

The logic of interrogation

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Logic and conversation

- ⦿ Standard logic deals with reasoning, entailment
- ⦿ Using standard logic, linguistic semantics deals with phenomena related to entailment
- ⦿ Information exchange more basic use of language than reasoning
- ⦿ Try to make cooperative information exchange a basic notion of logic

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Linguistic aims

- ⦿ Explain linguistic phenomena using the new logical notions
- ⦿ We will give some illustrations
- ⦿ By-product: a better notion of linguistic answerhood (within a partition semantics of questions)

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Overview

- ⦿ The game of interrogation
- ⦿ A query language
- ⦿ Semantics for the language
- ⦿ Logical notions to arbitrate the game
- ⦿ Answerhood
- ⦿ Illustration

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Game of Interrogation

- ⦿ Two players: the interrogator and the witness
- ⦿ The interrogator may only raise issues by asking the witness non-superfluous questions
- ⦿ The witness may only make credible (Quality), non-redundant (Quantity) statements which exclusively address the issues raised by the interrogator (Relation)

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Logic of interrogation

- ⦿ Define logical notions that arbitrate whether an interrogation proceeds in accordance with the rules
- ⦿ Like standard logic defines the notion of entailment to arbitrate whether an argumentation is in accordance with the rules of valid reasoning

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Query Language

differs a bit from paper

- Let PL be a language of predicate logic.
- If φ is a sentence of PL, then $!\varphi$ is a sentence of QL
- If φ is a formula of PL, then $?\varphi$ is a sentence of QL
 - The query operator binds all free variables in φ

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Examples

- Interrogatives ask for the specification of the denotation of an n-place relation ($n \geq 0$)
- $?\exists x Px$
- $?Px$
- $?x=a$
- $?Rxy$

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Proceedings of an interrogation

- Given the strict division of roles, the proceedings of an interrogation can be presented by a sequence of sentences $\varphi_1; \dots; \varphi_n$ from QL
- We don't have to indicate who said what

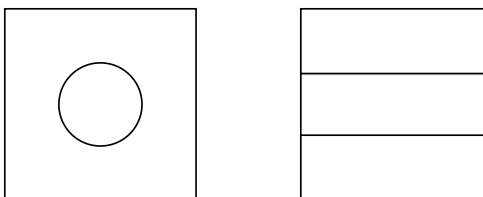
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Denotational semantics

- Standard truth definition for PL
 $\|\varphi\|_{w,g} \in \{1,0\}$
- Interpretation for QL
- $\|!\varphi\|_w = \|\varphi\|_{w,g}$
- $\|?\varphi\|_w = \{v \in W \mid \forall g: \|\varphi\|_{w,g} = 1\}$
- Partition semantics for interrogatives

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Proposition - Question



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Example

- $\|?\!Px\|_w$ is the set of worlds where the denotation of P is the same as in w
- $\|!\!Px\|_w$ is a proposition which exhaustively specifies which objects have the property P
 - So, what you get is the true and complete answer in w

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Update semantics

- In terms of the denotational semantics we define an update semantics for QL
- We define the notion $C[\varphi]$, the effect of updating a context C with an indicative or an interrogative sentence φ
- A context will consist of data (provided by the witness) and issues (raised by the interrogator)

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Data and Issues

- If we would only consider data, a context could be a subset of the set of possible worlds
- $C[!\varphi] \subseteq C$
- Interrogatives provide no data, they may only raise issues
- We model issues by structuring the context

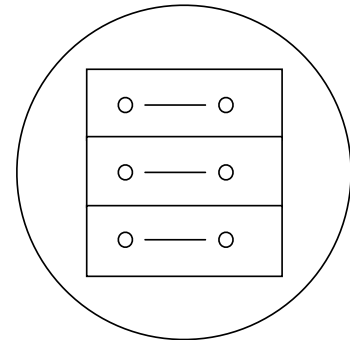
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Structured contexts

- A context C is a symmetric and transitive relation on the set of possible worlds W
- A context C is an equivalence relation on a subset of W
- If two worlds w and v are related in C , $\langle w, v \rangle \in C$, the difference between w and v is not an issue
- Notation: by $w \in C$ we mean $\langle w, w \rangle \in C$

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Picture of context



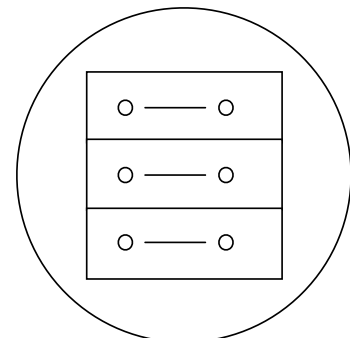
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Updating contexts

- $C[!\varphi] = \{ \langle w, v \rangle \in C \mid \|\varphi\|_w = \|\varphi\|_v = 1 \}$
- $C[?\varphi] = \{ \langle w, v \rangle \in C \mid \|\varphi\|_w = \|\varphi\|_v \}$
- For $\tau = \varphi_1; \dots; \varphi_n$, $C[\tau] = C[\varphi_1] \dots [\varphi_n]$

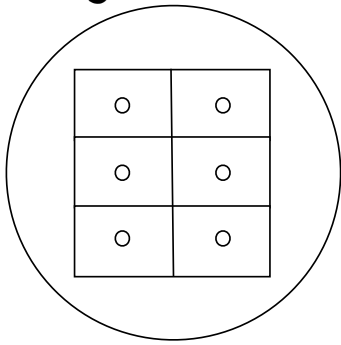
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Picture of context



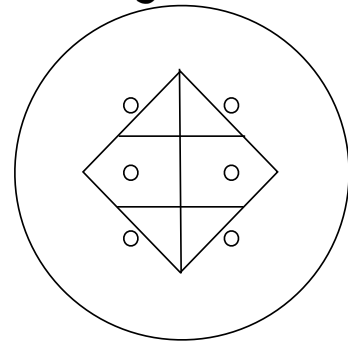
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Adding an issue



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Adding data



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Consistency

- ⦿ φ is consistent with τ iff $\exists C: C[\tau][\varphi] \neq \emptyset$
- ⦿ Only indicatives can be inconsistent with the context
- ⦿ Consistency is the logical notion used to arbitrate credibility of the witness
- ⦿ The witness is judged credible as long as he doesn't contradict himself

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Informativeness

- ⦿ τ entails φ iff $\forall C: C[\tau] = C[\tau][\varphi]$
- ⦿ φ informative after τ iff τ does not entail φ
- ⦿ Both indicatives and interrogatives can be uninformative
- ⦿ Informativeness is the logical notion used to arbitrate whether statements are non-redundant, and questions are not superfluous

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Examples entailment

- ⦿ $?Px$ entails $?Pa$ and $? \exists x Px$
- ⦿ $! \forall x (Px \leftrightarrow x=a)$ entails $?Px$
Corresponds to 'complete answerhood' in partition semantics
- ⦿ Note: allows for over-informative answers
- ⦿ $? \varphi$ entails $! \psi$ iff $! \psi$ is a tautology (or a presupposition of $? \varphi$)

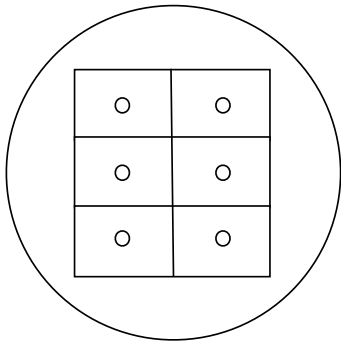
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Licensing

- ⦿ τ licenses φ iff $\forall C, w, v: \langle w, v \rangle \in C[\tau] \& w \notin C[\tau][\varphi] \Rightarrow v \notin C[\tau][\varphi]$
- ⦿ If φ eliminates a world from the context, it should eliminate the whole alternative to which that world belongs
- ⦿ Licensing is the logical notion used to arbitrate whether the witness exclusively addresses the issues raised by the interrogator

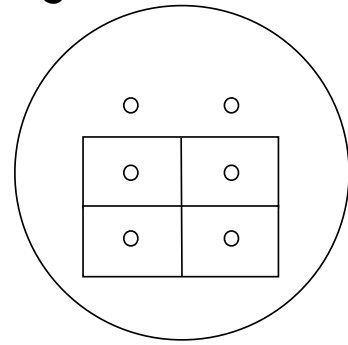
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Picture of context



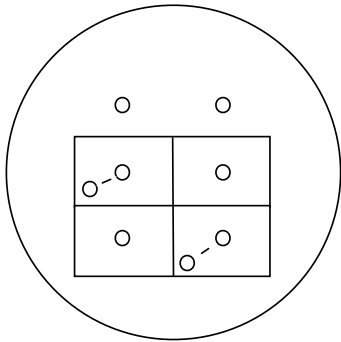
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Adding relevant data



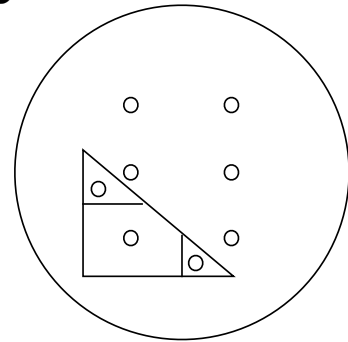
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Picture of context



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Being over-informative



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Remarks on Licensing

- ⦿ Licensing is the crucial new logical notion
- ⦿ It is typically the formulation of the semantics in update format that gives rise to it
- ⦿ The way the notion is defined here is inherently linked to the partition view
 - ⦿ With overlapping alternatives it does not work anymore

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Remarks on Licensing

- ⦿ Licensing only deals with relatedness of assertions to questions
- ⦿ Since questions do not eliminate worlds, questions are always licensed
- ⦿ Relatedness of one question to another is rather captured by entailment, which in partition semantics corresponds to the notion of a subquestion
- ⦿ Rules of the game prohibit subquestions

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Fact about Licensing

- ⦿ τ licenses $!\varphi$ iff τ entails $?\varphi$
- ⦿ An indicative is licensed by the context iff the corresponding polar interrogative is part of the issues raised in the context
 - ⦿ Note that this means that from a logical perspective the notion of licensing is superfluous, entailment can do the job

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Pertinence

- ⦿ φ pertinent after τ iff φ is consistent with τ ; φ is informative after τ ; and φ is licensed by τ
- ⦿ Quality, Quantity and Relation
- ⦿ The logical notion of pertinence arbitrates whether an interrogation is in accordance with the rules of the game

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Fact about pertinence

- ⦿ $!\varphi$ pertinent after τ iff $!\neg\varphi$ pertinent after τ
- ⦿ $!\varphi$ pertinent after τ iff τ entails $?\varphi$
- ⦿ Pertinence of an indicative presupposes the corresponding polar question

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Answerhood

- ⦿ $!\psi$ is a pertinent answer to $?\varphi$ iff $!\psi$ is pertinent after $?\varphi$
- ⦿ Allows for partial answers, but not for over-informative answers
- ⦿ Let $!\psi$ and $!\chi$ be pertinent answers to $?\varphi$. $!\psi$ is a more informative answer to $?\varphi$ than $!\chi$ iff ψ entails χ (and not vice versa)
- ⦿ Comparing answers nice and easy!

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Examples answers

- ⦿ Pertinent answers to $?Px$
- ⦿ $!Pa$
- ⦿ $!\neg Pa$
- ⦿ $!(Pa \wedge Pb)$
- ⦿ $!\forall x Px$
- ⦿ $!\forall x (Px \leftrightarrow x=a)$

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Illustration

Alf rescued Bea. And No-one else.

Ambiguous:

Rab; $\neg\exists x(Rxb \wedge x \neq a)$

Rab; $\neg\exists x(Rax \wedge x \neq b)$

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Illustration

(Who rescued Bea?)

Alf rescued Bea. And No-one else

Ambiguity resolved:

$Rab; \neg \exists x(Rxb \wedge x \neq a)$

$Rab; \neg \exists x(Rax \wedge x \neq b)$ Explanation:

Not licensed after ?Rxb; Rab

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Illustration

(Whom did Alf rescue?)

Alf rescued Bea. And No-one else

Ambiguity resolved:

$Rab; \neg \exists x(Rxb \wedge x \neq a)$

$Rab; \neg \exists x(Rax \wedge x \neq b)$ Explanation:

Not licensed after ?Rax; Rab

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Illustration

(Whom did Alf rescue?)

Alf rescued Bea. And, actually, no-one else

Ambiguity returns:

$Rab; \neg \exists x(Rxb \wedge x \neq a)$

$Rab; \neg \exists x(Rax \wedge x \neq b)$

Presupposition of addressing existing issue is cancelled

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Presupposing an issue

Alf rescued Bea

presupposes

Did Alf rescue Bea?

preserved under
negation

Alf rescued Bea

presupposes

Who rescued Bea?

Alf did not rescue Bea

presupposes

Who rescued Bea?

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Only

Who rescued Bea? Only Alf rescued Bea.

$?Rxb; Rab \wedge \neg \exists x(Rxb \wedge x \neq a)$

Who rescued Bea? *Alf rescued only Bea

$?Rxb; Rab \wedge \neg \exists x(Rax \wedge x \neq b)$

Not a pertinent answer

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A remaining issue?

Did someone rescue Bea?

Yes. Alf rescued Bea.

Is this equally correct if the 'Yes' is missing?

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Conclusion

- Enriching the notion of meaning to embody both information and issues opens a new perspective on dealing with pragmatic issues in rather standard logical terms
- The notion of licensing embodies a very strict logical notion of relatedness to the context, but the illustrations suggest that such a strict notion is linguistically relevant

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Looking ahead

- On all levels, the system is rather restricted
 - The game is very limited and artificial
 - Even as a first order query language the language is poor as compared to natural language
 - The idea that a new perspective on the notion of meaning is at stake does not really play a role

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Data and issues

- In our language providing data and raising issues is divided over two different categories of sentences
- It might be interesting to look at hybrid cases, where e.g. an indicative sentence (implicitly) raises an issue as well
 - Someone came to visit me yesterday
 - Who was it?

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Language

- Things that could be added:
 - Questions as subformulas
 - Conditional questions
 - Which questions
 - What happens to the partition view?

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Game

- Turn the game into a more realistic dialogue game, where really exchange of information plays a role
- Extend relatedness/licensing to questions as well
- Allow for critical moves in the game: denial, doubt

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