Hybrid, Classical, and Presuppositional Inquisitive Semantics

Ivano Ciardelli and Jeroen Groenendijk

Based on joint work with Floris Roelofsen

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Overview

Part One

1. Inquisitive meaning
2. Hybrid basic inquisitive semantics InqB
3. Classical erotetic languages
4. Classical inquisitive semantics InqA
5. Comparison of InqA and InqB

Part Two

1. Presuppositional inquisitive semantics InqP
2. The logic of InqP
3. A derivation system for InqP
4. Completeness proof
5. Conclusions

Informativeness and inquisitiveness

Two components of meaning

- Informative content — the information provided by a sentence
- Inquisitive content — the issue raised by a sentence

Informative content

- The informative content of a sentence φ is modeled, as usual, as a set of worlds $|\varphi|$.

Definition (Informativeness)

Let $\varphi \in \mathcal{L}$, and ω the set of suitable worlds for $\mathcal{L}$

- φ is informative iff $|\varphi| \neq \omega$

Inquisitive content (definitions)

Definition (Issues)

Let s be a set of worlds.

- An issue over s is a downward closed set $\mathcal{I}$ of subsets of s.
  I.e., if $t \in \mathcal{I}$ and $u \subseteq t$, then $u \in \mathcal{I}$.
- An issue $\mathcal{I}$ over s is unbiased iff $\mathcal{I}$ is a cover of s, i.e., $s = \bigcup \mathcal{I}$. Otherwise, $\mathcal{I}$ is called biased.

Definition (Inquisitive content)

- The inquisitive content of φ, $[\varphi]$ is an issue over $|\varphi|$.

Definition (Inquisitiveness)

- φ is inquisitive iff $|\varphi| \notin [\varphi]$ i.e., $[\varphi] \neq \wp(|\varphi|)$

Inquisitive content (motivation)

- An utterance of a sentence φ is a proposal to accept the information $|\varphi|$ it provides and to settle the issue $[\varphi]$ it raises.
- If a set of worlds $s \in [\varphi]$, then s embodies information that settles the issue raised by φ.

  If $t \subset s$, then t cannot fail to settle the issue as well. (Hence, downward closedness.)
- If $|\varphi| \in [\varphi]$, then nothing beyond accepting the information φ provides is needed to settle the issue it raises.
- So, φ is inquisitive iff more is needed to settle the issue it raises than accepting the information it provides.

Inquisitive meanings

- The inquisitive meaning of a formula φ is the pair $(|\varphi|, [\varphi])$.
- If the inquisitive content $[\varphi]$ of φ is an unbiased issue, then it fully determines its informative content: $|\varphi| = \bigcup[\varphi]$.
- The meaning of φ can then be identified with its inquisitive content $[\varphi]$.
- We call such inquisitive meanings non-presuppositional.
- A semantics is called non-presuppositional in case it assigns to each formula a non-presuppositional meaning.

Inquisitive support

Definition (Informativeness and inquisitiveness in a state)

- An information state s is a set of worlds.
- φ is informative in s iff $s \cap |\varphi| \neq s$
- φ is inquisitive in s iff $s \cap |\varphi| \notin [\varphi]$

Definition (support)

- s supports φ iff φ is neither informative nor inquisitive in s.

Inquisitive support and meaning

Fact (Support and meaning)

- *s supports* φ *iff s* $\in [\varphi]$

Inquisitive support semantics

- If our semantics is non-presuppositional, then $[\varphi]$ completely determines the meaning of φ;
- So, a support definition for a given language uniquely defines a non-presuppositional semantics.
- The meaning $[\varphi]$ of φ in such a system will be defined as the set of all supporting states.

Hybrid basic inquisitive semantics

Language is a standard propositional language

Definition (Semantics of InqB)

1. $s \models p \iff \forall w \in s : w(p) = 1$
2. $s \models \bot \iff s = \emptyset$
3. $s \models \varphi \to \psi \iff \forall t \subseteq s$: if $t \models \varphi$ then $t \models \psi$
4. $s \models \varphi \wedge \psi \iff s \models \varphi$ and $s \models \psi$
5. $s \models \varphi \vee \psi \iff s \models \varphi$ or $s \models \psi$

Definition (abbreviations)

1. $\neg\varphi := \varphi \to \bot$
2. $!\varphi := \neg\neg\varphi$ (non-inquisitive closure)
3. $?\varphi := \varphi \vee \neg\varphi$ (non-informative closure)

Inquisitive meanings and informative content in InqB

Definition (Meanings in InqB)

- The meaning of φ in InqB is $[\varphi] = \{s \subseteq \omega \mid s \models \varphi\}$;
- This determines the informative content $|\varphi| = \bigcup[\varphi]$.

Fact

- *Persistence: if $s \models \varphi$, then for every $t \subseteq s$: $t \models \varphi$.*
- *Classical behavior of singletons: $\{v\} \models \varphi$ iff $v \models_{cl} \varphi$.*

Information is treated classically

- These facts guarantee that $|\varphi|$ coincides with the set of worlds where φ is true.

Three semantic categories

Definition (Assertions, questions, and hybrids)

- φ is an assertion iff φ is not inquisitive
- φ is a question iff φ is not informative
- φ is a hybrid iff φ is informative and inquisitive

Tautologies

- φ is a question in InqB iff φ is a classical tautology.
- φ is a tautology in InqB iff φ is neither informative nor inquisitive.
- Inquisitive semantics enriches the notion of meaning (in a conservative way).
- Though being not informative, a sentence can still be meaningful in InqB by being inquisitive.

Disjunction is inquisitive

Fact (Hybrid disjunction)

- *$p \vee q$ is a hybrid sentence*
- *$p \vee q$ is informative: $|p \vee q| \neq \omega$*
- *$p \vee q$ is inquisitive: $|p \vee q| \notin [p \vee q]$*

Fact (Inquisitive question)

- *$?p = p \vee \neg p$ is an inquisitive question*
- *$p \vee \neg p$ is not informative: $|p \vee \neg p| = \omega$*
- *$p \vee q$ is inquisitive: $|p \vee \neg p| \notin [p \vee \neg p]$*

Closure operators

Fact (Negation, assertions, questions)

- *$\neg\varphi$ is an assertion*
- *$!\varphi$ is an assertion*
- *$?\varphi$ is a question*

Fact (Non-informative and non-inquisitive closure)

- *φ is an assertion iff $\varphi \equiv\, !\varphi$*
- *φ is a question iff $\varphi \equiv\, ?\varphi$*

Fact (Division)

- $\varphi \equiv\, !\varphi \wedge ?\varphi$

Conditional questions in InqB

Conditional assertion, question, and hybrid

- $s \models p \to q \iff s \subseteq |p \to q| \iff s \cap |p| \subseteq |q|$
- $s \models p \to ?q \iff s \models p \to q$ or $s \models p \to \neg q$
- $s \models p \to (q \vee r) \iff s \models p \to q$ or $s \models p \to r$

Conditional question with inquisitive antecedent

- $s \models (p \vee q) \to ?r \iff s \models (p \vee q) \to r$, or $s \models (p \vee q) \to \neg r$, or $s \models (p \to r) \wedge (q \to \neg r)$, or $s \models (p \to \neg r) \wedge (q \to r)$

Alternative and choice questions in InqB

Alternative question

- $s \models ?(p \vee q) \iff s \models p$ or $s \models q$ or $s \models \neg p \wedge \neg q$

Choice question

- $s \models ?p \vee ?q \iff s \models p$ or $s \models \neg p$ or $s \models q$ or $s \models \neg q$

Qualms

- Is InqB's representation of alternative questions fully adequate?
- Do choice questions surface in natural language as disjunctions of interrogative sentences?
- Is disjunction in natural language really semantically inquisitive?

The status of InqB

- InqB is a basic logical system to model inquisitiveness, on a par with informativeness, which is dealt with classically.
- There is no claim that a direct and perfect surface correspondence exists between specific sentences of the logical language and specific sentences of a specific natural language.
- The inherent claim is that there is a fundamental correspondence between the interpretation of the semantic operations in the logical language and constructions in natural language that involve informative and inquisitive content.
- Inquisitive semantics is to serve as a logical analytical tool in the study of meaning in natural language.

Classical erotetic languages

- In InqB the syntax of the logical language is standard, the meanings are enriched with inquisitive content.
- Unlike in most natural languages, and in most erotetic logics, in InqB no syntactic distinction is made between interrogatives and indicatives.

Indicatives and interrogatives

- We will consider a system InqA in which we do distinguish two syntactic categories of indicatives $\mathcal{L}_!$ and of interrogatives $\mathcal{L}_?$.
- For every sentence $\varphi \in \mathcal{L}$: $\varphi \in \mathcal{L}_! \cup \mathcal{L}_?$, and for no sentence $\varphi \in \mathcal{L}$: $\varphi \in L_! \cap \mathcal{L}_?$.
- In InqA all indicatives are assertions, all interrogatives are questions, and no no hybrid single sentences occur in $\mathcal{L}$.

Sufficient conditions for assertion- and questionhood in InqB

1. p is an informative assertion, for all atomic sentences p
2. $\bot$ is an informative assertion
3. If φ and ψ are assertions, then $\varphi \wedge \psi$ is an assertion
   If φ and ψ are questions, then $\varphi \wedge \psi$ is a question
4. If ψ is an assertion, then $\varphi \to \psi$ is an assertion
   If ψ is a question, then $\varphi \to \psi$ is a question
5. If either φ or ψ is a question, then $\varphi \vee \psi$ is a question

Fact (Disjunction is the only source of inquisitiveness in InqB)

In the disjunction-free fragment of InqB all sentences are assertions.

Notational convention

- α, β, γ denote indicatives, and Γ, Δ sets of indicatives;
- μ, ν, λ denote interrogatives, and Λ a set of interrogatives;
- φ, ψ, χ denote generic formulas, and Φ a set of generic formulas.

Classical erotetic language

Definition (Bi-categorial syntax of InqA)

1. $\alpha \in \mathcal{L}_!$, for all atomic sentences α
2. $\bot \in \mathcal{L}_!$
3. If Γ is a finite subset of $\mathcal{L}_!$, then $?\Gamma \in \mathcal{L}_?$
4. If $\alpha \in \mathcal{L}_!$ and $\varphi \in \mathcal{L}_{c \in \{!,?\}}$, then $(\alpha \to \varphi) \in \mathcal{L}_c$
5. If $\varphi, \psi \in \mathcal{L}_{c \in \{!,?\}}$, then $(\varphi \wedge \psi) \in \mathcal{L}_c$
6. If Φ is a finite subset of $\mathcal{L}_! \cup \mathcal{L}_?$, then $\Phi \in \mathcal{L}$

Hybrids can only be constructed in $\mathcal{L}$ as sets of non-hybrid single sentences. (Clause 6.)

Definition (Classical abbreviations)

1. $\neg\alpha := (\alpha \to \bot)$
2. $(\alpha \vee \beta) := \neg(\neg\alpha \wedge \neg\beta)$

Classical inquisitive semantics

Definition (Semantics of InqA)

1. $s \models p \iff \forall w \in s : w(p) = 1$
2. $s \models \bot \iff s = \emptyset$
3. $s \models ?\Gamma \iff \exists \alpha \in \Gamma : s \models \alpha$, or
   $\forall \alpha \in \Gamma : \forall t \subseteq s :$ if $t \models \alpha$, then $t = \emptyset$
4. $s \models \alpha \to \varphi \iff \forall t \subseteq s :$ if $t \models \alpha$ then $t \models \varphi$
5. $s \models \varphi \land \psi \iff s \models \varphi$ and $s \models \psi$
6. $s \models \Phi \iff \forall \varphi \in \Phi : s \models \varphi$

Basic questions

- $s \models ?\Gamma \iff \exists \alpha \in \Gamma : s \models \alpha$, or $\forall \alpha \in \Gamma : s \models \neg\alpha$

Inquisitive meanings and informative content in InqA

Definition (Meanings in InqA)

- The meaning of φ in InqA is $[\varphi] = \{s \subseteq \omega \mid s \models \varphi\}$;
- This determines the informative content $|\varphi| = \bigcup[\varphi]$.

Information is treated classically

- The informative content $|\alpha|$ of an indicative coincides with the set of worlds where α is true.
- The informative content $|\mu|$ of an interrogative is always trivial, that is, $|\mu| = \omega$.

Classical inquisitive semantics, simplified

Definition (Semantics of InqA)

1. $s \models p \iff s \subseteq |p|$
2. $s \models \bot \iff s = \emptyset$
3. $s \models ?\Gamma \iff \exists\alpha \in \Gamma: s \subseteq |\alpha|$, or $\forall\alpha \in \Gamma: s \cap |\alpha| = \emptyset$
4. $s \models \alpha \to \varphi \iff s \cap |\alpha| \models \varphi$
5. $s \models \varphi \wedge \psi \iff s \models \varphi$ and $s \models \psi$
6. $s \models \Phi \iff \forall\varphi \in \Phi: s \models \varphi$

The semantics of basic questions in InqA

Examples

- $s \models ?\{p\} \iff s \models p$ or $s \models \neg p$

  $?\{p\} \equiv ?\{p, \neg p\} \equiv ?\{\neg p\}$
- $s \models ?\{p, q\} \iff s \models p$ or $s \models q$, or ($s \models \neg p$ and $s \models \neg q$)

  $?\{p, q\} \equiv ?\{p, q, \neg p \wedge \neg q\}$

Comment

- Since the interrogative $?\{p, q\}$ is to be a question, is has to be non-informative. The disjunct marked in red takes care of that.
- If we read $?\{p, q\}$ as an alternative question, it may be observed that the answers p and q do not have the same status as the answer $\neg p \wedge \neg q$.
- Already for the polar questions $?\{p\}$ and $?\{\neg p\}$ it might be argued that they are not necessarily fully equivalent.

Comparison of InqA and InqB

Meaning preserving translations

- There is a straightforward translation procedure that transforms any finite set of sentences in InqA into a single equivalent conjunction of sentences in InqB
- Conversely, using the division fact $\varphi \equiv !\varphi \wedge ?\varphi$, any single sentence φ of InqB can be turned into an equivalent set $\{\alpha_\varphi, \mu_\varphi\}$ of two sentences of InqA, where:
    - α_φ is an indicative equivalent to $!\varphi$
    - μ_φ is an interrogative equivalent to $?\varphi$

Examples

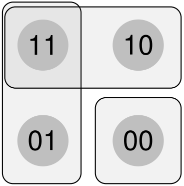
- The hybrid disjunction $p \vee q$ in InqB is equivalent with the set of two sentences $\{p \vee q, ?\{p, q\}\}$ in InqA.
- The conditional question $(p \vee q) \to ?r$ in InqB is equivalent with the basic question $?\{(p \vee q) \to r, (p \vee q) \to \neg r,$ $(p \to r) \wedge (q \to \neg r), (p \to \neg r) \wedge (q \to r)\}$ in InqA.

Conclusions first part

- Inquisitive semantics is a general erotetic semantic framework
- It is not inherently linked to a mono-categorial language or inquisitive disjunction
- It can just as well be used in combination with bi-categorial languages
- The inquisitive semantic framework can be used as a tool to compare different erotetic systems

Towards a presuppositional system

Consider an alternative question like $?\{p, q\}$.

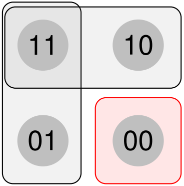


$[?\{p, q\}]$

Towards a presuppositional system

Consider an alternative question like $?\{p, q\}$.

- Unlike p and q, the response $\neg(p \vee q)$ does not seem to be invited by $?\{p, q\}$



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Towards a presuppositional system

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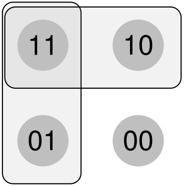
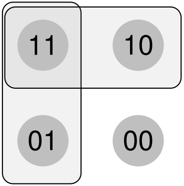


$[?\{p, q\}]$

Towards a presuppositional system

Consider an alternative question like $?\{p, q\}$.

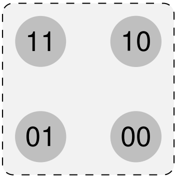
- Unlike p and q, the response $\neg(p \vee q)$ does not seem to be invited by $?\{p, q\}$
- The picture we would really like to have is this one.
- But then, since $|\varphi| = \bigcup[\varphi]$, $?\{p, q\}$ would turn out informative.



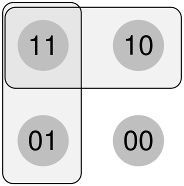
$[?\{p, q\}]$

Towards a presuppositional system

- We need to disassociate the informative content $|\varphi|$ of a formula from its inquisitive content $[\varphi]$.
- Meaning $[\![\varphi]\!]$ will consist of the pair $(|\varphi|, [\varphi])$.



$|?\{p, q\}|$



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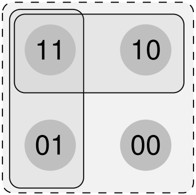

$[\![?\{p, q\}]\!]$

The system InqP

1. We leave untouched the notion of informative content.
   - Stipulating that an interrogative is true in any world, the informative content $|\varphi|$ can be seen as the truth-set of φ.
2. We simplify the support definition so that $?\Gamma$ is only satisfied by establishing one of the indicatives $\alpha \in \Gamma$.
   - $s \models p \iff s \subseteq |p|$
   - $s \models \bot \iff s = \emptyset$
   - $s \models ?\Gamma \iff \exists\alpha \in \Gamma : s \subseteq |\alpha|$, or $\forall\alpha \in \Gamma : s \cap |\alpha| = \emptyset$
   - $s \models \alpha \to \varphi \iff s \cap |\alpha| \models \varphi$
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   - $s \models \Phi \iff \forall \varphi \in \Phi : s \models \varphi$

We denote by $[\varphi]$ the set of states supporting φ.

The system InqP

Meanings

$$[\![\varphi]\!] = (|\varphi|, [\varphi])$$

Definitions

- $|\varphi|$ is the informative content of φ
- $[\varphi]$ is the inquisitive content of φ
- $\pi(\varphi) = \bigcup[\varphi]$ is the presupposition of φ

The system InqP

Definitions

- φ is informative if $|\varphi| \neq \omega$.
- φ is inquisitive if $|\varphi| \notin [\varphi]$.
- φ is a question if it is not informative.
- φ is an assertion if it is not inquisitive.

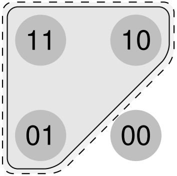
Fact

Indicatives are assertions, interrogatives are questions.

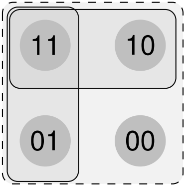
The system InqP

Definition

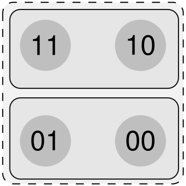
φ is presuppositional in case $|\varphi| \neq \pi(\varphi)$



$[\![p \vee q]\!]$



$[\![?\{p, q\}]\!]$



$?\{p, \neg p\}$

The logic of InqP

Entailment

- $\Phi \models_{\text{info}} \psi \iff$ whenever $w \models \varphi$ for all $\varphi \in \Phi$, $w \models \psi$.
- $\Phi \models_{\text{inq}} \psi \iff$ whenever $s \models \varphi$ for all $\varphi \in \Phi$, $s \models \psi$.
- $\Phi \models \psi \iff \Phi \models_{\text{info}} \psi$ and $\Phi \models_{\text{inq}} \psi$.

Deduction theorem

$\Phi, \alpha \models \psi \iff \Phi \models \alpha \rightarrow \psi$.

Compactness

If $\Phi \models \psi$ there is a finite $\Phi_0 \subseteq \Phi$ s.t. $\Phi_0 \models \psi$.

Split

If $\Gamma \models ?\Delta$, then $\Gamma \models \alpha$ for some $\alpha \in \Delta$.

The logic of InqP

What does entailment mean?

- $\Gamma \models \alpha$: amounts to classical entailment.
- $\Gamma \models \mu$: Γ provides enough information to settle μ.

$$p \wedge q \models ?\{p, q\}$$

- $\Lambda \models \mu$: μ can be reduced to Λ.

$$?\{p, \neg p\} \models q \rightarrow ?\{p, \neg p\}$$

- $\Gamma, \Lambda \models \alpha \iff \Gamma \models \alpha$.
- $\Gamma, \Lambda \models \mu$: Γ provides enough information to reduce μ to Λ.

$$\neg r, ?\{p, q, r\} \models ?\{p, q\}$$

A derivation system for InqP

Start from a natural deduction system for classical logic.

Conjunction

$$\frac{\alpha \quad \beta}{\alpha \wedge \beta} \qquad \frac{\alpha \wedge \beta}{\alpha} \quad \frac{\alpha \wedge \beta}{\beta}$$

Disjunction

$$\frac{\alpha}{\alpha \vee \beta} \quad \frac{\beta}{\alpha \vee \beta} \quad \frac{\begin{matrix}[\alpha] \\ \vdots \\ \gamma\end{matrix} \quad \begin{matrix}[\beta] \\ \vdots \\ \gamma\end{matrix} \quad \alpha \vee \beta}{\gamma}$$

Falsum

-

$$\frac{\bot}{\alpha}$$

Implication

$$\frac{\begin{matrix}[\alpha] \\ \vdots \\ \beta\end{matrix}}{\alpha \to \beta} \qquad \frac{\alpha \quad \alpha \to \beta}{\beta}$$

Negation

$$\frac{\begin{matrix}[\alpha] \\ \vdots \\ \bot\end{matrix}}{\neg\alpha} \qquad \frac{\alpha \quad \neg\alpha}{\bot}$$

Double negation

$$\frac{\neg\neg\alpha}{\alpha}$$

A derivation system for InqP

Extend the rules for conjunction and implication to deal with conjunctive and conditional interrogatives.

Conjunction

$$\frac{\alpha \quad \beta}{\alpha \wedge \beta} \qquad \frac{\alpha \wedge \beta}{\alpha} \quad \frac{\alpha \wedge \beta}{\beta}$$

Disjunction

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Double negation

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A derivation system for InqP

Extend the rules for conjunction and implication to deal with conjunctive and conditional interrogatives.

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$$\frac{\varphi \quad \psi}{\varphi \wedge \psi} \qquad \frac{\varphi \wedge \psi}{\varphi} \quad \frac{\varphi \wedge \psi}{\psi}$$

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$$\frac{\alpha}{\alpha \vee \beta} \quad \frac{\beta}{\alpha \vee \beta} \quad \frac{\begin{matrix}[\alpha] \\ \vdots \\ \gamma\end{matrix} \quad \begin{matrix}[\beta] \\ \vdots \\ \gamma\end{matrix} \quad \alpha \vee \beta}{\gamma}$$

Falsum

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$$\frac{\bot}{\varphi}$$

Implication

$$\frac{\begin{matrix}[\alpha] \\ \vdots \\ \varphi\end{matrix}}{\alpha \to \varphi} \qquad \frac{\alpha \quad \alpha \to \varphi}{\varphi}$$

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$$\frac{\begin{matrix}[\alpha] \\ \vdots \\ \bot\end{matrix}}{\neg\alpha} \qquad \frac{\alpha \quad \neg\alpha}{\bot}$$

Double negation

$$\frac{\neg\neg\varphi}{\varphi}$$

A derivation system for InqP

Give rules for the interrogative operator

Introduction

$$\frac{\alpha_i}{?\{\alpha_1, \ldots, \alpha_n\}}$$

Elimination

$$\frac{\begin{matrix}[\alpha_1] \\ \vdots \\ \mu\end{matrix} \quad \ldots \quad \begin{matrix}[\alpha_n] \\ \vdots \\ \mu\end{matrix} \quad ?\{\alpha_1, \ldots, \alpha_n\}}{\mu}$$

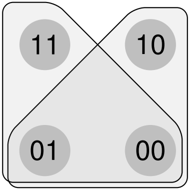
Remark

Logically, ? is *almost* a disjunction.

A derivation system for InqP

$$\alpha \rightarrow ?\{\beta_1, \ldots, \beta_m\} \equiv ?\{\alpha \rightarrow \beta_1, \ldots, \alpha \rightarrow \beta_m\}$$

- This is not provable using only the rules for ? and implication.



$[\![p \rightarrow ?\{q, \neg q\}]\!] =$
$[\![?\{p \rightarrow q, p \rightarrow \neg q\}]\!]$

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- We add the KP rule

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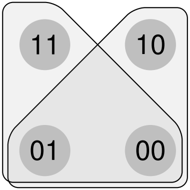

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A derivation system for InqP

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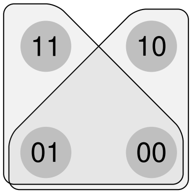
- We add the KP rule

$$\frac{\alpha \to ?\{\beta_1, \dots, \beta_m\}}{?\{\alpha \to \beta_1, \dots, \alpha \to \beta_m\}}$$

- Analogous to the Kreisel-Putnam rule of InqB:

$$\frac{\neg\varphi \to (\psi \vee \chi)}{(\neg\varphi \to \psi) \vee (\neg\varphi \to \chi)}$$



$[\![p \to ?\{q, \neg q\}]\!] =$
$[\![?\{p \to q, p \to \neg q\}]\!]$

Completeness proof

Lemma

Any interrogative μ is provably equivalent to a basic one.

Proof

By induction on μ.

1. μ basic: trivial.
2. $\mu = \nu \wedge \lambda$. If $\nu \equiv_P ?\{\alpha_1, \ldots, \alpha_n\}$ and $\lambda \equiv_P ?\{\beta_1, \ldots, \beta_m\}$, then:

$$\mu \equiv_P ?\{\, \alpha_i \wedge \beta_j \,|\, 1 \leq i \leq n, 1 \leq j \leq m \,\}$$

3. $\mu = \alpha \rightarrow \nu$. If $\nu \equiv_P ?\{\beta_1, \ldots, \beta_m\}$, then using the KP rule:

$$\mu \equiv_P ?\{\, \alpha \rightarrow \beta_1, \ldots, \alpha \rightarrow \beta_m \,\}$$

Completeness proof

- Suppose $\Phi \models \psi$.
- By compactness, we may assume Φ is finite. Write $\Phi = \Gamma \cup \Lambda$.
- We can immediately get rid of the case in which ψ is an assertion α.
- For, in this case $\Gamma, \Lambda \models \alpha$ is equivalent to $\Gamma \models \alpha$.
- Then $\Gamma \vdash \alpha$ by the completeness theorem for classical logic
- So we may assume that ψ is an interrogative μ.
- Let $\gamma = \bigwedge \Gamma$ and $\lambda = \bigwedge \Lambda$.
- Then $\Phi \models \mu$ is equivalent to $\gamma, \lambda \models \mu$.
- By the deduction theorem $\lambda \models \gamma \to \mu$.

Completeness proof

- By the previous lemma,
  - $\lambda \equiv_P ?\{\alpha_1, \ldots, \alpha_n\}$
  - $\gamma \to \mu \equiv_P ?\{\beta_1, \ldots, \beta_m\}$
- So, $?\{\alpha_1, \ldots, \alpha_n\} \models ?\{\beta_1, \ldots, \beta_m\}$.
- For any i, $\alpha_i \models ?\{\alpha_1, \ldots, \alpha_n\}$, so $\alpha_i \models ?\{\beta_1, \ldots, \beta_m\}$.
- By the split fact remarked above, there must be j such that $\alpha_i \models \beta_j$.
- But since α_i and β_j are indicatives, completeness for indicatives yields $\alpha_i \vdash \beta_j$.

Completeness proof

- By the rule of ?-introduction then, $\alpha_i \vdash ?\{\beta_1, \ldots, \beta_m\}$.
- Since $\alpha_i \vdash ?\{\beta_1, \ldots, \beta_m\}$ for all $1 \leq i \leq n$, the ?-elimination rule may be applied, yielding $?\{\alpha_1, \ldots, \alpha_n\} \vdash ?\{\beta_1, \ldots, \beta_m\}$.
- Recalling that $\lambda \equiv_P ?\{\alpha_1, \ldots, \alpha_n\}$ and $\gamma \to \mu \equiv_P ?\{\beta_1, \ldots, \beta_m\}$, we get $\lambda \vdash \gamma \to \mu$.
- Therefore, $\gamma, \lambda \vdash \mu$.
- But since γ and λ are conjunctions of formulas in Φ we have $\Phi \vdash \gamma$ and $\Phi \vdash \lambda$.
- Hence, $\Phi \vdash \mu$.

Conclusions: two types of meanings

- The goal of inquisitive semantics is to extend the notion of meaning to encompass inquisitive potential.
- A sentence φ provides information by specifying a set $|\varphi|$ of possible worlds.
- A sentence requests information by specifying an issue $[\varphi]$ over $|\varphi|$.
- The meaning of φ consists of the pair $[\![\varphi]\!] = (|\varphi|, [\varphi])$, embodying informative and inquisitive content of φ.

Conclusions: two types of meanings

Non-presuppositional systems

- In the systems InqB and InqA, meanings are assumed to be non-presuppositional: that is, $[\varphi]$ is assumed to be an unbiased issue over $|\varphi|$.
- Since this amounts to $|\varphi| = \bigcup[\varphi]$, the meaning $(|\varphi|, [\varphi])$ of φ in these systems is completely determined by the inquisitive component $[\varphi]$.

Conclusions: two types of meanings

Presuppositional systems

- The restriction to non-presuppositional meanings can be lifted to yield a richer semantic space.
- Presuppositional meanings can be useful to get a more accurate representation of certain NL meanings.
- In a presuppositional system, the issue $[\varphi]$ over $|\varphi|$ may be biased.
- Both components $|\varphi|$ and $[\varphi]$ are necessary to determine the meaning $[\![\varphi]\!] = (|\varphi|, [\varphi])$ of φ.

Conclusions: two types of languages

Once we choose what notion of meaning we want, we also have a choice about what language to use to express such meanings.

- Hybrid, or deep-structure languages:
  - allow for hybrid sentences;
  - connectives express the natural operations on the space of meanings.
- Classical or surface languages:
  - partition sentences into indicatives and interrogatives;
  - connectives are closer to their natural language counterpart.

Conclusions

We may distinguish four systems according to their notion of meaning and to their language.

Lang \ Mean	Non-presuppositional	Presuppositional
Hybrid	InqB	InqQ
Classical	InqA	InqP

Conclusions

We may distinguish four systems according to their notion of meaning and to their language.

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Thanks!