It then appeared clearly that a full treatment of this aspect requires a specific study. Important topics here are (i) the study of specific algorithms for choosing the questions to ask at the different steps of an interrogative inquiry and (ii) the study of interrogative inquiry from the point of view of complexity theory. Finally, scientific practice, and even conversations in daily life, show us that interrogative inquiry is often a *social process*, conducted by *groups* of cooperative agents. By combining the framework developed in this paper with logical approaches to multi-agent systems, one might hope having the necessary tools for studying the social dimension of interrogative inquiry.

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