Disjunctive Counterfactuals in Alternative and Inquisitive Semantics

Sarah Hiller

December 3, 2014

This is a report on a Master of Logic Research Project (6ECTS)
supervised by Floris Roelofsen

Contents

1 Introduction

The framework of *Alternative Semantics* is based on the idea that operators such as indefinites or disjunction generate a structured set of alternatives. It provides a great empirical improvement over Montague Grammar, in several respects. One relevant improvement concerns the interpretation of natural langage disjunction *or*. By viewing it as an operator which outputs a structured meaning containing the denotations of all disjuncts Alternative Semantics is able to account for several types of examples for which Montague Grammar makes false predictions. Notably, Alternative Semantics predicts a difference between sentences of type (A or B) and sentences of type (A or B or both), which have the same truth conditions and thus fail to be differentiated by Montague Grammar.
However, Alternative Semantics has conceptual drawbacks: by giving a special role to disjunction the connectives are no longer interpreted uniformly. That is, *or* is treated differently from *and*, which contradicts common intuition.
Inquisitive Semantics manages to overcome this impediment by incoporating inquisitiveness into the meaning of all sentences. A sentence is no longer limited in its meaning to the information provided, but in this framework also the issues which are raised become visible. This enables a uniform treatment of the connectives, while at the same time keeping many of the empirical advantages over Montague Grammar achieved before by using the framework of Alternative Semantics. Inquisitive Semantics together with the most basic translation of natural language disjunction *or* into the logical language fails, however, for examples of the type mentioned above: Viewing *or* inclusively we cannot differentiate between sentences of kind (A or B) and those of kind (A or B or both). It is therefore justified to think that we will need an extension of the Inquisitive Semantics framework with some kind of exclusive strengthening operator in the interpretation of *or*, such that (A or B) is interpreted as (A or B but not both) in the logical language. Introducing an operator to make sure that a framework is able to account for a certain type of example can easily be an inelegant ad hoc procedure. We want to find out wether a strengthening operator is also independently motivated.
The aim of this paper is therefore to investigate whether Alternative Semantics or Inquisitive Semantics together with an exclusive strengthening operator in the mapping of *or* to the logical language, Inq_{ES},[1] make better predictions in a specified set of examples. The set of examples chosen here are disjunctive counterfactuals, in reference to Alonso-Ovalle (2006). He presents these as one of the types of sentences Alternative Semantics makes the right predictions for while Montague Grammar does not. Concerning these examples Alternative Semantics is empirically stronger than the basic form of Inquisitive Semantics, thus the former was chosen as the point of comparison for Inq_{ES}.
The paper is organised as follows: In section 2 we will present elementary founda-

[1] The fact that we represent this process of interpretation into the logical language and consequent analysis via Inquisitive Semantics as Inq_{ES} is not intended to imply that we are modifying the framework of Inquisitive Semantics itself. Rather, we are modifying the way natural language sentences are mapped into the logical language before the semantic framework is applied.

tions and concepts of Alternative Semantics, with a specific emphasis on natural language disjunction *or*. In section 3 we will show how a basic framework of Inquisitive Semantics, Inq_B, is constructed starting from a modified notion of sentence meaning which includes inquisitiveness. We will see in section 3.3 that this approach cannot differentiate between nested alternatives. Structurally, this is a desired property. However, it also implies that with an inclusive mapping of *or* into our logical language we cannot account for the intuitive difference between natural language sentences containing disjunctions with nested alternatives and those that lack to nested possibility. In section 4 finally we will present the small pilot study we conducted to test people's intuitive understanding of *or*.

2 Disjunction in Alternative Semantics

In this section we will have a look at Alternative Semantics. First, at arguments brought up justifying its use against that of standard semantics. That is, sentences for which a standard analysis makes false predictions but which can be correctly analyzed using Alternative Semantics. Specifically, in our case these examples all contain natural language disjunction. Afterwards we will summarize how Alternative Semantics is concretely implemented. This immediately shows one disadvantage Alternative Semantics has against standard semantics. Inquisitive Semantics overcomes the issue mentioned here. At the same time however, basic Inquisitive Semantics $\mathsf{INQ_B}$ once again makes false predictions for some examples similar to those presented in the first subsection, which justified the use of alternative semantics. Looking at these inclined thinking about the same issues in the context of Inquisitive Semantics and showed that certain modifications in the interpreation of natural language disjunction are necessary. Thus, the following section is not only interesting in its own respect, it also clarifies the background against which the subsequent analysis takes place.

2.1 Justification

In his dissertation (Alonso-Ovalle, 2006) Luis Alonso-Ovalle presents Alternative Semantics as a solution to certain deficiencies of what he calls 'standard textbook semantics' with respect to the analysis of natural language disjunction. I consider 'textbook semantics' to refer to Montague Grammar. For readers not familiar with this theory appendix A contains a very short introduction. Montague Grammar and Alternative Semantics, in the way presented here, seem to differ in an important perspective. However, this difference can be overcome with a change of perspective. The apparent divergence is that in Montague Grammar the connectives are introduced via separate definitions for each different category, whereas Alonso-Ovalle only explicitly talks about the interpretation of connectives between sentences, implying a cross-categorial approach. Barbara Partee and Mats Rooth show how to recursively generalize a definition of the functioning of connectives on whole sentences to a definition of the functioning of connectives on any conjoinable type in Partee and Rooth (1982). Appendix B contains a summary of the relevant part of this paper.
After having made this explicit we can continue looking at the arguments Alonso-

Ovalle presents in favor of Alternative Semantics. He focuses on one central shortcoming of 'standard semantics' with respect to the interpretation of *or*.[2] To understand the argument he makes we first of all need to apply the above-mentioned procedure to get from the definition for the interpretation of *or* given in Montague Grammar to a possible world semantics. Thus an expression φ' of type t is no longer mapped to a truth condition. Instead, it is mapped to the set of possible worlds in the model $\mathbf{M}$ in which it holds. Explicitly, $\varphi' \mapsto \{w \mid [\![\varphi']\!]_{\mathbf{M},w,g} = 1\}$. Recall the rules in Montague Grammar for introduction of *or* between sentences:[3]

$S10$ If φ, ψ are expressions of category S, then $F_9(\varphi, \psi)$ is an expression of category S and $F_9(\varphi, \psi) = \varphi \textit{ or } \psi$

$T10$ If φ, ψ are expressions of category S and $\varphi \mapsto \varphi'$ and $\psi \mapsto \psi'$, then $F_9(\varphi, \psi) \mapsto (\varphi' \vee \psi')$

Now if $\varphi \mapsto \varphi' \mapsto \{w \mid [\![\varphi']\!]_{\mathbf{M},w,g} = 1\}$ and $\psi \mapsto \psi' \mapsto \{w \mid [\![\psi']\!]_{\mathbf{M},w,g} = 1\}$ then $F_9(\varphi, \psi) \mapsto \varphi' \vee \psi' \mapsto \{w \mid [\![\varphi' \vee \psi']\!]_{\mathbf{M},w,g} = 1\}$
But $\{w \mid [\![\varphi' \vee \psi']\!]_{\mathbf{M},w,g} = 1\} = \{w \mid [\![\varphi']\!]_{\mathbf{M},w,g} = 1\} \cup \{w \mid [\![\psi']\!]_{\mathbf{M},w,g} = 1\}$
So we can see that in 'standard semantics' *or* works as set union, combining several propositions into one. Alonso-Ovalle's criticism of this approach focuses on the fact that this entails inaccessibility of the single disjuncts in a later analysis. He illustrates why this leads to false conclusions using three sets of examples: counterfactuals with a disjunctive antecedent, unembedded disjunctions which have an exclusiveness implication and disjunctions which appear under the scope of a deontic modal. We will briefly discuss all three sets of examples in the following section but concentrate on disjunctive counterfactuals later in the article. Based on these grounds Alonso-Ovalle argues for an alternative semantics, in which *or* introduces a set of alternatives in contrast to one single proposition. The elements of this set are all accessible afterwards. He consequently shows how this can be used to solve the problems displayed beforehand. So now let us have a look at the examples he presents.

The first set of examples is concerned with disjunctive counterfactuals. We will focus here on *would*-counterfactuals, but mention in what ways *might*-counterfactuals differ. It will become apparent that the same problems that occur for *would*-counterfactuals still hold for *might* cases, so it suffices to elaborate one case for the argument to hold. In order to understand how 'standard semantics' leads to false predictions we need to know not only how *or* is interpreted, which was discussed above, but also how counterfactuals are evaluated.
Suppose we have a model $\mathbf{M}$ and a set of possible worlds W given and want to evaluate a counterfactual in a certain world w. Alonso-Ovalle uses a minimal change semantics for this purpose. That is, he presupposes a weak ordering $\leq$ on the set of possible worlds W, depending on the world of evaluation w. This weak ordering

[2]Precisely this should be called 'natural language disjunction', but I will often abbreviate to *or*, based on the fact that this text is written in english.

[3]See Appendix A.

shows how similar other possible worlds are to w. For example, if w is the world we are living in and it is currently raining, then a world w' in which it is not currently raining but apart from this everything stays the same is closer to w than a world w'' in which it is not raining and additionally the sea level has risen so high we all live under water. Using this weak ordering a *would*-counterfactual "If φ then would ψ" is then evaluated by checking whether ψ holds in all φ-worlds which are most similar to the current world.

$$[\![\text{If } \varphi, \text{ then would } \psi]\!]^{\leq}(w) \;\Leftrightarrow\; \forall w'[f_{\leq_w}([\![\varphi]\!])(w') \rightarrow [\![\psi]\!](w')]$$

Where $f_{\leq_w}$ is the function selecting the worlds closest to w according to the ordering $\leq_w$: $\quad f_{\leq_w}([\![\varphi]\!]) = \{w' \mid [\![\varphi]\!](w') \text{ and } \forall w''([\![\varphi]\!](w'') \rightarrow w' \leq_w w'')\}$
Also, considering that $[\![\varphi]\!]$ is a set and w a possible world, $[\![\varphi]\!](w)$ is simply a different notation for $w \in [\![\varphi]\!]$. For *might*-counterfactuals the consequent merely needs to be possible, given the antecedent, thus it only has to hold in *one* of the possible worlds.

$$[\![\text{If } \varphi, \text{ then might } \psi]\!]^{\leq}(w) \;\Leftrightarrow\; \exists w'[f_{\leq_w}([\![\varphi]\!])(w') \rightarrow [\![\psi]\!](w')]$$

So, continuing the example from above, let us look once more at the statement "If it weren't raining, then the ground would not be wet." In order to evaluate this in world w only world w' is taken into consideration, while world w'' is left out. Therefore the statement is correctly evaluated as true.[4]
Now one can construct rather artificial examples of counterfactuals with a disjunctive antecedent made up of two disjuncts, one of which is substantially closer to the real world than the other. Alonso-Ovalle uses an example given by Nute (1975), of a farmer complaining about the bad crop by saying "If we had had good weather this summer or the sun had grown cold, we would have had a bumper crop." Clearly if the sun had grown cold the crop would not have been good at all, so the statement should be evaluated as false. Standard semantics however falsely predicts it to be true. We saw above that *or* is interpreted as set union. Thus the good-weather worlds are joined with the cold-sun worlds to give one single set. Evidently way more things would have to be different for the sun to grow cold than for the weather to be good in summer. Hence inside the single set created by *or* all worlds closest to the actual world are worlds in which the weather was good during summer. And all worlds in which the weather was good during summer, and in which not much else changed, will be worlds in which there was also a good crop. Hence the counterfactual is falsely evaluated as true. This is how a standard interpretation of *or* together with a minimal-change semantics for counterfactuals yields a wrong prediction for examples of this type. To generalize, with the constituents of the disjunction not being accessible in the later analysis, as is the case in standard semantics, only the more likely part of the antecedent gets taken into consideration. In contrast to this, one would naturally think that the consequent has to hold in the possibilities created by each of the disjuncts in order for the counterfactual to be true. It is easy to see that all of this holds equivalently in *might*-counterfactuals.

[4] That is, the current world w belongs to the set of possible worlds associated with this statement, if we stick to a possible world semantics.

In alternative semantics expressions no longer denote single objects, but instead sets of objects. In most basic cases these sets will be singletons containing the standard denotation of the expression that is being interpreted. One main novelty is that applying *or* keeps the interpretation of both disjuncts and creates a new set of possibilities containing both these denotations. Functional application is effected pointwise, so this set of alternatives does not collapse into a singleton unless it reaches an existential or universal closure operator. That is, an expression which is interpreted as a function that quantifies over the elements of this set of alternatives. Therefore using alternative semantics makes it possible for the consequent of a disjunctive counterfactual to operate on both constituent parts of the antecedent. Consequently an appropriate analysis of counterfactuals has to be chosen, which makes use of this possibility. To this end, Alonso-Ovalle views counterfactuals as correlative constructions, with *then* acting as a pronoun ranging over propositions which picks up the denotation of the *if*- phrase and makes it accessible to the *would*- or *might*-functions. Here Alonso-Ovalle's analysis is not quite explicit, as he points out that in alternative semantics sentences no longer denote propositions, but sets of propositions. However, when talking about the reference of the pronoun *then* he states that *then* ranges over propositions, but at the same time picks up the denotation of the *if*-clause.[5] This can be clarified by stating that *then* ranges over sets of proposition. This is in line with Alonso-Ovalles argument which links the analysis of correlatives, where a pronoun in the main clause picks up the denotation of a relative clause, to the analysis of conditionals, where *then* picks up the denotation of the *if*-clause, i.e. a set of propositions. Aside from this, the main idea in the interpretation of conditionals is that *then* makes the denotation of the *if*-clause available in the main sentence. *Would* and *might* work as was described before and thus provide the closure operator which quantifies over the possibilites created by *or*. After the above adjustment we also need to clarify how $f_{\leq_w}$ works when applied to sets of propositions. As we can clearly see from the examples above we would want a world w' to belong to $f_{\leq_w}(\llbracket\varphi\rrbracket)$ if w' is the closest world to w in one of the possibilities in $\llbracket\varphi\rrbracket$. That is

$$f_{\leq_w}(\llbracket\varphi\rrbracket) \;=\; \{w' \mid \exists p \in \llbracket\varphi\rrbracket \text{ s.t. } (p(w') \text{ and } \forall w''[p(w'') \rightarrow w' \leq_w w''])\}$$

If $\llbracket\varphi\rrbracket$ is a singleton this simply coincides with the above definition, whereas if $\llbracket\varphi\rrbracket$ contains several alternatives this function selects the closest worlds out of every alternative.

To summarize, the analysis of *or* in alternative semantics together with an interpreation of counterfactuals as correlative constructions with *then* referring to the *if*-clause and finally *would* or *might* operating on this set of alternatives is a solution to the problem presented concerning interpretation of disjunctive counterfactuals.[6]

Although we will concentrate on disjunctive counterfactuals in the later part of this paper we will nevertheless briefly explore the two other sets of examples Alonso-

[5] All this is stated in Alonso-Ovalle (2006), pp.22 - 25

[6] Alonso-Ovalle also presents solutions to this issue which change the interpretation of counterfactuals, instead of that of *or*, but I will not discuss these here.

Ovalle brings up in justification of Alternative Semantics for *or* over 'Standard Semantics'. The second type of sentences for which a standard analysis of *or* has trouble making the right predictions is the derivation of exclusiveness implicatures in unembedded disjunctions.[7] That is, given a disjunction "φ or ψ", we can derive that only one of the two disjuncts holds. In the case of two disjuncts a standard analysis for *or* still yields the right results. *Or* is standardly assumed to form a Horn-scale with *and*. That is, "φ and ψ" is the related stronger statement to "φ or ψ". Exclusivity of a given disjunction then follows as a conversational implicature: if the speaker stated "φ or ψ", and "φ and ψ" is a stronger statement on the same scale, this must mean that the speaker did not have enought evidence to state "φ and ψ". Therefore the interpretation of the disjunction must be restricted to "φ or ψ, but not both."[8] It is important to note that even though this is a pragmatic derivation the semantics needs to be correct for it to work. It is the semantic analysis that provides the constituents on which a following pragmatic analysis can work.
Now that we have seen how one can derive exclusiveness on a disjunction with two atomic disjuncts using 'standard semantics' we can continue to the case Alonso-Ovalle shows to be problematic: disjunctions with more than two disjuncts. If we try to apply the above procedure to a disjunction "φ or ψ or χ" it becomes clear that the only statements we can pragmatically derive to be false are the following.[9]

- (φ and ψ) or χ
- (φ or ψ) and χ
- (φ and ψ) and χ

However, the negation of the first statement is pragmatically inconsistent with the basic disjunction as it derives that χ must be false, so there was no reason for the speaker to state χ as an option to begin with. Furthermore the negations of the other two statements do not exclude the possiblity of two disjuncts being true at the same time. Thus the algorithm for deriving an exlusiveness implicature applied above does not extend to disjunctions with more than one atomic disjunct.
Instead, the competitors needed to derive exclusiveness are the following.

- φ and ψ
- φ and χ
- ψ and χ

To arrive at these while sticking with the standard analysis of *or* Sauerland proposed an algorithm that asserts the existence of two otherwise silent binary connectives in the same scale as *and* and *or*, called $\mathbb{L}$ and $\mathbb{R}$. These extract from a disjunction its

[7] This set of examples is also elaborated in Alonso-Ovalle (2008)

[8] We will not discuss how to derive the epistemic step. That is, how to get from the fact that the speaker did not know that "φ and ψ" holds to the implicature that she knows that "φ and ψ " does not hold. A nice paper discussing this issue is for example Westera (2013).

[9] Keeping in mind that the interpretation of a statement is executed increasingly, from left to right.

left and right disjunct, thus making those once more available for further analysis. Fox then made use of this method to derive exclusiveness for disjunctions with more than two atomic disjuncts. Suppose we have a disjunction "φ or ψ or χ". Then using Sauerland's algorithm the set of competitors is no longer made up only of the three insufficient alternatives mentioned above, but of all combinations of applying *and*, $\mathbb{L}$ and $\mathbb{R}$ to the disjuncts instead of *or*. This set also contains the competitors needed to derive exclusiveness. Fox then proceeds using a method that is similar to the one we saw for deriving exclusiveness in two disjuncts: add as many negations of these competitors to the meaning of the disjunction as is consistently possible. This does give the right analysis. However, asserting the existence of two otherwise silent connectives to achieve this is a rather ad hoc procedure which could be improved. This is exactly what Alonso-Ovalle does using Alternative Semantics for *or*.
With Alternative Semantics for *or* the disjuncts are accessible without having to revert to Sauerland's algorithm. Consequently the set of all scalar competitors can be constructed by taking the atomic disjuncts and all possible conjunctions of them. Alonso-Ovalle calls this the 'set of conjunctive competitors'. Similar to Fox's procedure all those alternatives in this set which can be 'innocently' excluded from the disjunction, i.e. without leading to inconsistency, are in fact excluded from the interpretation. This procedure yields exclusiveness without making use of Sauerland's supposed silent connectives.
Furthermore, what also drops out of using alternative semantics for *or* in unembedded disjunctions is the right analysis of sentences like "φ or ψ or both", in contrast to "φ or ψ". These have the same truth conditions, thus Montague Grammar fails to differentiate between them. Alternative Semantics however correctly predicts a divergence, as all two or three disjuncts are available as alternatives, constituting two different sets.

The third type of sentences presented by Alonso-Ovalle in justifying Alternative Semantics for *or* are disjunctions under the scope of deontic modals, may and must.[10] In both cases an analysis which aims at correctly representing the natural understanding has to yield that all disjuncts need to be permitted in order for the whole expression to come out true. Alonso-Ovalle calls this the 'distribution requirement', in reference to Kratzer and Shimoyama (2002). In the case of *must* additionally making neither of the disjuncts true is not permitted. We will focus on the modal *may*, but mention in what ways *must* differs and how the analysis can be adjusted to this operator. To illustrate the problem 'standard semantics' has Alonso-Ovalle gives two concrete examples, one for each of the modal operators.[11] Both are set in a family with Mom and the daughter Sandy. The first is in discussion of dessert options, when Mom says "You may have either cake or ice cream." This statement implies that Sandy may have cake and that she may have ice-cream, but not both. The modal operator is distributed over both disjuncts.
The second example is in discussion of home chores, where Mom says "You must either clean your bedroom or mow the lawn." Once more this statement implies that

[10] Next to Alonso-Ovalles PhD thesis his paper Alonso-Ovalle (2006) also explains these examples.
[11] I will use slight modifications which make the examples shorter.

both options are permitted, that Sandy may clean her bedroom and that she may mow the lawn.
Now, as we have seen before, 'standard semantics' for *or* creates one unstructured set containing the denotations of both disjuncts, such that we cannot differentiate between the two any longer. Furthermore, the interpretation of deontic modals is standardly the following:

If $[\![\varphi]\!] \in \mathbf{D}_{\langle s,t\rangle}$ and $\mathcal{D}_w$ is the set of worlds deontically accessible from w, then

$$[\![\text{may } \varphi]\!](w) = 1 \ \Leftrightarrow \ \exists w' \in \mathcal{D}_w : \ [\![\varphi]\!](w) = 1$$

If $[\![\varphi]\!] \in \mathbf{D}_{\langle s,t\rangle}$ and $\mathcal{D}_w$ is the set of worlds deontically accessible from w, then

$$[\![\text{must } \varphi]\!](w) = 1 \ \Leftrightarrow \ \forall w' \in \mathcal{D}_w : \ [\![\varphi]\!](w) = 1$$

Now consider an example where φ = "φ_1 or φ_2" and the same disjunct, φ_1, holds in every world accessible from w, while the other, φ_2, is prohibited. In this case the standard analysis of *or* falsely predicts that $[\![\text{may } \varphi]\!](w) = [\![\text{must } \varphi]\!](w) = 1$ whereas the distribution requirement is not met. It would therefore be possible for the two above statements to hold if for example Sandy is not allowed to have ice cream and not allowed to mow the lawn, as long as she can have cake and clean her bedroom. Once again, an alternative semantics makes atomic disjuncts accessible, but this opportunity has to be joined with a correct analysis of deontic modals and derivation of the distribution requirement to yield an appropriate exhaustive analysis.
Alonso-Ovalle first presents one approach, 'Analysis 1', of interpreting deontic modals which enables one to derive the distribution requirement truth-conditionally, but claims that it is insufficient. Therefore he continues to present a different approach, 'Analysis 2', which derives the distribution requirement pragmatically, as a quantity-implicature. It is worthwile to note that Alonso-Ovalle's perception is not the only possible option. For example, Maria Aloni did pursue a semantic account using an explicit exhaustification operator for the analysis of free choice indefinites in a paper published after Alonso-Ovalle's (Aloni, 2007).[12] However, the main point we need to see here is that Alternative Semantics for *or* enables us to make the right predictions. Therefore it suffices to present one possible derivation of the distribution requirement using one possible analysis of deontic modals. Thus, we will present Alonso-Ovalle's 'Analysis 2', using a pragmatic reasoning.[13]
Alonso-Ovalle refers to Kratzer and Shimoyama (Kratzer and Shimoyama, 2002) as an inspiration for his 'Analysis 2'. They analyse the German prefix *irgend* in combination with indeterminate pronouns, for example *ein* (any). Like *or*, an indeterminate pronoun creates a set of alternatives. The way this is concretely implemented will be covered in the next section 2.2. In combination with a deontic modal the distribution requirement appears equivalently for indeterminate pronouns as it does for disjunctions. That is, any of the alternatives is a permitted option. Consider the example of Mom telling Sandy "You may marry anybody." Then clearly, for every

[12] As we have seen, free choice indefinites resemble *or* and can likewise be interpreted using a Hamblin Style Semantics.

[13] This is not intended to imply that the author has a higher affiliation with this account than with any of the others mentioned.

human h Sandy is allowed to marry him/her. With *must* the case for indeterminate pronouns is slightly different from conjunction, as every alternative an indeterminate pronoun creates must be required. Consider the example "You must help anybody." Then really for every human h, Sandy must help them. But also, for every human h Sandy is allowed to help them, which was what the distribution requirement states. Now that we have seen that the distribution requirement also occurs for indeterminate pronouns let us proceed by looking at the analysis Kratzer and Shimoyama present for these, together with the prefix *irgend* and how Alonso-Ovalle applies it to *or*.

The prefix *irgend* induces a maximal widening of the set of alternatives over which the indeterminate pronoun ranges. Similarly, *or* widens the domain of alternatives by adding disjuncts. This weakens the total statement, as more possibilities are available. Kratzer and Shimoyama then propose an analysis which derives the distribution requirement as a quantity implicature. They state that in order for this domain widening to be justified any stronger statement must have been false. Equivalently, Alonso-Ovalle argues that stating a disjunction inside the scope of a deontic modal implies that any stronger statement, i.e. less disjuncts being permitted, must be false.

Semantically this can be seen from the following elaboration. The modals receive a standard interpretation:

$$[\![\text{may}(A)]\!] = \{\lambda w.\exists w' : [\![\exists A]\!](w')\} = \{\lambda w.\exists w' \in \mathcal{D}_w.\exists p \in [\![A]\!]\&\ p(w') = 1\}$$

$$[\![\text{must}(A)]\!] = \{\lambda w.\forall w' : [\![\exists A]\!](w')\} = \{\lambda w.\forall w' \in \mathcal{D}_w.\exists p \in [\![A]\!]\&\ p(w') = 1\}$$

where $\mathcal{D}_w$ is once more the set of worlds deontically accessible from w.

That is, the set of alternatives A is 'caught' by an existential closure operator before the interpretation of the modals works on it. The modal operators do not have access to A. Hence the effect of using a disjunction, i.e. enlarging the set of alternatives, is solely to change the domain over which the existential closure operator quantifies. Now making this domain bigger by adding disjuncts weakens the overall statement. If Sandy may have ice cream then she may also have ice cream or cake, but not the other way around. Note that we focus only on the strict semantic meaning of the sentence and disregard the distribution requirement, which we still need to derive pragmatically. The fact that domain widening weakens the overall statement holds equivalently for *must*.

At a next step the pragmatic reasoning described above can take place. That is, as adding disjuncts weakens the statement, it follows as a quantity implicature that if the speaker did only state the weaker assertion, then the stronger version cannot hold. To come back to the examples, if Sandy was not allowed to have ice cream, then stating "You may have cake or ice cream" instead of "You may have cake" is not in accordance with Grice's quantity maxim, as it would not give all the information known to the speaker. Clearly a similar reasoning holds for *must*.

To conclude, the problem of accurately deriving the distribution requirement for disjunctions which appear under the scope of a deontic modal can likewise be solved using alternative semantics for "or".

2.2 Implementation

Now that we have seen why Alonso-Ovalle employs Alternative Semantics let us have a look at how it is implemented. Alternative Semantics was first developed by Hamblin (Hamblin, 1973), in an attempt to represent questions. His idea was taken up by Kratzer and Shimoyama who give a concise summary of the implementation in Kratzer and Shimoyama (2002). The following section is therefore taken from Alonso-Ovalle (2008) as well as from Kratzer and Shimoyama (2002).
First, let us recall some notions of Montague Grammar. Every expression is of a certain type σ, with the basic types s, e and t and functional types $\langle\sigma, \tau\rangle$. Expressions of type σ are mapped to elements of the corresponding domain $\mathbf{D}_\sigma$.[14] Entities are mapped to elements of D, sentences to sets of possible worlds and all functional types to corresponding functions.
In Hamblin Semantics however, expressions are mapped to sets containing elements of their standard domain of interpretation. Most lexical items, like the following examples, are simply mapped to the singleton containing their standard denotation.

1. (a) $[\![\text{ Mary }]\!] = \{\mathbf{m}\}$
   (b) $[\![\text{ John }]\!] = \{\mathbf{j}\}$
2. (a) $[\![\text{ sleep }]\!] = \{\lambda x.\lambda w.\mathbf{sleep}_w(x)\}$
   (b) $[\![\text{ love }]\!] = \{\lambda y.\lambda x.\lambda w.\mathbf{love}_w(x, y)\}$

The main difference lies in the interpretation of indeterminate expressions. These are mapped to the set containing all their possible references. For example, consider the pronoun "who" in the question "Who sleeps?".[15] It is unclear which person is the reference of this pronoun, it is indefinite. Every human could be the person in question, thus Hamblin semantics maps this pronoun to the set of all humans $[\![who]\!] = \{x \mid \mathbf{human}_w(x)\}$. Next we need to know how functional application is implemented. Hamblin's solution is to proceed pointwise:

If $[\![\alpha]\!] \in \mathbf{D}_{\langle\sigma,\tau\rangle}$ and $[\![\beta]\!] \in \mathbf{D}_\sigma$, then $[\![\alpha(\beta)]\!] = \{c \in \mathbf{D}_\tau \mid \exists a \in [\![\alpha]\!] \exists b \in [\![\beta]\!](c = a(b))\}$

To put it in the pictural way Kratzer and Shimoyama describe this: "sets of alternatives [...] keep 'expanding' until they meet an operator that selects them."(Kratzer and Shimoyama, 2002; **?**) That is, continuing the example of the question "Who sleeps?", where "sleep" is interpreted as the function stated above, this function is applied to every element of the set of alternatives created by "who".

$$[\![\text{ Who sleeps }]\!] = \{p \mid \exists x(\mathbf{human}_w(x) \text{ and } p = \lambda w.\mathbf{sleep}_w(x))\}$$

This example also shows the analysis of questions Hamblin achieved with this approach: a question is interpreted as the set of all its possible answers. In this case that is the set containing the statement "x sleeps" for every human x.

[14]Assume the domain of interpretation for individual expressions, D, and the set of possible worlds W to be fixed. Reference to these sets will be omitted.

[15]This example and its analysis is taken from Kratzer and Shimoyama (2002)

It is also important to note that the alternatives can be of different types, for example individuals like in the example above, but also propositions. Thus there will have to be various existential closure operators that quantify over the elements of sets containing different types of alternatives. For example, considering propositions:

$$\text{If } [\![A]\!] \subseteq \mathbf{D}_{\langle s,t\rangle}, \text{ then } [\![\exists\ A]\!] = \{\lambda w.\exists p(p \in [\![A]\!] \text{ and } p(w))\}$$

Note that $[\![A]\!]$ is not an element of $\mathbf{D}_{\langle s,t\rangle}$ but a subset, reflecting that propositions are sets of sets of possible worlds. We can see immediately that after an existential closure operator has been applied, the alternatives in A are no longer individually available.

Considering that we are very much concerned with *or* we clearly still need to specify its interpretation in Alternative Semantics. *Or* can be applied to expressions of different types, but the two expressions it links need to be of the same type. Thus Alonso-Ovalle gives the following rule for its interpretation:

$$\text{If } [\![B]\!], [\![C]\!] \subseteq D_\tau, \text{ then } [\![B \text{ } or \text{ } C]\!] = [\![B]\!] \cup [\![C]\!] \subseteq D_\tau$$

Now this might at first sight seem confusing. Did we not take all this effort to make sure that *or* is no longer set union? The answer lies in the interpretation of the expressions B and C. They are not elements of $\mathbf{D}_\tau$ but subsets of it. Therefore the set union keeps the interpretation of both B and C. Let us illustrate this with another example.

Take the two entities John and Mary, where $[\![$ John $]\!] = \{\mathbf{j}\} \subseteq \mathbf{D}_e$ and $[\![$ Mary $]\!] = \{\mathbf{m}\} \subseteq \mathbf{D}_e$, as described above. Then the expression "John or Mary" is interpreted as follows:

$$[\![\text{ John or Mary }]\!] = [\![\text{ John }]\!] \cup [\![\text{ Mary }]\!] = \{\mathbf{j}\} \cup \{\mathbf{m}\} = \{\mathbf{j}, \mathbf{m}\}$$

Now let us use functional application, effected pointwise as defined above, on this set of alternatives. Consider for example the interpretation of 'sleep', which, applied to $[\![$John or Mary $]\!]$, gives the interpreation of the statement "John or Mary sleep." Recall that $[\![$ sleep $]\!] = \{\lambda x.\lambda w.\mathbf{sleep}_w(x)\} \subseteq \mathbf{D}_{\langle e\langle s,t\rangle\rangle}$. Thus the functional application yields the following interpretation:

$$\begin{aligned}[\![\text{sleep(John or Mary)}]\!] &= \{c \in \mathbf{D}_{\langle s,t\rangle} \mid \exists a \in [\![\text{sleep}]\!] \exists b \in [\![\text{John or Mary}]\!](c = a(b))\}\\ &= \{c \in \mathbf{D}_{\langle s,t\rangle} \mid \exists b \in \{\mathbf{j}, \mathbf{m}\}(c = (\lambda x.\lambda w.\mathbf{sleep}_w(x))(b))\}\\ &= \{c \in \mathbf{D}_{\langle s,t\rangle} \mid \exists b \in \{\mathbf{j}, \mathbf{m}\}(c = \lambda w.\mathbf{sleep}_w(b))\}\\ &= \{\lambda w.\mathbf{sleep}_w(\mathbf{j}),\ \lambda w.\mathbf{sleep}_w(\mathbf{m})\} \subseteq \mathbf{D}_{\langle s,t\rangle}\end{aligned}$$

That is, the sentence "John or Mary sleep." is interpreted as the set containing the alternative that John sleeps and the alternative that Mary sleeps.

The interpretation of *and* however cannot follow this same pattern. Consider for example the sentence "John sleeps and Mary sleeps"[16] Now if *and* were interpreted

[16]Recall that we omit the treatment of expressions like "John and Mary" because these induce plurality.

as intersection then this statement would be interpreted as inconsistent (unless John sleeps if and only if Mary sleeps):

$$\{\lambda w.\mathbf{sleep}_w(\mathbf{j})\} \cap \{\lambda w.\mathbf{sleep}_w(\mathbf{m})\} = \emptyset$$

Each of the two sets above contains exactly one element, and unless this element is the same their intersection will be empty. Thus *and* is instead interpreted pointwise:

$$\text{If } [\![B]\!], [\![C]\!] \subseteq D_{\langle s,t\rangle}, \text{ then } [\![B \textit{ and } C]\!] = \{\lambda w.\exists b \in [\![B]\!] \exists c \in [\![C]\!](b(w) \text{ and } c(w))\}$$

Recall that we can generalise a definition of the interpretation of quantifiers only on whole sentences to other expressions, as explained in Appendix B. Now applying this definition to the sentence "John sleeps and Mary sleeps." gives us:

$$\{\lambda w.\exists b \in \{\lambda w'.\mathbf{sleep}_{w'}(\mathbf{j})\}\exists c \in \{\lambda w'.\mathbf{sleep}_{w'}(\mathbf{m})\}(b(w) \text{ and } c(w)\}$$

$$= \{\lambda w.(\mathbf{sleep}_w(\mathbf{j}) \text{ and } \mathbf{sleep}_w(\mathbf{m}))\}$$

That is, this definition yields exactly the interpretation that we want for the sentence "John sleeps and Mary sleeps.", namely the set containing the set of worlds in which both of them sleep. Thus overall with this implementation of alternative semantics and the interpreations for counterfactuals and modals explained above, Alonso-Ovalle is able to account for all the examples he mentions and for which standard semantics fails.

2.3 Critical Discussion

We have seen examples of sentences for which 'standard semantics', Montague Grammar, makes false predictions. We have also seen how the use of Alternative Semantics enables a correct interpretation of these examples. It is thus clear that empirically Alternative Semantics has a great advantage over Montague Grammar.
However, in the above section it became apparent that in Alternative Semantics the two connectives *and* and *or* are interpreted as two functions which seem to have nothing in common. *Or* works on the whole sets of propositions, whereas *and* acts on the elements of the propositions in the two sentences it joins.[17] This contradicts the intuition one has when speaking a language, which tells us that *and* and *or* are somehow similar. In a theory which aims at coming close to representing the actual processes taking place in the human mind when analysing natural language one would therefore expect the connectives to receive a uniform treatment. This is what happens in Inquisitive Semantics, which we will have a look at in the following section.

3 Inquisitive Semantics

We have seen in section 2.2 above how Alternative Semantics is implemented. It is based on Hamblin semantics, which was developed to analyse questions in natural

[17]We defined *or* generally on all types, but *and* only on whole sentences. Therefore in this comparison I only refer to the behaviour on whole sentences.

language and puts special emphasis on expressions which introduce several alternatives to be chosen from, such as indefinites. These alternatives can be seen as possible answers, namely to the question which one of them is the *right* choice, that is, makes the surrounding sentence come out as true. This shows the connection from indefinites to questions. Consequently the connective *or*, equivalently introducing inquisitiveness into an expression, receives an interpretation which differs largely from that of the other connectives, such as *and*. A framework which circumvents this distinguished treatment of *or* while keeping many of the empirical advantages Alternative Semantics has over Montague Grammar is Inquisitive Semantics. In Inquisitive Semantics every expression is analysed with respect to both its informative and its inquisitive content, and the connectives receive a uniform procedure of interpretation. We will start out by looking at the new definition of meaning in Inquisitive Semantics for which we will need the notion of *discourse context*. Afterwards we will look at the entailment order we can derive from it and finally at a complete definition of the semantics for a propositional logic.[18]

3.1 Meaning

Inquisitive Semantics takes a dynamic perspective on the meaning of sentences. Expressions are not interpreted in isolation but in view of the discourse in which they are embedded. Uttering a sentence has an effect on the discourse context and consequently the meaning of a sentence is its *context change potential*, that is, a function from discourse contexts to discourse contexts. Clearly the next question is what precisely we take a discourse context to be. We will first of all look at how to embed the classical view of propositions as sets of possible worlds into this dynamic perspective. Thus we start out with the definition of discourse context appropriate for this enterprise:

Definition 1 *(Discourse context - classical view)*
A ***discourse context*** *c is a set of possible worlds.*

In the beginning of the discourse the context is trivially the set of all possible worlds W; consequently the participants try to locate the actual world more precisely inside this set by exchanging utterances. Every sentence denotes a set of possible worlds, thus the update achieved by its utterance is to delete all those worlds from the context which do not belong to its denotation. Consider as an example the statement "John or Mary are sleeping.". Uttering it in a conversation has the effect that all worlds in which both John and Mary are awake are deleted from the context. Thus the discourse context can be viewed as the information established so far. It coincides with an information state, as an information state is simply also a collection of information given in the form of a set of possible worlds.

Definition 2 *(Information states)*
An ***information state*** *s is a set of possible worlds, i.e. $s \subseteq W$*

[18] The following exposition follows Ciardelli et al. (2013).

The discourse context is that set of worlds which are still an active option for being the actual one. Utterances add more information to the discourse context.
Contrastingly, in Inquisitive Semantics the discourse context contains not only the information that was established, an information state, but also the issues which have been raised. This refinement is motivated by the clearly interactive nature of dialogue. Discourse participants do not solely act alternately by tellin each other information, they interact in asking and resolving issues.
In order to be able to define a discourse context made up of an information state and a set of issues we first need to clarify how to model this second notion.
For a given context c we will let info_c denote its information state and issues_c its set of issues. Thus $c = \langle \mathsf{info}_c, \mathsf{issues}_c \rangle$.
Suppose $s' \subseteq s$ for certain information states s, s'. Then clearly s' locates the actual world more precisely than s, thus s' is called an *enhancement* of s. Now a set of enhancements $\mathcal{I}$ of an information state s can be seen as a request for information: to locate the actual world more precisely in one of the enhancements. But requests for information are exactly what issues are intended to model. Thus we can see a non-empty set of enhancements $\mathcal{I}$ of the information given in a certain context c as an *issue* in it. However, we need a few restrictions: First, if $t \in \mathcal{I}$ and t' is a further enhancement of t, then t' also satisfies the request for information given by $\mathcal{I}$. Thus an issue has to be a downward closed set of enhancements.[19]
We can see that this notion of an issue is similar to the one proposed for questions in Alternative Semantics. Namely, a question in Alternative Semantics is modeled as the set containing its possible answers. But as propositions are sets of possible worlds, these answers are also simply sets of possible worlds. The same general idea is visible in Inquisitive Semantics: issues are modeled as sets of information states, which are once more simply sets of possible worlds. However, the above-mentioned restriction shows an important distinction between the two approaches. Namely, in Inquisitive Semantics an issue is not modeled as the set containing only all possible answers to it but instead as the set containing all information states which *resolve* the issue. That is, those states which give enough information to locate the actual world inside only one of the answers.
In addition to the above restriction of an issue being downward closed we need to give a second one: The set of issues in a given context must contain all the worlds in its information state info_c. If any of the possible worlds is not a possible answer to the issues in question, then clearly this possible world must already be excluded from the set of live options. Therefore it cannot be part of info_c anymore.[20] Thus, as a second restriction, issues_c should form a cover of info_c.

Definition 3 *(Issues)*
Let s be an information state and $\mathcal{I}$ a nonempty set of enhancements of s. Then we

[19] Where a set of enhancements I of an information state s is downward closed iff. $t \in I$ and $t' \subseteq t$ implies that $t' \in I$.

[20] For example, suppose the current issue is "Were the curtains green or blue?" Then it is already clear that they were not red. The red-curtain worlds are not part of info_c. Only the green-curtain and blue-curtain worlds are live options, and the question is in which of those sets the actual world lies.

say that $\mathcal{I}$ is an issue over s if and only if:

- *$\mathcal{I}$ is downward closed: if $t \in \mathcal{I}$ and $t' \subseteq t$ then also $t' \in \mathcal{I}$*
- *$\mathcal{I}$ forms a cover of s: $\bigcup \mathcal{I} = s$*

As we discussed before, the elements of an issue are exactly those information states which contain enough information to settle it.

Definition 4 *(Settling an Issue)*
Let s be an information state, t an enhancement of s and $\mathcal{I}$ and issue over s. Then we say that t settles $\mathcal{I}$ if and only if $t \in \mathcal{I}$.

Now that we know how to model information states and issues a first idea for modeling a discourse context could be to take $c = \langle \mathsf{info}_c, \mathsf{issues}_c \rangle$, where issues_c is a set of issues over an information state info_c. However, any set of issues $\mathcal{I}$ over an information state s can be identified with just one single issue: the one which asks all the questions at once. Formally, $\mathcal{I}$ can be identified with the single issue $I_{\mathcal{I}} = \{t \subseteq s \mid t \in I \text{ for each } I \in \mathcal{I}\}$ which is settled iff. every issue in $\mathcal{I}$ is settled.[21] Thus we do not need a set of issues to define a discourse context but only a single issue. Thus a context could be modeled as $c = \langle \mathsf{info}_c, \mathsf{issue}_c \rangle$, where issue_c is a single issue over an information state info_c. Yet by definition an issue over an information state forms a cover of this state. Thus we can recover the information state if we have only the issue given, by $\mathsf{info}_c = \bigcup \mathsf{issue}_c$. Overall this means we can identify a discourse context with one issue.

Definition 5 *(Discourse context in Inquisitive Semantics)*

- *A **discourse context** c is a non-empty, downward closed set of states.*
- *The set of all discourse contexts will be denoted by $\mathcal{C}$.*

Where a state is a set of possible wolds, i.e. a proposition. The following equivalence states exactly what was discussed above.

Definition 6 *(Information in a discourse context)*
For any discourse context c:

$$\mathsf{info}_c := \bigcup c$$

In Alternative Semantics a discourse context was viewed as a proposition, here it is defined as an issue. We have discussed orderings on propositions, information states. A state is more informed if it contains less worlds and uttering an informative statement enhances the context by deleting those worlds which are incompatible with it. Similarly, we can define an order on the discourse contexts in Inquisitive Semantics, which compares how *informed* the contexts are.

[21] It is easily seen that this set is once more an issue, i.e. fulfills the abovementioned restrictions.

Definition 7 *(Informative order on discourse contexts)*
Let $c, c' \in \mathcal{C}$. Then:

$$c \geq_{\mathsf{info}} c' \quad \textit{iff.} \quad \mathsf{info}_c \subseteq \mathsf{info}_{c'}$$

In addition to this informative order on contexts we can also ask when a context is more *inquisitive* than another. As we discussed, how much can be asked in a context depends largely on what is already known. Thus the question of which context is more inquisitive can only reasonably be asked if both contexts are equally informative. In that case, a context c asks for more information than another context c' if every state which settles c also settles c'.

Definition 8 *(Inquisitive order on discourse contexts)*
Let $c, c' \in \mathcal{C}$ and $\mathsf{info}_c = \mathsf{info}_{c'}$. Then

$$c \geq_{\mathsf{inq}} c' \quad \textit{iff.} \quad c \subseteq c'$$

Nevertheless, one would like to be able to compare informativeness and inquisitiveness at the same time in contexts which are not equally informative. To do this we define the notion of *extension.* A context c is an extension of another context c' if c is at least as informed as c' and c is at least as inquisitive as $c' \upharpoonright_{\mathsf{info}_c}$. That is, c is at least as inquisitive as the context obtained by restricting c' to the information available in c. These two conditions coincide simply with $c \subseteq c'$.

Definition 9 *(Extension of a discourse context)*
Let $c, c' \in \mathcal{C}$. Then

$$c \textit{ is an extension of } c', \textit{ written } c \geq c' \quad \textit{iff.} \quad c \subseteq c'$$

This relation forms a partial order on $\mathcal{C}$. The absurd discourse context $\{\emptyset\} =: c_\perp$ is an extension of every other discourse context and every context is an extension of the initial context $\wp(W) =: c_\top$.
As a final remark on contexts in general, note that the intersection of every two discourse contexts is once more a discourse context. We will call it the *merge* of its two constituents.

Definition 10 *(Merge of two discourse contexts)*
For any $c, c' \in \mathcal{C}$, $c \cap c' \in \mathcal{C}$ is called the ***merge*** *of c and c'.*

Now that we have defined what a discourse context is and discussed a few of the properties this definition entails we can continue to look at what a sentence means against this background. We said in the beginning that a sentence's meaning is its *context change potential*, that is, the way in which it changes the discourse context when uttered in it. Thus a meaning will be a function $f : \mathcal{C} \to \mathcal{C}$ such that $f(c) \geq c$, where $\geq$ is the extension relation we just defined. Uttering a sentence cannot reduce information or inquisitiveness. Additionally f should be compatible with extensions of contexts. That is, for two contexts $c \geq c'$ it should hold that $f(c) = f(c') \cap c$. This is called the *compatibility condition.*

Definition 11 *(Compatibility condition)*
A function $f: \mathcal{C} \to \mathcal{C}$ satisfies the compatibility condition if and only if for every $c, c' \in \mathcal{C}$ such that $c \geq c'$ we have that $f(c) = f(c') \cap c$.

Thus a first definition for meanings in general can be given as follows:

Definition 12 *(Meanings - first definition)*
A ***meaning*** *is a function $f: \mathcal{C} \to \mathcal{C}$ which maps every discourse context c to a new discourse context $f(c) \geq c$, in compliance with the compatibility condition.*

However, considering that $c \geq c_\top$ for every discourse context $c \in \mathcal{C}$ the compatibility condition yields that every such function f is uniquely defined by the way it works on $c_\top$. Namely, for every $c \in \mathcal{C}$ and for every meaning function $f: \mathcal{C} \to \mathcal{C}$:

$$f(c) = f(c_\top) \cap c$$

Thus $f(c_\top$, which is itself a discourse context, suffices to recover all of f. Therefore the meaning of a sentence can simply be defined as a discourse context.

Definition 13 *(Meanings - simplified)*

- *The meaning of a sentence is a non-empty, downward closed set of states.*
- *The set of all sentence-meanings is denoted by* Π.

Using this analogy between discourse contexts and sentence meaning we can naturally extend the notions defined so far in this section for contexts to sentence meanings. Accordingly informative content, inquisitive content, settling, tautology and contradiction are defined in the obvious way on sentence meanings.
Moreover, the extension relation on contexts can be viewed as an entailment order on sentences.

Definition 14 *(Informative order on sentence meanings)*
Let $A, B \in \Pi$. Then:

$$A \models_{\mathsf{info}} B \quad \textit{iff.} \quad \mathsf{info}(A) \subseteq \mathsf{info}(B)$$

Definition 15 *(Inquisitive order on sentence meanings)*
Let $A, B \in \Pi$ such that $\mathsf{info}(A) = \mathsf{info}(B)$. Then:

$$A \models_{\mathsf{inq}} B \quad \textit{iff.} \quad A \subseteq B$$

Definition 16 *(Entailment)*
Let $A, B \in \Pi$. Then:

$$A \models B \quad \textit{iff.} \quad A \subseteq B$$

Once more this entailment relation $\models$ forms a partial order on the set of all sentence meanings Π, and the tautology $\wp(W) =: A_\top$ and contradiction $\emptyset =: A_\bot$ form the extrema of this ordering.
This exposition should suffice for now as an introduction to the conception of discourse contexts and sentence meanings in Inquisitive Semantics, so that we can now continue by defining the actual Semantics.[22]

[22] The following section is based on Ciardelli et al. (2012).

3.2 Semantics

The definition of the semantic interpretation of the connectives in Inquisitive Semantics is based on the algebraic structure induced on the set of all sentence-meanings Π by the entailment relation $\models$. To understand it better we will first of all have a look at how the interpretation of connectives in standard semantics relates to the entailment relation on propositions there. In standard semantics the meaning of a sentence is a proposition, a set of worlds. One proposition A entails another B if it is more informative, that is, if $A \subseteq B$. This entailment induces a partial order on the set of all propositions $\wp(W)$.

The pair $\langle \wp(W), \subseteq \rangle$ is not only ordered, it has an even richer structure. First, there is a minimal and a maximal element in the order, the contradiction $\emptyset$ and the trivial proposition W. Furthermore every set of propositions Σ has a *meet*, a greatest lower bound, as well as a *join*, a least upper bound. The meet of a set Σ is $\bigcap \Sigma$, its join is $\bigcup \Sigma$. Thus $\langle \wp(W), \subseteq \rangle$ is a *complete, bounded lattice*. Additionally for every two propositions A and B there is a *pseudo-complement of A relative to B*. That is, a unique weakest proposition C such that $A \cap C \models B$. In this possible worlds semantics C amounts to $\overline{A} \cup B$. The pseudo-complement of A relative to the bottom element $\emptyset$ is reffered to as A^*. With the existence of relative pseudo-complements $\langle \wp(W), \subseteq \rangle$ is a *Heyting-Algebra*.

Now the interpretation of the connectives in a propositional logic can be given in relation to the connectives we just saw.

Definition 17 *(Algebraic Semantics - the standard case)*

- $[\![\neg\varphi]\!] = [\![\varphi]\!]^*$
- $[\![\varphi \wedge \psi]\!] = [\![\varphi]\!] \cap [\![\psi]\!]$
- $[\![\varphi \vee \psi]\!] = [\![\varphi]\!] \cup [\![\psi]\!]$
- $[\![\varphi \rightarrow \psi]\!] = [\![\varphi]\!] \Rightarrow [\![\psi]\!]$

Where $\cup$ denotes the join, $\cap$ the meet and $\Rightarrow$ relative pseudo-complementation. As before, $[\![\varphi]\!]$ is the interpretation of the sentence φ, thus a set of possible worlds.

This same scheme can be applied to the inquisitive take on sentence meaning presented above. However, we first need to show that we can indeed find the meet, join and pseudo-complements for any elements of the domain of interpretation of sentences ordered by the new entailment relation, i.e. $\langle \Pi, \subseteq \rangle$. We already saw that we once again have a bottom element, $A_\perp = \{\emptyset\}$, which entails every other sentence-meaning, and a top element $A_\top = \wp(W)$, which is entailed by every other element. Furthermore meet and join are also easily seen to be available.

Fact 1 *(Meet, Join)*

- *For any set $\mathcal{F} \subseteq \Pi$, $\bigcap \mathcal{F} \in \Pi$ and it is the meet of $\mathcal{F}$.*
- *For any set $\mathcal{F} \subseteq \Pi$, $\bigcup \mathcal{F} \in \Pi$ and it is the join of $\mathcal{F}$.*

Where $\bigcap \emptyset = \wp(W)$ and $\bigcup \emptyset = \{\emptyset\}$. Thus $\langle \Pi, \subseteq \rangle$ is also a complete, bounded lattice. Next we need to show the existence of relative pseudo-complements. Recall that the pseudo-complement of A relative to B is the weakest sentence-meaning C such that, when we add C to A - i.e. intersect them - we get a meaning which is at least as strong as B - i.e. is a subset of B. We wil define the notation for this, from which it can easily be seen that the given definition fulfils the requirements.

Definition 18 *($\Rightarrow$)*
Let $A, B \in \Pi$. Then:

$$A \Rightarrow B := \{s \mid \textit{ for every } t \subseteq s, \textit{ if } t \in A, \textit{ then } t \in B\}$$

By definition, if $s \in A \Rightarrow B$ and $s \in A$, then $s \in B$, thus $A \cap A \Rightarrow B \models B$. Thus $C := A \Rightarrow B$ is a meaning such that, when we add it to A the emerging meaning entails B. Now we have to show that it is the weakest meaning with this property. Let $C' \in \Pi$ with $C' \supset C$. Then there must be a state $s \in C'$ such that $s \notin C$, i.e. there is a $t \subseteq s$ with $t \in A$ but $t \notin B$. But $C' \in \Pi$, thus especially C' is downward closed and $t \in C'$. Therefore $t \in A \cap C'$ but $t \notin B$, and $A \cap C' \not\models B$.

Fact 2 *(Relative pseudo-complement)*
For any $A, B \in \Pi$, $A \Rightarrow B$ is the pseudo-complement of A relative to B.

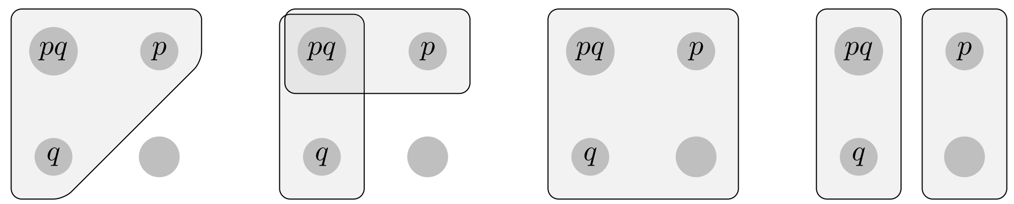
Just as in the classical case, the set of all sentence meanings ordered by entailment $\langle \Pi, \subseteq \rangle$ forms a Heyting algebra. Therefore we can employ the same relation from the operations in the Heyting algebra to the interpretation of the connectives which we saw before. We assume a language $\mathcal{L}_{\mathcal{P}}$ of propositional logic, containing an infinite set of propositional variables $\mathcal{P}$ and the logical constants $\neg$, $\wedge$, $\vee$, $\rightarrow$.[23]

Definition 19 *(Inquisitive Semantics for a propositional logic)*

- $s \in [\![p]\!]_{\mathsf{inq}}$ *iff.* $\forall w \in s: \; w \in [\![p]\!]$
- $[\![\bot]\!]_{\mathsf{inq}} := \{\emptyset\}$
- $[\![\neg\varphi]\!]_{\mathsf{inq}} := [\![\varphi]\!]^*_{\mathsf{inq}}$
- $[\![\varphi \wedge \psi]\!]_{\mathsf{inq}} := [\![\varphi]\!]_{\mathsf{inq}} \cap [\![\psi]\!]_{\mathsf{inq}}$
- $[\![\varphi \vee \psi]\!]_{\mathsf{inq}} := [\![\varphi]\!]_{\mathsf{inq}} \cup [\![\psi]\!]_{\mathsf{inq}}$
- $[\![\varphi \rightarrow \psi]\!]_{\mathsf{inq}} := [\![\varphi]\!]_{\mathsf{inq}} \Rightarrow [\![\psi]\!]_{\mathsf{inq}}$

[23]This differs from the perspective taken in Ciardelli et al. (2012), where a first-order language in predicate logic is analysed. I chose to follow Ciardelli and Roelofsen (2011) here, as the use of a propositional language makes the connection to Montague Grammar and Alternative Semantics in the way discussed above more easily visible.

Where $[\![p]\!]$ is the standard interpretation of p, i.e. a set of possible worlds. We can see from this definition that every interpretation of a propositional variable $[\![p]\!]_{\mathsf{inq}}$ is given as a sentence meaning, a non-empty downward closed set of states.[24] $[\![\varphi]\!]_{\mathsf{inq}}$ is called the *meaning* in Inquisitive Semantics of φ,[25] and the elements of $[\![\varphi]\!]$ are called the *possibilities* for φ.
Let us illustrate this with a few examples. Suppose we have four possible worlds which can be distinguished by two propositional variables p, which is the statement “John is sleeping”, and q, “Mary is sleeping”. In figure 1 the interpretations of several disjunctive clauses constructed from these basic expressions are depicted. For better readability only the maximal elements are drawn in the Inquisitive Semantics cases. Therefore the picture would look the same for Alternative Semantics. However, in Alternative Semantics that means downward closure is actually not present, whereas in Inquisitive Semantics we solely do not depict every possibility to avoid overloaded diagrams.



(a) $p \vee q$ - standard semantics

(b) $p \vee q$ - inquisitive semantics

(c) $p \vee \neg p$ - standard semantics

(d) $p \vee \neg p$ - inquisitive semantics

Figure 1: Interpretation of disjunction in Standard and Inquisitive Semantics

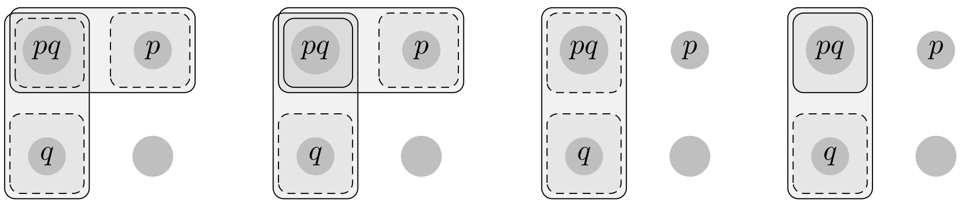
Thus we can see that in Inquisitive Semantics, as in Alternative Semantics, the interpretation of disjunction yields structured possibilities instead of mixing all possibilities into one set, as in the standard case. Therefore Inquisitive Semantics keeps many of the empirical advantages Alternative Semantics has over Montague Grammar, which we saw in section 2. Additionally the connectives are treated uniformly in Inquisitive Semantics. The definition of their interpretation is symmetrical, unlike in Alternative Semantics, where *or* was treated differently from the other connectives. Hence Inquisitive Semantics has a structural advantage over Alternatvie Semantics while keeping many of its empirical benefits.

3.3 Motivation for a strengthening operator

The basic implementation of Inquisitive Semantics presented so far has empirical advantages over Montague Grammar in making correct predictions for certain examples for which Montague grammar fails. It has structural advantages over Alternative Semantics by offering a uniform treatment of the logical connectives. However, downward closure of the set of states which form the meaning of a sentence has one possible

[24] $\{\emptyset\}$ is not empty as it contains the empty set.
[25] *Not* the inquisitive meaning. We will omit reference to the semantic framework when no confusion can arise.

disadvantage: we can no longer distinguish between nested possibilities, which Alternative Semantics can. This is depicted in figure 2 for the same set of worlds as in the earlier examples. We can see that if natural language disjunction is regarded in its most basic interpretation, mapped to the disjunction of our logical language, "p or q or both" cannot be distinguished from the sentence "p or q". The same holds for "p" and "p or (p and q)". To emphasize the point we are making we need to depict all possibilities, not only the maximal ones. Maximal possibilities are drawn with solid lines, subpossibilities with dotted lines. This serves only to emphasize the intuitive difference there could be between the several interpretations. Looking at the diagrams it is clearly visible that the basic Inquisitive Semantics described so far is incapable of recognizing a possibility as distinct if it is contained within another possibility. Inquisitive Semantics restricts the structure which alternatives can have, which enriches expressive power of the framework but discards nested possibilities.[26] This feature of the basic version of Inquisitive Semantics motivates the move of enhancing this framework with an exclusive strengthening operator in the interpretation of *or* which makes it possible for this connective to be interpreted exclusively. An exclusive mapping of *or* into the logical lanugage would restore the difference in interpretation in the depicted examples between the "p or q" and the "p or q or both"-sentences.



(a) p or q (b) p or q or both (c) q (d) q or (p and q)

Figure 2: Nested Alternatives in Inquisitive Semantics

4 Inq_{ES} vs. Alternative Semantics in disjunctive counterfactuals

We have seen above that despite all the advantages Inquisitive Semantics has there are still certain types of examples the framework presented so far cannot account for if *or* is interpreted in its most basic form.

Nested possibilities in disjunctions are not recognized in Inquisitive Semantics. This is due to the fact that a possibility included in another one already appears in the issue without having to be explicitly mentioned, as we have defined issues to be

[26] It is not a prori clear that the framework should be able to distinguish between nested possibilities. We will see below one way of enhancing the interpretation of natural lanugage disjunction which makes it possible for the Inquisitive Semantics to distinguish "p or q" from "p or q or both" sentences without having to represent nested alternatives. We show that actually this way of interpretation is empirically more accurate than the way proposed in Alternative Semantics.

downward closed. Thus if *or* is mapped to inclusive disjunction "p or q or both" sentences are modeled exaclty the same way that "p or q" sentences are. In this section we will suppose an extended version of Inquisitive Semantics including an exclusive strengthening operator for the mapping of *or* into our logical language, Inq_{ES}. We want to test empirically whether this analysis or Alternative Semantics makes better predictions. Our hypothesis is that Inq_{ES} is stronger.
First we will present in section 4.1 certain properties this extended framework will have. In section 4.2 we will explain the type of examples we have developed to empirically compare the performance of Alternative Semantics and Inq_{ES}. Section 4.3 presents the predictions made by the two frameworks on the given examples. Finally in section 4.4 we present and discuss the results of the investigation.

4.1 Exclusive strengthening operator

We will not give a full definition of an exclusive strengthening operator in the framework of Inquisitive Semantics here. We will also not discuss when such an operator will be applied and when it will not.[27] Instead we will now look at properties such a strengthening operator will definitely have, and which we will therefore be able to work with in the coming sections. In order to get there we will as a preliminary step have a look at existing definitions of exclusive strengthening operators in the standard case.
Several existing accounts defining exclusive strengthening operators for disjunctions have been developed in response to a set of characteristics caused by *Hurfords constraint*. We will have a look at the arguments brought up in the developtment of those approaches to see how these results in standard semantics are transferable to our case.
Hurford observed that disjunctions are infelicitous in cases where one disjunct entails the other. Consider the following example

> John lives in Paris or in France.

Such a statement would be considered odd in natural conversation. The same holds for the reversed disjunction:

> John lives in France or in Paris.

These observations sparked Hurford to formulate a constraint on the felicity conditions of disjunctive statements.

> **Hurford's constraint:** A sentence that contains a disjunctive phrase of the form 'S or T' is infelicitous if S entails T or T entails S.

Where 'entails' means 'is included in', to generalise to cases where S and T are not themselves sentences.
However, this constraint does not universally hold. For example, the statement

[27] In addition to the examples presented here which call for an exclusive reading of *or* there are also examples which show that in certain cases an inclusive reading of *or* is necessary.

John or Mary or both are sleeping.

is felicitous, even though in a standard interpretation "both John and Mary" entails "John or Mary": the former is contained in the latter. Hurford argued that therefore *or* must be ambiguous between an inclusive and an exclusive reading. Reading the first disjunct exclusively reinstates the constraint. Yet, a similar problem comes up in other instances, for example in disjunctions containing scalar items:

Mary ate some or all of the cookies.

This statement is once more felicitous although "all of the cookies" implies "some of the cookies". The unifying feature between both these examples is that Hurford's constraint does apply to them if we strengthen the meaning of the first disjunct to contain its conversational implicatures. That is, if we read "John or Mary" as "John or Mary but not both" and "some" as "some but not all".[28] These readings clearly break the entailment relation between the disjuncts.
Chierchia et al. (2007) argued that therefore conversational implicatures should be computed locally. Singh (2008) uses the infelicity of the statement

Both John and Mary or John or Mary are sleeping.

to additionally argue for an incremental computational procedure in checking Hurford's constraint. That is, he proposes the following procedure for the interpretation of disjunctive sentences:

Procedure for the interpretation of disjunctive sentences:

(i) Let $\psi = \varphi_1 \vee \ldots \vee \varphi_n$ be a disjunctive sentence.

(ii) Begin with the empty proposition $[\![\psi]\!] = A = \emptyset$, i.e. a set with no worlds.[29]

(iii) Take each disjunct in the left-right order in which it appears, and check that its basic meaning is in no entailment relation with A, i.e. check that $[\![\varphi_i]\!] \not\models A$ and $A \not\models [\![\varphi_i]\!]$.

(iv) If one of the two conditions comes out as true, halt and output infelicity.

(v) If neither of the two conditions comes out as true, compute $exh([\![\varphi_i]\!])$, add it to A, and continue with φ_{i+1}

Where $exh([\![\varphi_i]\!])$ is the basic meaning of φ_i plus its conversational implicatures. Singhs procedure computes the strengthened meaning after Hurford's constraint was checked. Thus, for the first disjunct the strengthened meaning is taken into consideration whereas for the last disjunct only its basic meaning, without conversational implicatures added, contributes. This procedure correctly predicts the felicity of "John or Mary or both" and the infelicity of "Both John and Mary or John or Mary", given that we have the right way of computing implicatures which yields that the

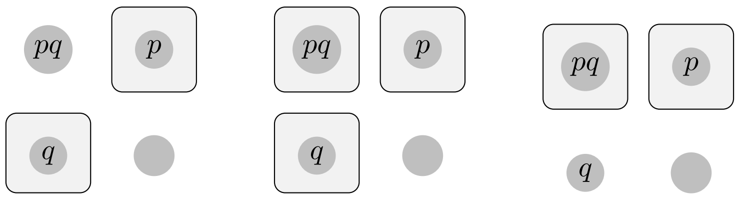
[28] We will go more deeply into how exactly to derive these implicatures below.

[29] Recall that this interpretation procedure is performed in a possible world semantics.

strengthened meaning of “John” includes “and not Mary”. Note that the asymmetry of the strengthening operator only applies when we check Hurfords constraint; generally also the last disjunct is strengthened before its meaning is added to that of the whole sentence. Several of the above mentioned arguments apply to our case:

- Postulating an ambiguity of *or* between an inclusive and an exclusive reading is inelegant and conceals possible generalisations
- A better way of strengthening the meaning of the disjuncts is to include conversational implicatures
- The strengthening procedure is computed locally and incrementally

Hence we will assume a strengthening operator in Inquisitive Semantics which is computed locally and incrementally by including conversational implicatures into the meaning of the disjuncts. Such an operator will yield the interpretations of disjunction depicted in figure 3.



(a) p or q (b) p or q or both (c) p or (p and q)

Figure 3: Interpretation of disjunctions with an exclusive strengthening operator

4.2 Examples

In this section we describe the types of examples we constructed for the questionnaire to test whether Alternative Semantics or Inq_{ES} make better predictions. In total we composed a questionnaire with 13 questions. The complete questionnaire is attached in appendix C. It will become clear why these examples serve to differentiate between these two frameworks in the next section 4.3, where we show that they systematically make different predicitons on the truth values of these examples evaluated in the actual world. The examples we construct are disjunctive counterfactuals “If φ then would ψ” where the antecedent has either the form $\varphi_1 \vee \varphi_2$, $\varphi_1 \vee \varphi_2 \vee (\varphi_1 \wedge \varphi_2)$ or $\varphi_1 \vee (\varphi_1 \wedge \varphi_2)$. Below we will reference these three forms as A or B, A or B or both, and A or (A and B), going by the convention that $[\![\varphi_1]\!] = A$, $[\![\varphi_2]\!] = B$ and writing the natural language expressions *and* and *or* as the interpretations of the connectives for easy readibility.
In all examples it will be the case that the $\varphi_1 \wedge \varphi_2$-worlds are closest to the actual world:

- $f_{\leq_w}(\bigcup[\![\varphi_1]\!]_{AS}) \quad \subseteq \quad \bigcup[\![\varphi_1 \wedge \varphi_2]\!]_{AS}$
- $f_{\leq_w}(\bigcup[\![\varphi_2]\!]_{AS}) \quad \subseteq \quad \bigcup[\![\varphi_1 \wedge \varphi_2]\!]_{AS}$

- $f_{\leq_w}(\bigcup[\![\varphi_1]\!]_{\mathsf{Inq}_B}) \quad \subseteq \quad \bigcup[\![\varphi_1 \wedge \varphi_2]\!]_{\mathsf{Inq}_B}$
- $f_{\leq_w}(\bigcup[\![\varphi_2]\!]_{\mathsf{Inq}_B}) \quad \subseteq \quad \bigcup[\![\varphi_1 \wedge \varphi_2]\!]_{\mathsf{Inq}_B}$

Note that this formalisation specifically applies to the examples under discussion, as we know that φ_1 and φ_2 here contain only one alternative, one maximal proposition. Thus this proposition is exactly $\bigcup[\![\varphi_i]\!]$. The same holds for $\varphi_1 \wedge \varphi_2$. Furthermore note that in Inquisitive Semantics this only holds in the basic case, without a strengthening operator, as the strengthening operator would make sure $[\![\varphi_1]\!]_{\mathsf{Inq}_{ES}}$ excludes $\varphi_1 \wedge \varphi_2$. Additionally in almost all of the examples $B \models A$, i.e. $[\![\varphi_2]\!] \subseteq [\![\varphi_1]\!]$.
The crucial point in the examples is that the consequent holds in all (φ_1 and φ_2)-worlds, but does not hold in (φ_1 but not φ_2)-worlds.

- $\bigcup[\![\psi]\!] \supseteq \bigcup[\![\varphi_1 \wedge \varphi_2]\!]$
- $\bigcup[\![\psi]\!] \not\supseteq \bigcup[\![\varphi_1]\!]$

4.3 Predictions

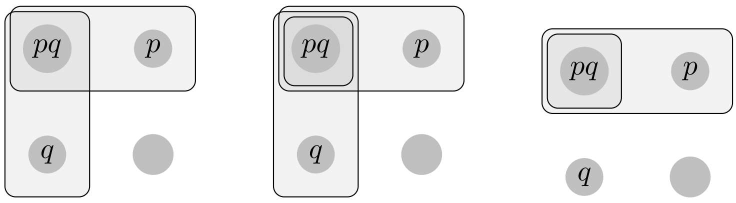
In this section we compute the predictions Alternative Semantics and Inquisitive Semantics extended with an exclusive strengthening operator make for the presented examples. We will follow the interpretation of disjunctive counterfactuals in Alternative Semantics used by Alonso-Ovalle, presented in section 2.1. That is, we presuppose a partial order $\leq_w$ on the set of possible worlds W depending on the world of interpretation w. If $w' \leq_w w''$ then w' is more similar to w than w'', less things change between w and w' than between w and w''. Another interpretation is that w' is more likely than w'' if we are in w. Using this partial order a counterfactual statement "If φ then would ψ" is analysed in standard semantics as *true* in a world w if the consequent holds in all possible worlds which are closest to w and in which the antecedent holds , $f_{\leq_w}([\![\varphi]\!])$.
For both Alternative and Inquisitive Semantics we need to adjust this view: sentence meanings are sets of propositions and no longer sets of worlds. Additionally, an important feature we need to keep in the analysis is the structure of the propositions inside the meaning. Thus we cannot, for example, execute the comparison simply on the informative content of an expression, even though this would give us a set of possible worlds. Instead, as Alonso-Ovalle convincingly showed, we want the consequent to hold in the worlds closest to the actual world from every single possibility to be worlds in which the consequent holds for the counterfactual to be evaluated as *true*. For Inquisitive Semantics this means that the consequent has to hold in all of the worlds closest to w (in the sense of $\leq$) in maximal (in the sense of inclusion) propositions in $[\![\varphi]\!]$. Thus we get to the following truth conditions of counterfactuals:[30]

Definition 20 *(Truth conditions of counterfactuals in AS)*
$w \in \bigcup[\![\textit{If } \varphi \textit{ then would } \psi]\!]_{AS} \quad \Leftrightarrow \quad \forall s \textit{ maximal in } [\![\varphi]\!]_{AS}\ f_{\leq_w}(s) \subseteq \bigcup[\![\psi]\!]_{AS}$

[30]Note that this does not give us the complete interpretation of the counterfactuals. However, the truth conditions suffice for the current investigation.

Definition 21 *(Truth conditions of counterfactuals in* Inq_{ES}*)*
$w \in \bigcup[\![\textit{If } \varphi \textit{ then would } \psi]\!]_{\mathsf{Inq}_{ES}} \Leftrightarrow \forall s \textit{ maximal in } [\![\varphi]\!]_{\mathsf{Inq}_{ES}}\ f_{\leq_w}(s) \subseteq \bigcup[\![\psi]\!]_{\mathsf{Inq}_{ES}} = \mathsf{info}(\psi)$

Where w is the world of interpretation.
Furthermore, as we described in section 4.2 the examples "If φ_1 or φ_2, then would ψ", "If φ_1 or φ_2 or both, then would ψ " and "If φ_1 or (φ_1 and φ_2), then would ψ" are all constructed in such a way that the $\varphi_1 \wedge \varphi_2$ worlds are closest to the actual world, w. Now let us analyse which predictions the frameworks make for the presented examples.



(a) φ_1 or φ_2 (b) φ_1 or φ_2 or both (c) φ_1 or (φ_1 and φ_2)

Figure 4: Disjunctions in Alternative Semantics

Alternative Semantics

Figure 4 shows the interpretations in Alternative Semantics of the disjunctions used in the antecedents of our examples. It is visible that the conjunction of φ_1 and φ_2 is not excluded from the interpretation of φ_1 alone or φ_2 alone.
A or B cases: The two alternatives in the antecedent are φ_1 and φ_2. Thus ψ has to hold in $f_{\leq_w}([\![\varphi_1]\!]_{AS})$ and in $f_{\leq_w}([\![\varphi_2]\!]_{AS})$ for the counterfactual to come out true.
We constructed the examples in such a way that both $f_{\leq_w}([\![\varphi_1]\!]_{AS})$ and $f_{\leq_w}([\![\varphi_2]\!]_{AS})$ are exactly the (φ_1 and φ_2)-worlds. But we also constructed the examples in such a way that ψ specifically holds in exactly these worlds.
Hence Alternative Semantics predicts that these examples should be evaluated as *true.*
A or B or both cases: The alternatives in the antecedent are φ_1, φ_2 and $\varphi_1 \wedge \varphi_2$. The same reasoning as above goes for φ_1 and φ_2, and by construction ψ holds in all $\varphi_1 \wedge \varphi_2$ worlds, thus especially in the ones closest to w.
Hence Alternative Semantics predicts that these examples should be evaluated as *true.*
A or (A and B) cases: The alternatives in the antecedent are $\varphi 1$ and $\varphi_1 \wedge \varphi_2$. The same reasoning as above holds.
Hence Alternative Semantics predicts that these examples should be evaluated as *true.*

Inq_{ES}

The interpretation in Inq_{ES} of the disjunctions used in the antecedents of our examples is what we depicted above in figure 3.
<u>A or B cases:</u> The maximal issues in the antecedent are (only φ_1) and (only φ_2). Thus ψ has to hold in $f_{\leq_w}([\![\varphi_1]\!]_{\mathsf{Inq}_{ES}})$ and in $f_{\leq_w}([\![\varphi_2]\!]_{\mathsf{Inq}_{Es}})$ for the counterfactual to come out true.
We constructed the examples in such a way that $f_{\leq_w}([\![\varphi_1]\!]_{\mathsf{Inq}_{ES}})$ is contained inside the (φ_1 but not φ_2)- worlds and $f_{\leq_w}([\![\varphi_2]\!]_{\mathsf{Inq}_{ES}})$ is contained inside the (φ_2 but not φ_1)- worlds.
Furthermore we made sure in the construction that in both the (only φ_1) and the (only φ_2) worlds the consequent does not hold.
Hence Inq_{ES} predicts that these examples should be evaluated as *false*.
<u>A or B or both cases:</u> The maximal issues in the antecedent are (only φ_1), (only φ_2) and $\varphi_1 \wedge \varphi_2$. The same reasoning as above holds for φ_1 and $varphi_2$.
Hence Inq_{ES} predicts that these examples should be evaluated as *false*.
<u>A or (A and B) cases:</u> The maximal issues in the antecedent are φ_1 and $\varphi_1 \wedge \varphi_2$. The same reasoning as above holds for φ_1.
Hence Inq_{ES} predicts that these examples should be evaluated as *false*.

For all of the examples we constructed Alternative Semantics and Inq_{ES} make diverging predictions concerning the truth conditions. Therefore the examples we constructed serve to evaluate which one of the two frameworks is empirically stronger in these cases.

4.4 Results

We tested the questionnares on 10 German and 7 English native speakers, in their native language. The outcome of the investigation is depicted in figures 4 to 6. Furthermore the basic numeric outcomes are presented in tables 7 and 8. It is apparent that there are large differences between the two languages. This is most likely due to the fact that all native English speakers who filled in the questionnaire were Master of Logic students at the University of Amsterdam, which presumably influences intuition towards a logical point of view. In contrast, none of the German native speakers who participated in the survey has a background in logic or linguistics. Therefore the average taken over both languages presumably renders the most meaningful results.

Looking at the results obtained over both languages a few things are immediately apparent: firstly, for all three types of examples the percentage of people who interpreted them as *false* is considerably higher than the percentage of those who interpreted them as *true*. Testing for statistical significance using an independent t-test with the null-hypothesis that judgements are evenly distributed between *true* and *false* shows that these differences are significant in all three cases.
Furthermore, the difference is a lot smaller for examples of type (A or B) than for the other types of examples. These differences between types of examples turned out not to be significant however.

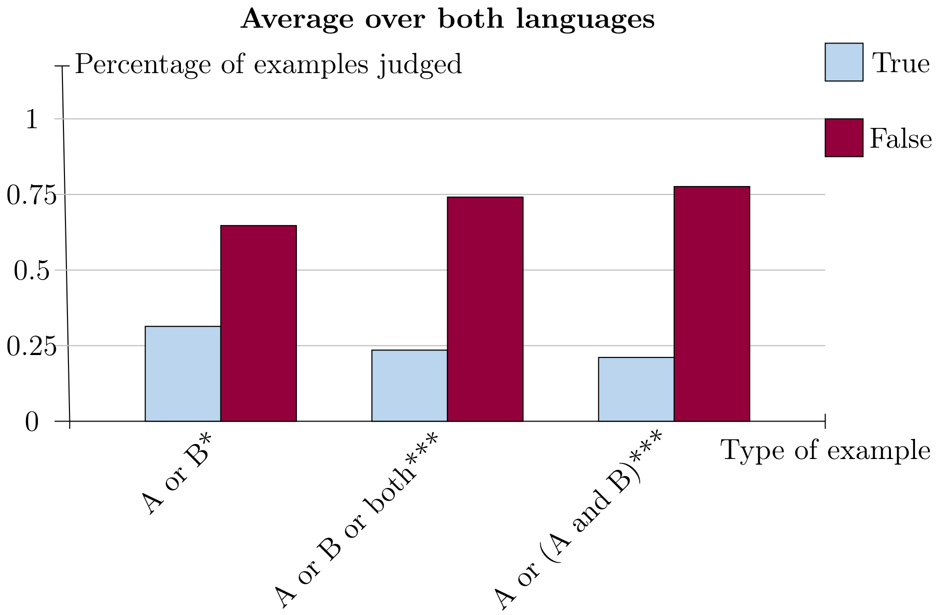


Figure 5: Percentage of Examples interpreted as true or false, according to the type of the example: A or B, A or B or both, A or (A and B). These results show the average over both English and German responses. Neither of the observed differences in judgement between the types of examples is statistically significant. However, all three types of exmaples significantly refute the hypothesis that P(True)=P(False)=0.5; *='p<0.1', ***='p<0.001'. Numbers missing for the percentages to sum up to 1 are due to the option of ticking "I don't know", used 5 times in all 221 responses.

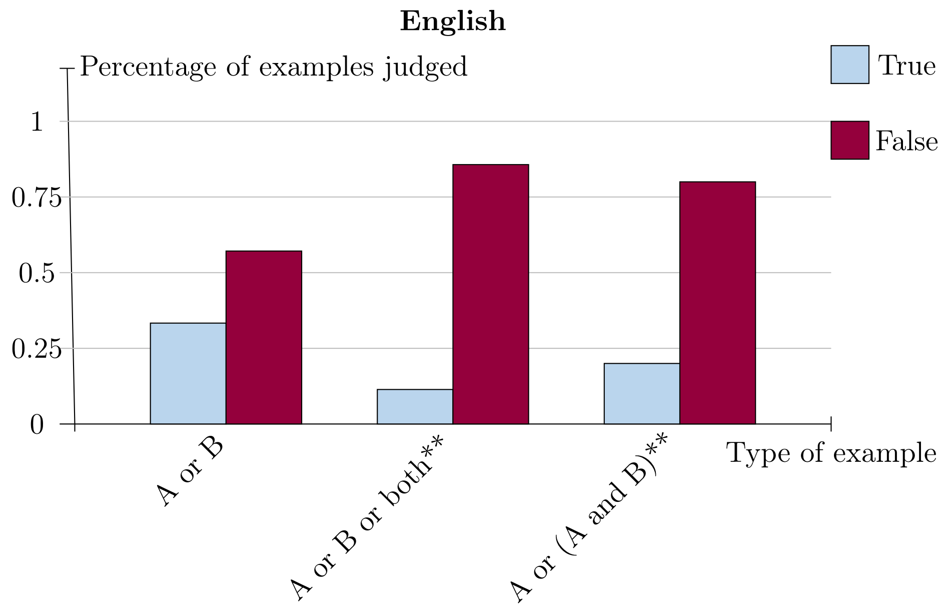


Figure 6: Percentage of Examples interpreted as true or false, only the English questionnaires. There is a significant difference ($p < 0.05$) between examples of type A or B and examples of type A or B or both. Examples of type A or B do not significantly refute the hypothesis that P(True) = P(False) = 0.5. The other types of examples do, **='p<0.01'.

Both these outcomes support our hypothesis. First, as we already described, the fact that most people intuitively interpret the counterfactuals as false supports our claim. Second, the fact that there is no significant difference between the interpretation of (A or B)-type examples and (A or B or both)-type examples shows that disjunction is interpreted exclusively in the (A or B) cases, even if the conjunct is not mentioned as an explicit alternative. Thus adding an exclusive strengthening operator for disjunction to the basic Inquisitive Semantics framework is justified.
The results for native English speakers show a great discrepancy for the interpretations of (A or B)-type examples in comparison to the interpretation of (A or B or both)-type examples. This once more fits in with what was said before, namely that the English speakers participating in this survey all have a background in logic. In first order logic, and in most other logical language, *or* is interpreted inclusively. Thus logic students' intuition could be skewed towards this interpretation, which would explain the higher number of *true* answers for examples of type (A or B).

Contrastingly to the answers of the English native speakers, the German native speakers show little difference in the interpretation of (A or B) versus (A or B or both)-type examples. Due to the close proximity of the two languages this divergence is most likely not caused by structural differences in the understanding of the two languages but rather by the fact that the German speakers in this experiment are non-logicians. The difference between the interpretations of (A or B or both)-type

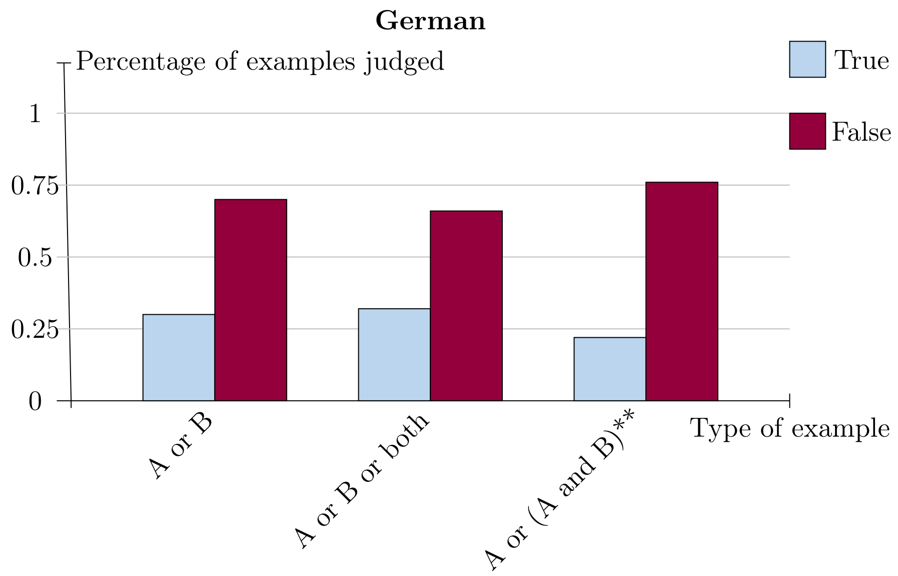


Figure 7: Percentage of Examples interpreted as true or false, only the German questionnaires. None of the observed differences in judgement between the types of examples are statistically significant. Examples of type A or B and of type A or B or both do not significantly refute the hypothesis that P(True) = P(False) = 0.5. Examples of type A or (A and B) do, **='p<0.01'.

Example Number	Label	Evaluated as True	Evaluated as False	Don't know
1	A or B or both	1	6	-
2	A or (A and B)	1	6	-
3	A or B	3	4	-
4	A or B	1	5	1
5	A or B or both	1	5	1
6	A or (A and B)	2	5	-
7	A or B or both	1	6	-
8	A or (A and B)	1	6	-
9	A or (A and B)	1	6	-
10	A or B or both	0	7	-
11	A or B	3	3	1
12	A or (A and B)	2	5	-
13	A or B or both	1	6	-

Figure 8: Numerical results for the English questionnaires.

Example Number	Label	Evaluated as true	Evaluated as false	Don't know
1	A or B or both	8	2	-
2	A or (A and B)	3	7	-
3	A or B	3	7	-
4	A or B	2	8	-
5	A or B or both	3	6	1
6	A or (A and B)	4	6	-
7	A or B or both	1	9	-
8	A or (A and B)	2	8	-
9	A or (A and B)	1	8	1
10	A or B or both	3	7	-
11	A or B	4	6	-
12	A or (A and B)	1	9	-
13	A or B or both	1	9	-

Figure 9: Numerical results for the German questionnaires.

examples is statistically significant, with an independent t-test resulting in $p<0.05$. This definitely raises questions concerning how far the intuition Linguists have about the interpretation of natural language coincides with the intuition most "people on the street" have, which should be investigated further in future research.
Another remark concerns the high number of *true* interpretations of the first example, which is of type A or B or both, visible in the table showing numerical results for the single example. As this diverges from the interpretation of the other examples of this type it can be assumed that the participants had to get used to the overall type of questions. Nevertheless, this question was not excluded from the evaluation. However, to get back to our overall topic, and to sum up, we motivated an exclusive strengthening operator in Inquisitive Semantics empirically by showing that this extended framework makes the correct predictions on a set of examples for which Alternative Semantics fails.

5 Conclusion

We set out in this paper to compare the performance of Alternative Semantics with that of Inq_{ES}, Inquisitive Semantics extended with an exclusive strengthening operator. To understand this we first of all presented Alternative Semantics, with some motivating examples and consequently the implementation. Afterwards we presented the basic implementation of Inquisitive Semantics without an exclusive strengthening operator, and motivated why such an operator is needed. To make sure this strengthening operator is not some ad hoc procedure defined to save the Inquisitive Semantics framework in face of examples it is, in its basic form, unable to account for, we investigated empirically whether this strengthening operator is also independently motivated. To this end we constructed a questionnaire made up of examples that are able to differentiate between an Alternative Semantics interpretation of *or*

and an interpretation using Inquisitive Semantics plus the strengthening operator in such a way that the two frameworks predict diverging truth conditions. The investigation showed that over all types of examples we looked at Inquisitive Semantics with a strengthening operator makes the correct predictions, whereas Alternative Semantics fails. Thus we showed that in our cases *or* is indeed interpreted the way that Inq_{ES} analyses it. Our current examination gives an empirical support for this framework. Therefore we motivated its use in further conceptual work.
Additionally future research can include a more thorough investigation of the observed phenomena with a bigger set of participants. The detected side effect of a divergence between the intuition of logicians versus non-logicians also provides in interesting starting point for further investigations.

A Montague Grammar

In his dissertation Alonso-Ovalle presents Alternative Semantics as a solution to certain shortcomings of what he calls *standard textbook semantics*. I consider this to refer to Montague Grammar, based on the statement in Gamut (1991), p.193: "Montague grammar still serves as the standard model for a logical grammar." In order to better understand the contrasts Alonso-Ovalle draws between this model and Alternative Semantics I will now give a short summary of the part of Montague Grammar which is relevant here. In contrast to Montagues original formulation, the approach presented here uses the improved *two-sorted* type theory, which allows for direct reference to possible worlds.[31]
Quite generally, Montague Grammar is a logical grammar. That is, it uses semantic methods from formal logic to analyse natural language. To be able to do so certain presuppositions and constraints are necessary. First, the linguistic meaning of a sentence is identified with its truth conditions. Formal logic operates with expressions that are mapped to truth values, therefore this view permits a straightforward link from this discipline to linguistics. Second, only a part of natural language is treated. Let us say that the language under discussion is English.[32] Then there is a truth predicate in the natural language, "is true", as well as names for its own sentences, which together enable self-reference. It is well known that self-reference, allowing for statements like "This sentence is false.", causes logical paradoxes or inconsistencies. Therefore only the non self-referential (non *semantically closed*) part of the language is analyzed. The third presupposition is compositionality of meaning, that is, the idea that the meaning of a sentence is derived in a principled and exact way from the meanings of its constituent parts. A semantic analysis must be able to provide interpretations for an infinite number of expressions via a finite number of rules. Therefore only the interpretations of basic words are explicitly given and every syntactic rule, permitting the construction of a more complex expression, is correlated with a semantic rule, giving the interpretation of the compound expression. Without compositionality of meaning this approach would certainly fail. Having made these

[31] The following is taken mainly from Gamut (1991), chapters 4 - 6., with the adjustments to two-sorted type theory based on Muskens (2011)

[32] Using other languages, or even multiple ones, does not remedy the issue that is presented.

$$\boxed{\text{natural language}} \xrightarrow{\text{translation}} \boxed{\text{logical language}} \xrightarrow{\text{interpretation}} \boxed{\text{model}}$$

(categorial syntax) (intensional type theory)

Figure 10: The interpretation process in Montague Grammar

prerequesites explicit we can now look at how Montague Grammar works.
In order to achieve a link between natural language expressions and a model for a semantic theory certain intermediate steps are employed. Figure A depicts these. As a very first step the basic syntactic construction tree is assumed to be known. Consider for example a structurally ambiguous sentence like "John sees old men and women". This can be read as one of the two following statements.

- John sees [old [men and women]]
- John sees [[old men] and women]

It is assumed that we know which one of the possibilities holds and the further analysis works on expressions which are already equipped with a basic syntactic structure. Next a proper syntactic analysis using categorial syntax takes place. This syntax ascribes a certain category to every natural language expression and allows for expressions of corresponding categories to be combined into more complex expressions of simpler categories. Formally, we first define the set CAT of categories.

Definition 22 *(The set of categories)*
CAT*, the set of categories, is the smallest set such that:*

- S, CN, IV $\in \mathsf{CAT}$
- *If* A,B $\in \mathsf{CAT}$*, then* A/B $\in \mathsf{CAT}$

The basic categories are S, the category of sentences, CN, the category of common noun phrases and IV, the category of intransitive verb phrases. An element of category A/B takes an element of category B as argument and outputs an element of category A. Only finitely many of the categories will actually be used. Each of these categories contains certain basic expressions, which are given in a lexicon. For example, 'man' and 'woman' are common nouns, 'walk' and 'talk' are intransitive verbs. 'John' and 'Mary' are terms, of category T=S/IV, and 'love' is a transitive verb, of category TV = IV/T = IV/(S/IV).
Next a variety of syntactic rules defines how to construct new expressions from given ones, according to their affilitation with the categories. For example, the following rule allows for the construction of the sentence "John walks" from the term 'John' and the intransitive verb 'walk'.

S2 If δ is an expression of category IV and α is an expression of category T=S/IV, then $F_1(\alpha, \delta)$ is an expression of category S and $F_1(\alpha, \delta) = \alpha\delta'$, where δ' is the result of replacing the main verb in δ by its third-person singluar present form

As another example, there is a separate rule for each determiner, combining it with common nouns to give terms. We can see that the syntactic category of an expression already represents its semantic function. Generally, the way these syntactic rules are constructed can be roughly understood from this easy example. This is all we need here, so I will not go into more detail.
After having defined the categorial syntax on the natural language these structured expressions are translated into a logical language, intensional type theory. Let us first of all define the syntax and semantics of intensional type theory before going on with the translation process. To be able to define the syntax, we first need to define the set of intensional types.

Definition 23 *(The set of types)*
$\mathbf{T}$*, the set of intensional types, is the smallest set such that:*

- $s, e, t \in \mathbf{T}$
- *If* $a, b \in \mathbf{T}$*, then* $\langle a, b \rangle \in \mathbf{T}$

There are three basic types,s for possible worlds, e for entities and t for truth values. Expressions of type $\langle a, b \rangle$ are of a function-type, mapping expressions of type a to expressions of type b. For example, elements of type $\langle e, \langle s, t \rangle \rangle$ map entities to truth conditions according to possible worlds. They refer to (characteristic functions of) a relation between entities and possible worlds. Expressions of type $\langle s, a \rangle$ refer to intensions of expressions of type a. An expression which is a function from possible worlds to expressions of another type is an intensional entity. Generally, the truth of a sentence is evaluated with respect to a possible world. That is, sentences are objects of type $\langle s, t \rangle$. They are interpreted as functions from possible worlds to truth values. Thus a sentence is not simply true or false, it is true or false in a certain world. In that way the interpretation of a sentence is a set of possible worlds: those worlds in which the sentence is true.
The vocabulary of a type-theoretical language L is

1. for every type a, an infinite set VAR_a of variables of type a
2. the connectives $\wedge, \vee, \rightarrow, \neg$ and $\leftrightarrow$
3. the quantifiers $\exists$ and $\forall$
4. the identity symbol $=$
5. the operators $\Box$ and $\Diamond$
6. the brackets (and)
7. for every type a, a (possibly empty) set CON_a^L of constants of type a

Unlike in first-order predicate logic, the description of the syntax does not only include a definition of well-formed formulas, we generally give a recursive definition of well-formed expressions of any type a in a given language L, WE_a^L. Formulas are then expressions of type t.

Definition 24 *(Well-formed expressions)*

1. *If* $\alpha \in VAR_a$ *or* $\alpha \in CON_a^L$*, then* $\alpha \in WE_a$
2. *If* $\alpha \in WE^L_{\langle a,b\rangle}$ *and* $\beta \in WE^L_a$ *then* $(\alpha(\beta)) \in WE^L_b$
3. *If* $\varphi, \psi \in WE^L_t$*, then* $\neg\varphi,\ (\varphi \wedge \psi),\ (\varphi \vee \psi),\ (\varphi \to \psi)$ *and* $(\varphi \leftrightarrow \psi)\ \in WE^L_t$
4. *If* $\varphi \in WE^L_t$ *and* $v \in VAR_a$ *then* $\forall v\varphi \in WE^L_t$ *and* $\exists v\varphi \in WE^L_t$
5. *If* $\alpha, \beta \in WE^L_a$*, then* $\alpha = \beta \in WE^L_t$
6. *If* $\alpha \in WE^L_a$ *and* $v \in VAR_b$*, then* $\lambda v.\alpha \in VAR_{\langle b,a\rangle}$
7. *If* $\varphi \in WE^L_t$*, then* $\Box\varphi,\ \Diamond\varphi \in WE^L_t$
8. *Every element of* WE_a*, for any* a*, is constructed in a finite number of steps using the above rules*

The variables and constants of a certain type are well-formed expressions of this type. Expressions of type $\langle a, b\rangle$ are functions, applied to elements of type a they yield an element of type b. Not surprisingly, this rule is called application. Formulas are the elements of type t, only these can be combined via the logical connectives, to give another formula. Quantifiers can range over variables of any type. The statement that two expressions of the same type are equal always gives a formula. Rule 6 introduces lambda-abstraction. Rule 7 introduces the modal operators. These work in the standard way.

Now that we have defined the syntax we can continue with the semantics. A model $\mathbf{M}$ for two-sorted intensional type theory consists of a set of possible worlds W, an accessibility relation R and an interpretation function I for the constants of each type. This interpretation function returns the *intension* of a constant. The accessibility relation R is chosen to be total. Additionally we will need an assignment function g for the variables of each type.

Furthermore, we need to know in which domain $\mathbf{D}_a$ elements of type a are evaluated. As elements of type $\langle a, b\rangle$ will be functions we can determine their domain, given the domains of the types a and b. Thus, to initiate this inductive definition we need to specify the domain for elements of the basic types s, e and t. As we stated above, expressions of type s refer to possible worlds, their domain of interpretation is thus W. Expressions of type t refer to formulas, so they are evaluated as truth values 0 or 1. The domain for elements of type e is called D. This domain is universal, every possible world is equipped with the same domain D. The general notation for the domain of interpretation of elements of type a, given the domain D for entities and a set of possible world W, is $\mathbf{D}_{a,D,W}$. The index W is needed as the domains of intensional objects depend on the set of possible worlds.

Definition 25 *(Domains of interpretation)*

- $\mathbf{D}_{s,D,W} = W$
- $\mathbf{D}_{e,D,W} = D$
- $\mathbf{D}_{t,D,W} = \{0,1\}$
- $\mathbf{D}_{\langle a,b\rangle,D,W} = \mathbf{D}_{b,D,W}^{\mathbf{D}_{a,D,W}}$

Generally, Y^X is the set of all functions mapping a set X to a set Y, so the above definition represents exactly what was explained before. Namely, that elements of type $\langle a,b\rangle$ are functions from elements of type a to elements of type b. Expressions of type $\langle s,t\rangle$, sets of possible worlds, are called *propositions*, while expressions of type $\langle e,\langle s,t\rangle\rangle$ are called *properties*.
Next, we can define the extension $[\![\alpha]\!]_{\mathbf{M},w,g}$ of an expression α depending on the model $\mathbf{M}$, the world of evaluation w and the assignment function g.

Definition 26 *(Extensions)*

1. *If* $\alpha \in CON_a^L$, *then* $[\![\alpha]\!]_{\mathbf{M},w,g} = I(\alpha)(w)$

   If $\alpha \in VAR_a$, *then* $[\![\alpha]\!]_{\mathbf{M},w,g} = g(\alpha)$

2. *If* $\alpha \in WE_{\langle a,b\rangle}^L$ *and* $\beta \in WE_a^L$, *then* $[\![\alpha(\beta)]\!]_{\mathbf{M},w,g} = [\![\alpha]\!]_{\mathbf{M},w,g}([\![\beta]\!]_{\mathbf{M},w,g})$

3. *If* $\varphi,\psi \in WE_t^L$, *then* $[\![\neg\varphi]\!]_{\mathbf{M},w,g} = 1$ *iff.* $[\![\varphi]\!]_{\mathbf{M},w,g} = 0$

   $[\![\varphi \wedge \psi]\!]_{\mathbf{M},w,g} = 1$ *iff.* $[\![\varphi]\!]_{\mathbf{M},w,g} = [\![\psi]\!]_{\mathbf{M},w,g} = 1$

   $[\![\varphi \vee \psi]\!]_{\mathbf{M},w,g} = 1$ *iff.* $[\![\varphi]\!]_{\mathbf{M},w,g} = 1$ *or* $[\![\psi]\!]_{\mathbf{M},w,g} = 1$

   $[\![\varphi \rightarrow \psi]\!]_{\mathbf{M},w,g} = 0$ *iff.* $[\![\varphi]\!]_{\mathbf{M},w,g} = 1$ *and* $[\![\psi]\!]_{\mathbf{M},w,g} = 0$

   $[\![\varphi \leftrightarrow \psi]\!]_{\mathbf{M},w,g} = 1$ *iff.* $[\![\varphi]\!]_{\mathbf{M},w,g} = [\![\psi]\!]_{\mathbf{M},w,g}$

4. *If* $\varphi \in WE_t^L$ *and* $v \in VAR_a$, *then*

   $[\![\forall v\varphi]\!]_{\mathbf{M},w,g} = 1$ *iff. for all* $d \in \mathbf{D}_a$: $[\![\varphi]\!]_{\mathbf{M},w,g[v/d]} = 1$

   $[\![\exists v\varphi]\!]_{\mathbf{M},w,g} = 1$ *iff. for some* $d \in \mathbf{D}_a$: $[\![\varphi]\!]_{\mathbf{M},w,g[v/d]} = 1$

5. *If* $\alpha,\beta \in WE_a^L$, *then* $[\![\alpha = \beta]\!]_{\mathbf{M},w,g} = 1$ *iff.* $[\![\alpha]\!]_{\mathbf{M},w,g} = [\![\beta]\!]_{\mathbf{M},w,g}$

6. *If* $\alpha \in WE_a^L$ *and* $v \in VAR_b$, *then* $[\![\lambda v.\alpha]\!]_{\mathbf{M},w,g}$ *is that function* $h \in \mathbf{D}_a^{\mathbf{D}_b}$ *such that for all* $d \in \mathbf{D}_b$: $h(d) = [\![\alpha]\!]_{\mathbf{M},w,g[v/d]}$

7. *If* $\varphi \in WE_t^L$, *then*

   $[\![\Box\varphi]\!]_{\mathbf{M},w,g} = 1$ *iff. for all* $w' \in W$: $[\![\varphi]\!]_{\mathbf{M},w',g} = 1$

   $[\![\Diamond\varphi]\!]_{\mathbf{M},w,g} = 1$ *iff. for some* $w' \in W$: $[\![\varphi]\!]_{\mathbf{M},w',g} = 1$

All these rules represent exactly what was explained above.
Now that we know the syntax and semantics of two-sorted intensional type theory we can continue the short exposition of Montague grammar. We were about to look at the translation from natural language expressions equipped with a categorial syntax to expressions in type theory. Recall that the syntactic category of an expression is already intended to indicate its semantic function. Thus the translation should develop a link between the categories and the types. An apparent analogy between the construction of categories and that of types is that we have some basic ones and the rest are composed as functions. Thus the following definition maps CAT onto $\mathbf{T}$.

Definition 27 *(Interpretation function)*

f is a function from CAT to $\mathbf{T}$ such that:

1. $f(S) = \langle s, t\rangle$
2. $f(CN) = f(IV) = \langle e, \langle s, t\rangle\rangle$
3. $f(A/B) = \langle\langle s, f(B)\rangle f(A)\rangle$

Sentences are mapped to elements of type $\langle s, t\rangle$, truth conditions. Common nouns like 'man' and intransitive verbs like 'walk' form sentences when applied to entity-type expressions like 'John': "The property of being a man holds of John", i.e "John is a man", and "The property of walking holds of John", i.e. "John walks". Functional application semantically works on the *intension* of the expression. Next the categorial expressions are translated into well-formed expressions of type theory in accordance with the above function. To this end the translation of the basic elements, which were specified in a lexicon, is explicitly given. Afterwards each syntactic rule, which allows for the construction of a more complex natural language expression, is paired with a corresponding translation rule. This translation rule specifies how to translate the compound expression given the translations of the constituent parts and the rule by which they were combined.
This shows very roughly how the interpretation of natural language expressions in Montague Grammar takes place. As a very last step I will give the definitions for the introduction of connectives.[33] Sx is the syntactic rule, Tx the corresponding translation rule.

$S9$ If φ, ψ are expressions of category S, then $F_8(\varphi, \psi)$ is an expression of category S and $F_8(\varphi, \psi) = \varphi\ \mathit{and}\ \psi$

$T9$ If φ, ψ are expressions of category S and $\varphi \mapsto \varphi'$ and $\psi \mapsto \psi'$, then $F_8(\varphi, \psi) \mapsto (\varphi' \wedge \psi')$

[33] The introduction of *and* between terms is left out, as this would cause plurality. Additionally we are more concerned with *or* in the rest of the paper so this limitation should not cause too much confusion.

$S10$ If φ, ψ are expressions of category S, then $F_9(\varphi, \psi)$ is an expression of category S and $F_9(\varphi, \psi) = \varphi$ *or* ψ

$T10$ If φ, ψ are expressions of category S and $\varphi \mapsto \varphi'$ and $\psi \mapsto \psi'$, then $F_9(\varphi, \psi) \mapsto (\varphi' \vee \psi')$

$S11$ If γ, δ are expressions of category IV, then $F_8(\gamma, \delta)$ is an expression of category IV

$T11$ If γ, δ are expressions of category IV and $\gamma \mapsto \gamma'$ and $\delta \mapsto \delta'$, then $F_8(\gamma, \delta) \mapsto \lambda x.(\gamma'(x) \wedge \delta'(x))$

$S12$ If γ, δ are expressions of category IV, then $F_9(\gamma, \delta)$ is an expression of category IV

$T12$ If γ, δ are expressions of category IV and $\gamma \mapsto \gamma'$ and $\delta \mapsto \delta'$, then $F_9(\gamma, \delta) \mapsto \lambda x.(\gamma'(x) \vee \delta'(x))$

$S13$ If α, β are expressions of category T, then $F_9(\alpha, \beta)$ is an expression of category T

$T13$ If α, β are expressions of category T and $\alpha \mapsto \alpha'$ and $\beta \mapsto \beta'$, then $F_9(\alpha, \beta) \mapsto \lambda X.(\alpha'(X) \vee \beta'(X))$

As we can see, all instances of *or* generate a new, homogeneous entity containing both of the disjuncts.

B Uniform treatment of connectives

We have seen above that Montague introduced the connectives separately for each different type. Thus, for example, the connective "and" linking two intransitive verbs is introduced separately from the connective "and" linking two sentences. However, Alonso-Ovalle only treats the way "or" operates on whole sentences. Barbara Partee and Mats Rooth show in their article "Generalized Conjunction and Type Ambiguity" (Partee and Rooth, 1982) that this limitation is justified: the type-specific definition given by Montague can be generalized by recursively reducing the functioning of connectives for conjoinable types to the functioning of the connectives for sentences. Conjoinable types are types "which end in a t".

Definition 28 *(Conjoinable Types)*

- *t is a conjoinable type*
- *if b is a conjoinable type, then for all a, $\langle a, b\rangle$ is a conjoinable type*

That is, if an expression is of a conjoinable type then it takes a certain number of arguments and outputs an element of type t, truth values.[34] Sentences are of type t, thus every expression which can form a sentence given certain other expressions as an input is of a conjoinable type. Furthermore, suppose we already know how the connectives work on sentences. Then a recursive definition of how to relate the functioning of a connective on an element of type b with the functioning of this connective on elements of the type $\langle a, b\rangle$, which map elements of type a to elements of type b, suffices to know how the connectives work on all conjoinable types. Partee and Rooth show that Montague's definition can be simplified by using this approach. Concretely, they give the following definition of how to interpret the connectives according to the type of element they operate on.
The interpretation of the connectives "and" and "or" on sentences is given by the logical connectives $\wedge$ and $\vee$, respectively, on truth values. This is equivalent in Montague grammar. The logical connectives can be defined via finite truth tables. Subsequently the interpretation of connectives on function-types $\langle a, b\rangle$ is defined via pointwise recursion. As the symbols $\wedge$ and $\vee$ are already used for the logical connectives Partee and Rooth use the symbols $\sqcap$ and $\sqcup$ to refer generally to the interpretation of the connectives on various types.

Definition 29 *(Pointwise definition of $\sqcap$ and $\sqcup$)*

- $X \sqcap Y = X \wedge Y \quad$ *if X and Y are truth values.*
- $X \sqcap Y = \{(z, x \sqcap y) \mid (z, x) \in X \text{ and } (z, y) \in Y\} \quad$ *if X and Y are functions.*
- $X \sqcup Y = X \vee Y \quad$ *if X and Y are truth values.*
- $X \sqcup Y = \{(z, x \sqcup y) \mid (z, x) \in X \text{ and } (z, y) \in Y\} \quad$ *if X and Y are functions.*

Note that this definition still yields, via a small detour, a boolean algebra in which $\sqcap$ is the join and $\sqcup$ is the meet. To see this, we need to switch from identifying a sentence with a truth value to a possible-world semantics. This is easily done by relating a sentence to the set of worlds in which it is true. With the above definition "and", applied to sentences, then amounts to set intersection and "or" to set union. These are exactly the join and meet operators in the standard boolean structure on the powerset of the set of all possible worlds. Thus the fact that we have an algebraic structure in which the connectives are uniformly represented holds both for Montague Grammar as well as for this slight modification.

C The questionnaires

These are the full questionnaires used in the interviews. Some of the examples are modifications of instances presented in Veltman (2005). Only the English questionnaires have been attached; the German ones are made up in the same order and contain the same examples, translated by the author.

[34]Strictly speaking, elements of type t *are* not truth values, they *refer to* truth values. However, I will sometimes use this abbreviative denotation as it does not cause any confusion.

Examples

1. A man called Jones, who lives in California, always wears his hat if a die he throws in the morning shows a 6. If the sun shines Jones throws the die once. If it is raining (which rarely happens in California) he throws the die until he gets a 6. So he always wears his hat if it is raining.
   Now suppose we see Jones wearing his hat, and we know that it is raining. I say to you:

   "If it wasn't raining, or he hadn't thrown a six, or both, then Jones wouldn't be wearing his hat."

   - Is this sentence true or false ? true □ false □
   - How natural do you think this sentence is ?
     very odd □ – □ – □ – □ – □ – □ – □ very natural
   - Comments:

2. Charlie and his little sister Mary play the following game every day to determine what Charlie gets for dessert: They look outside the window for 5 minutes, to see if a red car passes. Afterwards Charlie asks his sister what he gets for dessert. If there was no red car she will always only let him have an apple. If there was a red car she will most of the times let him have cake, but sometimes he still only gets an apple and she keeps the cake to herself.
   Charlie's mother, who knows that today no red car drove by and that Charlie is having an apple, says to his father:

   "If there had been a red car passing by, or there had been a red car and Mary had said 'cake', then Charlie would be having cake."

   - Is this sentence true or false ? true □ false □
   - How natural do you think this sentence is ?
     very odd □ – □ – □ – □ – □ – □ – □ very natural
   - Comments:

3. A man called Jones always wears his hat if a die he throws in the morning shows a 6. If the sun shines Jones throws the die once. If it is raining he throws the die until he gets a 6. So he always wears his hat if it is raining. In addition, if it is raining, Jones also wears his bow-tie.
We know that it is not raining, and we see Jones wearing neither his hat nor his bow-tie. I say:

   "If Jones was wearing his hat, or his bow-tie, then it would be raining."

   - Is this sentence true or false ? true □ false □
   - How natural do you think this sentence is ?
     very odd □ – □ – □ – □ – □ – □ – □ very natural
   - Comments:

4. King Louis of France has a castle. If he is at home then the flag in the castle is up and torches are lit around the castle. If he is not at home then the flag is down and the torches are out. On his birthday King Louis always goes to his friend's, King Arthur's, castle to celebrate, but the flag is nevertheless up. The torches however are not lit in that case.
Now suppose two farmers, who know all this, walk by the castle. They don't know whether it is the King's birthday or not, but they see the flag being down and the torches out. One farmer says to the other:

   "If the flag was up, or the torches were lit, then the King would be home."

   - Is this sentence true or false ? true □ false □
   - How natural do you think this sentence is ?
     very odd □ – □ – □ – □ – □ – □ – □ very natural
   - Comments:

5. Charlie and his little sister Mary live next to the fire brigade, so the red fire brigade cars often pass in front of their house with their sirens on. Mostly the cars leave the station with their sirens on, only very rarely do they leave without an emergency. Every day Charlie and Mary play the following game to determine what Charlie gets for dessert: They look outside the window for 5 minutes, to see if a red fire brigade car passes with its sirens on. Afterwards Charlie asks his sister what he gets for dessert. If there was a red car with the sirens on she will let him have cake. If there was no red car, or the car didn't

have it's sirens on, he only gets an apple and she keeps the cake to herself. However, if they heard sirens in the distance Charlie still gets cake.
Charlie's parents know that today no red car drove by and no sirens were ringing. They see Charlie eating an apple, and his mother says:

"If there had been a red car passing by, or if they had heard sirens, or both, then he would be having cake."

- Is this sentence true or false ? true □ false □
- How natural do you think this sentence is ?
  very odd □ – □ – □ – □ – □ – □ – □ very natural
- Comments:

6. A man called Jones, who lives in California, always wears his hat if a die he throws in the morning shows a 6. If the sun shines Jones throws the die once. If it is raining (which rarely happens in California) he throws the die until he gets a 6. So he always wears his hat if it is raining.
   Now suppose we see Jones wearing his hat, and we know that it is raining. I say to you:

   "If it wasn't raining, or it wasn't raining and he hadn't thrown a six, then Jones wouldn't be wearing his hat."

   - Is this sentence true or false ? true □ false □
   - How natural do you think this sentence is ?
     very odd □ – □ – □ – □ – □ – □ – □ very natural
   - Comments:

7. King Louis of France has a castle. If he is at home then the flag in the castle is up and torches are lit around the castle. If he is not at home then the flag is down and the torches are out. On his birthday King Louis always goes to his friend's, King Arthur's, castle to celebrate, but the flag is nevertheless up. The torches however are not lit in that case. Additionally, if the King dies then the torches are on, but the flag stays down.
   Two farmers walk by the castle and see the flag being down and the torches out. One farmer says:

   "If the flag was up, or the torches were lit, or both, then the King would be home."

- Is this sentence true or false ? true □ false □
- How natural do you think this sentence is ?
  very odd □ – □ – □ – □ – □ – □ – □ very natural
- Comments:

8. A man called Jones always wears his hat if a die he throws in the morning shows a 6. If the sun shines Jones throws the die once. If it is raining he throws the die until he gets a 6. So he always wears his hat if it is raining. In addition, if it is raining, Jones also wears his bow-tie.
   We know that it is not raining, and we see Jones wearing neither his hat nor his bow-tie. I say:

   "If Jones was wearing his hat, or his hat and his bow-tie, then it would be raining."

   - Is this sentence true or false ? true □ false □
   - How natural do you think this sentence is ?
     very odd □ – □ – □ – □ – □ – □ – □ very natural
   - Comments:

9. Charlie meets a witch who can transform any object into sweets. Most of the time she will make chocolate cake, but sometimes she doesn't concentrate properly and produces another type of cake, or even ice-cream. Charlie only eats chocolate cake, no other type of cake and no ice-cream, but he adores chocolate cake.
   The witch just transformed a piece of wood into strawberry ice-cream for him. His little sister Mary says:

   "If she had turned it into cake, or even chocolate cake, he would have eaten it up immediately."

   - Is this sentence true or false ? true □ false □
   - How natural do you think this sentence is ?
     very odd □ – □ – □ – □ – □ – □ – □ very natural
   - Comments:

10. A man called Jones always wears his hat if a die he throws in the morning shows a 6. If the sun shines Jones throws the die once. If it is raining he throws the die until he gets a 6. So he always wears his hat if it is raining. Additionally, he wears his bow-tie if it is raining, or if the sun shines but it is his birthday.
We know that it is not raining, and we see Jones wearing neither hat nor bow-tie. I say:

"If Jones was wearing his hat, or his bow-tie, or both, then it would be raining."

- Is this sentence true or false ? true □ false □
- How natural do you think this sentence is ?
very odd □ – □ – □ – □ – □ – □ – □ very natural
- Comments:

11. Charlie and his little sister Mary play the following game every day to determine what Charlie gets for dessert: They look outside the window for 5 minutes, to see if a red car passes. Afterwards Charlie asks his sister what he gets for dessert. If there was no red car she will always only let him have an apple. If there was a red car she will most of the times let him have cake, but sometimes he still only gets an apple and she keeps the cake to herself.
Charlie's mother, who knows that today no red car drove by and that Charlie is having an apple, says to his father:

"If there had been a red car passing by, or Mary had said 'cake', then he would be having cake."

- Is this sentence true or false ? true □ false □
- How natural do you think this sentence is ?
very odd □ – □ – □ – □ – □ – □ – □ very natural
- Comments:

12. King Louis of France has a castle. If he is at home then the flag in the castle is up and torches are lit around the castle. If he is not at home then the flag is down and the torches are out. On his birthday King Louis always goes to his friend's, King Arthur's, castle to celebrate, but the flag is nevertheless up. The torches however are not lit in that case.
Now suppose two farmers, who know all this, walk by the castle. They don't

know whether it is the King's birthday or not, but they see the flag being down and the torches out. One farmer says to the other:

"If the flag was up, or the flag was up and the torches were lit, then the King would be home."

- Is this sentence true or false ? true □ false □
- How natural do you think this sentence is ?
  very odd □ – □ – □ – □ – □ – □ – □ very natural
- Comments:

13. On Mondays, Charlie will only eat white-chocolate-blueberry cake. He meets a witch who most of the time transforms things into white-chocolate-blueberry cake but sometimes she misses and produces only white-chocolate or only blueberry cake, or even ice-cream.
    It is a Monday, and the witch just transformed a piece of wood into ice-cream. Charlie's little sister Mary says:

    "If she had transformed it into a white-chocolate or a blueberry cake, or a white-chocolate-blueberry cake, he would have eaten it up immediately."

    - Is this sentence true or false ? true □ false □
    - How natural do you think this sentence is ?
      very odd □ – □ – □ – □ – □ – □ – □ very natural
    - Comments:

References

Martin Aher (2012). Free Choice in Deontic Inquisitive Semantics (DIS). In: Maria Aloni et. al. (ed.), *Logic, Language and Meaning*, Lecture Notes in Computer Science, pp. 22 – 31. Springer Berlin-Heidelberg.

Maria Aloni (2007). Free choice and exhaustification: an account of subtrigging effects. In: Estela Puig-Waldmueller (ed.), *Proceedings of Sinn und Bedeutung 11.*

Luis Alonso-Ovalle (2005). Distributing the disjuncts over the modal space. *Proceedings of the North-Eastern Linguistic Society (NELS)*, 35(1).

Luis Alonso-Ovalle (2006). *Disjunction in alternative semantics.* Ph.D. thesis, University of Massachussets Amherst.

Luis Alonso-Ovalle (2008). Innocent exclusion in an alternative semantics. *Natural Language Semantics*, 16(2):115 – 128.

Gennaro Chierchia, Danny Fox and Benjamin Spector (2007). Hurford's constraint and the theory of scalar implicatures: Evidence for embedded implicatures. In: Paul Egre and Giorgio Magri (eds.), *Presuppositions and Implicatures.*

Ivano Ciardelli, Jeroen Groenendijk and Floris Roelofsen (2012). Inquisitive semantics. *NASSLLI 2012 lecture notes.*

Ivano Ciardelli, Jeroen Groenendijk and Floris Roelofsen (2013). Inquisitive semantics: A new notion of meaning. *Language and Linguistics Compass*, 7(9):459 – 476.

Ivano Ciardelli and Floris Roelofsen (2011). Inquisitive logic. *Journal of Philosophical Logic*, 40:55 – 94.

L. T. F. Gamut (1991). *Logic, Language and Meaning*, vol. 2. University of Chicago Press.

Charles L. Hamblin (1973). Questions in montague english. *Foundations of Language*, 10:41 – 53.

Roni Katzir (2007). Structurally-defined alternatives. *Linguistics and Philosophy*, 30(6):669 – 690.

Angelika Kratzer and Junko Shimoyama (2002). Indeterminate pronouns: The view from japanese. *The Proceedings of the Third Tokyo Conference on Psycholinguistics (TCP 2002)*, pp. 1 – 25.

Reinhard Muskens (2011). Type-Logical Semantics. In: E. Craig (ed.), *Routledge Encyclopedia of Philosophy Online*. Routledge.

Donald Nute (1975). Counterfactuals. *Notre Dame Journal of Formal Logic*, 16(4):476 – 482.

Barbara Partee and Mats Rooth (1982). Generalized Conjunction and Type Ambiguity. In: Arnim von Stechow (ed.), *Meaning, Use and Interpretation.* de Gruyter.

Floris Roelofsen (2013a). Algebraic foundations for the semantic treatment of inquisitive content. *Synthese*, pp. 1–24.

Floris Roelofsen (2013b). A bare bone attentive semantics for might. In: Maria Aloni, Michael Franke and Floris Roelofsen (eds.), *The dynamic, inquisitive, and visionary life of œÜ, ?œÜ, and ,óäœÜ: a festschrift for Jeroen Groenendijk, Martin Stokhof, and Frank Veltman*, pp. 190 – 215.

Raj Singh (2008). On the interpretation of disjunction: Asymmetric, Incremental and eager for Inconsistency. *Linguistics and Philosophy*, 31(2):245 – 260.

Frank Veltman (2005). Making counterfactual assumptions. *Journal of Semantics*, 22:159 – 180.

Matthijs Westera (2013). Exhaustivity through the maxim of relation. *Proceedings of Logic and Engineering of Natural Language Semantics (LENLS).*