Suppositional inquisitive semantics

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1. Suppositional inquisitive semantics

1.1. Basic motivation: support, reject, dismiss

Support

- Inquisitive semantics takes sentences to express a proposal to update the common ground of the conversation (CG) in one or more ways.

- The question in (1a) proposes two alternative ways to update the CG, which correspond to the two responses (1b-c).

(1)
  a. If Alf goes to the party, will Bea go too? $p \rightarrow ?q$
  b. If Alf goes, then Bea will go as well. $p \rightarrow q$
  c. If Alf goes, then Bea will not go. $p \rightarrow \neg q$

- Basic inquisitive semantics (InqB) accounts for the intuition that (1b-c) are responses that, *if accepted by the other conversational participants*, yield a CG that supports the question in (1a), settling the proposal that it expresses.

Support and reject

- InqB does not account for the intuition that (1c) rejects the proposal expressed by (1b), and vice versa.

(1) a. If Alf goes to the party, will Bea go too? $p \rightarrow ?q$
  b. If Alf goes, then Bea will go as well. $p \rightarrow q$
  c. If Alf goes, then Bea will not go. $p \rightarrow \neg q$

- Radical inquisitive semantics (InqR) does account for this.

- It achieves this by not only specifying support-conditions, as InqB does, but simultaneously also rejection-conditions.

Support, reject, dismiss

- InqB and InqR do not account for the intuition that (1d) dismisses a supposition that is shared by (1a)-(1c).

(1) a.	If Alf goes to the party, will Bea go too?	$p \rightarrow ?q$
b.	If Alf goes, then Bea will go as well.	$p \rightarrow q$
c.	If Alf goes, then Bea will not go.	$p \rightarrow \neg q$
d.	Alf will not go to the party.	$\neg p$

- This is just as much a way of settling the proposals that these sentences express, on a par with support and rejection.
- Suppositional inq semantics (InqS) aims to characterize when a response suppositionally dismisses a given proposal.
- To achieve this, it does not only specify conditions for support and rejection, but also for supposition dismissal.

Reject and dismiss

- in InqR $\neg p$ both supports and rejects $p \to q$.
- Couldn't *that* mean that $\neg p$ suppositionally dismisses $p \to q$?
- This does not work for slightly more complex examples:

(2)
a. If Alf or Cor goes, Bea will go too. $(p \vee q) \to r$
b. Alf will not go. $\neg p$
c. And if Cor goes, then Bea will not go. $q \to \neg r$

- Intuitively, (2c) rejects (2a), but (2b) does not reject it, but dismisses a supposition of (2a).
- In InqR (2b) does reject (2a), but does not support it.
- Taking: suppositional dismissal = support + rejection, does not account for the fact that (2b) dismisses a supposition of (2a).
- InqS accounts for this, plus for that once (2b) is *accepted*, (2a) is no longer supportable, but is still rejectable, as (2c) shows.

1.2. Basic semantic notions

Some basic notions

- We consider a language $\mathcal{L}$ of propositional logic.
- We let $?\varphi$ be an abbreviation of $\varphi \vee \neg\varphi$
- Sentences are evaluated relative to information states.
- An information state s is set of possible worlds.
- A possible world w is a valuation function that assigns the value 1 or 0 to each atomic sentence in $\mathcal{L}$.
- We use ω to denote the set of all worlds, the ignorant state.
- We refer to the empty set as the absurd or inconsistent state.

Global structure of the semantics

- The semantics for $\mathcal{L}$ is given by a simultaneous recursive definition of three basic semantic relations:

  1. $s \models^{+} \varphi$ state s supports φ InqB
  2. $s \models^{-} \varphi$ state s rejects φ InqR
  3. $s \models^{\circ} \varphi$ state s dismisses a supposition of φ InqS

- By $[\varphi]^{\dagger}$ we denote $\{s \subseteq \omega \mid s \models^{\dagger} \varphi\}$.

- In InqS the proposition expressed by φ, $[\varphi]$, is determined by the triple $\langle [\varphi]^{+}, [\varphi]^{-}, [\varphi]^{\circ} \rangle$.

- In *presenting* the semantics, we will often quantify over the maximal elements of $[\varphi]^{\dagger}$, called $\dagger$-alternatives.

- For any set of states $\mathcal{S}$: $\textsc{alt}\,\mathcal{S} = \{s \in \mathcal{S} \mid \neg\exists t \in \mathcal{S} : s \subset t\}$

Some derived semantic relations

- In terms of the three basic semantic relations, we can define other ones, such as:

Suppositionally dismissing supportability

- $s \models^{\oplus} \varphi$ iff $s \models^{\circ} \varphi$ and $\forall t \subseteq s : t \not\models^{+} \varphi$.

Suppositionally dismissing rejectability

- $s \models^{\ominus} \varphi$ iff $s \models^{\circ} \varphi$ and $\forall t \subseteq s : t \not\models^{-} \varphi$.

Suppositionally dismissing (supportability and rejectability)

- $s \models^{\otimes} \varphi$ iff $s \models^{\oplus} \varphi$ and $s \models^{\ominus} \varphi$.

Some responsehood relations

- We can define a range of logical responsehood relations according to the following scheme, filling in different semantic relations for †:
    - $\psi \models^{\dagger} \varphi$ iff $\forall s$: if $s \models^{+} \psi$, then $s \models^{\dagger} \varphi$
- Three basic responsehood relations are:
    - ψ supports φ: $\psi \models^{+} \varphi$
    - ψ rejects φ: $\psi \models^{-} \varphi$
    - ψ dismisses a supposition of φ: $\psi \models^{\circ} \varphi$
- Three derived responsehood relations are:.
    - ψ suppositionally dismisses supportability of φ: $\models^{\oplus} \varphi$
    - ψ suppositionally dismisses rejectability of φ: $\models^{\ominus} \varphi$
    - ψ suppositionally dismisses φ: $\models^{\otimes} \varphi$

Inquisitive and suppositional sentences

- φ is support inquisitive iff there are at least two support-alternatives for it, i.e., $\textsc{alt}[\varphi]^+$ contains at least two elements
- Rejection inquisitiveness and suppositional inquisitiveness are defined similarly
- We call a sentence φ suppositional iff there is a non-absurd state s such that $s \models^\circ \varphi$

Notation convention for representing states

- Let $|\varphi|$ denote the set of worlds where φ is classically true
- This gives us a convenient notation for states. For instance:

$$
\begin{array}{lll}
|p| & \models^{+} & p \vee q \\
|\neg p| & \models^{-} & p \wedge q \\
|\neg p| & \models^{\circ} & p \rightarrow q
\end{array}
$$

1.3. Suppositional inquisitive meaning postulates

Downward closure / persistence

- A distinctive feature of InqB is that $[\varphi]^+$ is downward closed
  - If $s \models^+ \varphi$, then for any $t \subseteq s$: $t \models^+ \varphi$

  That is, in InqB support is persistent

- In InqR, both $[\varphi]^+$ and $[\varphi]^-$ are downward closed
  - If $s \models^+ \varphi$, then for any $t \subseteq s$: $t \models^+ \varphi$
  - If $s \models^- \varphi$, then for any $t \subseteq s$: $t \models^- \varphi$

  That is, in InqR both support and rejection are persistent

- Underlying idea: if s supports/rejects a sentence φ, then any more informed state $t \subseteq s$ will support/reject φ as well

- Information growth cannot lead to retraction of support/reject

Persistence and suppositional dismissal

- As soon as we take suppositional dismissal into account this central idea from InqB and InqR is no longer defensible
- For instance, we want that:

$$|p \to q| \models^{+} p \to q$$

But we also want that:

$$\begin{array}{lll} |\neg p| & \models^{\circ} & p \to q \\ |\neg p| & \not\models^{+} & p \to q \end{array}$$

- So: information growth can lead to suppositional dismissal, and thereby to retraction of support (or retraction of rejection)

Persistence modulo suppositional dismissal

- Fortunately, there is a natural way to adapt the idea that support and rejection are persistent to the setting of InqS
- Namely, in InqS we postulate that support and rejection are persistent modulo dismissal of a supposition, and that dismissal itself is fully persistent:
    - If $s \models^{+} \varphi$ and $t \subseteq s$, then $t \models^{+} \varphi$ or $t \models^{\circ} \varphi$
    - If $s \models^{-} \varphi$ and $t \subseteq s$, then $t \models^{-} \varphi$ or $t \models^{\circ} \varphi$
    - If $s \models^{\circ} \varphi$ and $t \subseteq s$, then $t \models^{\circ} \varphi$

Two more postulates

Second postulate

- The inconsistent state suppositionally dismisses any sentence φ, and never supports or rejects it. That is, for any φ:

$$\emptyset \models^{\circ} \varphi$$
$$\emptyset \not\models^{+} \varphi$$
$$\emptyset \not\models^{-} \varphi$$

Third postulate

- Support and rejection are mutually exclusive : $[\varphi]^{+} \cap [\varphi]^{-} = \emptyset$
- The postulates do not exclude that for some φ and $s \neq \emptyset$:
    - $s \models^{+} \varphi$ and $s \models^{\circ} \varphi$
    - $s \models^{-} \varphi$ and $s \models^{\circ} \varphi$

Finally

- Final postulate: any completely informed consistent state $\{w\}$ supports, rejects, or suppositionally dismisses any sentence:

$$\forall \varphi \in \mathcal{L} : \forall w \in \omega : \{w\} \in ([\varphi]^+ \cup [\varphi]^- \cup [\varphi]^\circ)$$

Propositions as conversational issues

- The postulates imply that the three components of a proposition jointly form a non-empty downward closed set of states that cover the set of all worlds:

$$\bigcup([\varphi]^+ \cup [\varphi]^- \cup [\varphi]^\circ) = \omega$$

- In terms of InqB, our propositions are issues over ω.
- The issue embodied by $[\varphi]$ is a conversational issue, it specifies several appropriate ways of responding to φ.

1.4. Recursive statement of the semantics

Atomic sentences

- $s \models^{+} p$ iff $s \neq \emptyset$ and $\forall w \in s\colon w(p) = 1$
  $s \models^{-} p$ iff $s \neq \emptyset$ and $\forall w \in s\colon w(p) = 0$
  $s \models^{\circ} p$ iff $s = \emptyset$

- Atomic sentences are not suppositional, since only the inconsistent state can dismiss a supposition of *p*.
- Atomic sentences are not inquisitive, since there is only a single support-alternative and a single rejection-alternative:

$$\begin{array}{lcl} \textsc{alt}[p]^{+} & = & \{|p|\} \\ \textsc{alt}[p]^{-} & = & \{|\neg p|\} \end{array}$$

Negation

$$s \models^{+} \neg\varphi \quad \text{iff} \quad s \models^{-} \varphi$$
$$s \models^{-} \neg\varphi \quad \text{iff} \quad s \models^{+} \varphi$$
$$s \models^{\circ} \neg\varphi \quad \text{iff} \quad s \models^{\circ} \varphi$$

- The suppositional content of φ is inherited by its negation $\neg\varphi$
- Unlike in InqB: $\neg\neg\varphi \equiv \varphi$

Disjunction

- $s \models^+ \varphi \vee \psi$ iff $s \models^+ \varphi$ or $s \models^+ \psi$
  $s \models^- \varphi \vee \psi$ iff $s \models^- \varphi$ and $s \models^- \psi$
  $s \models^\circ \varphi \vee \psi$ iff $s \models^\circ \varphi$ or $s \models^\circ \psi$

- The suppositional content of φ and ψ is inherited by the disjunction $\varphi \vee \psi$

- The disjunction $p \vee q$ is support-inquisitive: there are two support-alternatives for $p \vee q$:

$$\textsc{alt}[p \vee q]^+ = \{|p|, |q|\}$$

Conjunction

- $s \models^{+} \varphi \wedge \psi$ iff $s \models^{+} \varphi$ and $s \models^{+} \psi$
  $s \models^{-} \varphi \wedge \psi$ iff $s \models^{-} \varphi$ or $s \models^{-} \psi$
  $s \models^{\circ} \varphi \wedge \psi$ iff $s \models^{\circ} \varphi$ or $s \models^{\circ} \psi$

- The suppositional content of φ and ψ is inherited by the conjunction $\varphi \wedge \psi$

- The conjunction $p \wedge q$ is reject-inquisitive: there are two rejection-alternatives for $p \wedge q$:

$$\textsc{alt}[p \wedge q]^{-} = \{|\neg p|, |\neg q|\}$$

Triggering and projection of suppositional content

- None of the clauses in the semantics we have met so far trigger suppositional content.
- Atomic sentences are not suppositional, and negation, disjunction and conjunction only project suppositional content of their subformulas in a cumulative way.
- For the language at hand, implication is the only trigger of suppositional content.
- Implication also projects the suppositional content of its consequent, but relativized to its antecedent.

Supposition triggered by implication

- The supposition that is triggered by an implication concerns the supposability of its antecedent.
- The supposability of a sentence is determined by:
  (a) the existence of support-alternatives for it.
  (b) the supposability of its support-alternatives.
- Suppositional dismissal of an implication occurs in s, when there is no support-alternative for its antecedent, or when there is some support-alternative that is not supposable in s.

Supporting an implication: InqB versus InqS

- The clause for implication in InqB is as follows:

  $s \models \varphi \to \psi$ iff $\forall t$: if $t \models \varphi$, then $t \cap s \models \psi$

- We can also formulate this in terms of the alternatives for φ:

  $s \models \varphi \to \psi$ iff $\forall u \in \textsc{alt}[\varphi]$: $u \cap s \models \psi$

- Since in InqB support is fully persistent, it makes no difference whether we consider just the support-alternatives for φ or all states that support it.

- In InqS, where support is only persistent modulo suppositional dismissal, it does potentially make a difference.

- We should only consider the support-alternatives for φ, because other states that support φ may contain additional information which causes suppositional dismissal of ψ.

- This should not be a reason for support of $\varphi \to \psi$ to fail.

Implication in InqS: the intuitive idea

- s supports $\varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and for every $u \in \textsc{alt}[\varphi]^+$:

  (a) u is supposable in s, and

  (b) $s \cap u$ supports ψ

- s rejects $\varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and for some $u \in \textsc{alt}[\varphi]^+$:

  (a) u is supposable in s, and

  (b) $s \cap u$ rejects ψ

- s dismisses $\varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^+ = \emptyset$, or for some $u \in \textsc{alt}[\varphi]^+$:

  (a) u is is not supposable in s, or

  (b) $s \cap u$ dismisses a supposition of ψ

Implication in InqS: supposability

When is it possible to suppose a support-alternative u for φ?

- Normally, to suppose a piece of information u in a state s is thought of as going from s to the more informed state $s \cap u$
- Thus, we could say that u is supposable in s iff in going from s to $s \cap u$ our state remains consistent
- However, in the present setting, u is not just an arbitrary piece of information: it is a piece of information that supports φ
- This property should be maintained in going from u to $s \cap u$:

$$\forall t \text{ from } u \text{ to } u \cap s : \; t \models^{+} \varphi$$

In words: support should persist in restricting u to s

Persisting support and suppositional dismissal

- Recall our first general postulate:

  Support should be persistent modulo suppositional dismissal

- Given this postulate, the only reason why support of φ may fail to persist in restricting u to s is that somewhere along the way, suppositional dismissal occurs

Persisting support and consistency

- Our persisting support condition: $\forall t$ from u to $u \cap s$: $t \models^{+} \varphi$ entails the basic requirement that $s \cap u$ should be consistent.
- Just requiring consistency is not always sufficient.
- Example: $p \rightarrow q$ has a single support-alternative $u = |p \rightarrow q|$. Let $s = |\neg p|$, then $u \cap s \neq \emptyset$. But $p \rightarrow q$ is not supposable in s.

Persisting support versus support in $u \cap s$

- We require persisting support all the way from u to $u \cap s$:

$$\forall t \text{ from } u \text{ to } u \cap s: \; t \models^+ \varphi$$

- Just requiring support at $u \cap s$ is not always sufficient.

- Example:

  - Let $\varphi = (p \to q) \vee r$
  - Then φ has two support-alternatives: $|p \to q|$ and $|r|$
  - Let $u = |p \to q|$ and let $s = |\neg p \wedge r|$
  - Then $u \cap s = s$, and $s \models^+ (p \to q) \vee r$, because $s \models^+ r$
  - However, $(p \to q) \vee r$ should not count as supposable in s

Implication in InqS fully spelled out

- $s \models^+ \varphi \to \psi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^+$:
  1. $\forall t$ from u to $u \cap s$: $t \models^+ \varphi$, and
  2. $u \cap s \models^+ \psi$
- $s \models^- \varphi \to \psi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and $\exists u \in \textsc{alt}[\varphi]^+$:
  1. $\forall t$ from u to $u \cap s$: $t \models^+ \varphi$, and
  2. $u \cap s \models^- \psi$
- $s \models^\circ \varphi \to \psi$ iff $\textsc{alt}[\varphi]^+ = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^+$:
  1. $\exists t$ from u to $u \cap s$: $t \not\models^+ \varphi$, or
  2. $u \cap s \models^\circ \psi$

Non-suppositional reductions

Reduction: φ not suppositional

- $s \models^{+} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^{+} : u \cap s \models^{+} \psi$
- $s \models^{-} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\exists u \in \textsc{alt}[\varphi]^{+} : u \cap s \models^{-} \psi$
- $s \models^{\circ} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^{+} : u \cap s \models^{\circ} \psi$

Reduction: φ and ψ not suppositional

- $s \models^{+} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^{+} : u \cap s \models^{+} \psi$
- $s \models^{-} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\exists u \in \textsc{alt}[\varphi]^{+} : u \cap s \models^{-} \psi$
- $s \models^{\circ} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^{+} : u \cap s = \emptyset$

Non-suppositional reductions

Reduction: φ not suppositional

- $s \models^{+} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^{+} : u \cap s \models^{+} \psi$
- $s \models^{-} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\exists u \in \textsc{alt}[\varphi]^{+} : u \cap s \models^{-} \psi$
- $s \models^{\circ} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^{+} : u \cap s \models^{\circ} \psi$

Reduction: φ and ψ not suppositional

- $s \models^{+} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^{+} : u \cap s \models^{+} \psi$
- $s \models^{-} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\exists u \in \textsc{alt}[\varphi]^{+} : u \cap s \models^{-} \psi$
- $s \models^{\circ} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^{+} : u \cap s = \emptyset$

Non-inquisitive reductions

- Now suppose that besides being non-suppositional, φ is not support-inquisitive either (though still supportable)
- In this case, $\textsc{alt}[\varphi]^+$ consists of a single alternative, call it α_φ
- The clauses for $\varphi \to \psi$ then simply reduce to:

$$s \models^+ \varphi \to \psi \quad \text{iff} \quad s \cap \alpha_\varphi \models^+ \psi$$

$$s \models^- \varphi \to \psi \quad \text{iff} \quad s \cap \alpha_\varphi \models^- \psi$$

$$s \models^\circ \varphi \to \psi \quad \text{iff} \quad s \cap \alpha_\varphi \models^\circ \psi$$

- If ψ is non-suppositional, dismissal further reduces to:

$$s \models^\circ \varphi \to \psi \quad \text{iff} \quad s \cap \alpha_\varphi = \emptyset$$

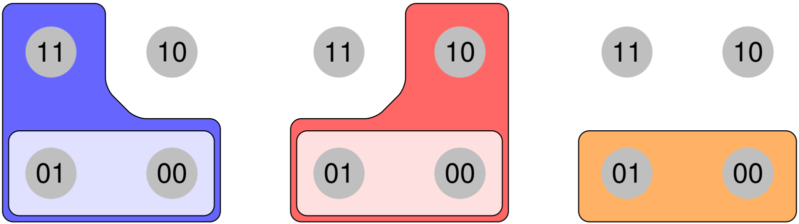
1.5. Examples

Our initial example: $p \to q$

$$s \models^{+} p \to q \quad \text{iff} \quad s \cap |p| \models^{+} q$$

$$s \models^{-} p \to q \quad \text{iff} \quad s \cap |p| \models^{-} q$$

$$s \models^{\circ} p \to q \quad \text{iff} \quad s \cap |p| = \emptyset$$



(a) support (b) reject (c) dismiss

How to read the pictures

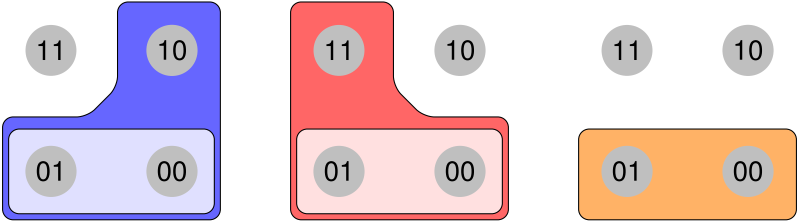
- Support is persistent modulo suppositional dismissal.
  - We depict maximal states that support φ, and if necessary also the maximal substates of these states that no longer support φ.
  - We think of these substates as support holes.
- Rejection is persistent modulo suppositional dismissal.
  - We depict maximal states that reject φ, and if necessary also the maximal substates of these states that no longer reject φ.
  - We think of these substates as rejection holes.
- Dismissal is fully persistent.
  - We depict only maximal states that dismiss a supposition of φ.
  - All substates thereof also dismiss a supposition of φ.

Our initial example: $p \to \neg q$

$$s \models^{+} p \to \neg q \quad \text{iff} \quad s \cap |p| \models^{+} \neg q$$

$$s \models^{-} p \to \neg q \quad \text{iff} \quad s \cap |p| \models^{-} \neg q$$

$$s \models^{\circ} p \to \neg q \quad \text{iff} \quad s \cap |p| = \emptyset$$



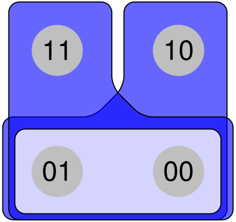
(a) support (b) reject (c) dismiss

Our initial example: $p \to ?q$

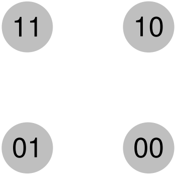
$s \models^{+} p \to ?q$ iff $s \cap |p| \models^{+} q$ or $s \cap |p| \models^{+} \neg q$

$s \models^{-} p \to ?q$ iff $s \cap |p| \models^{-} q$ and $s \cap |p| \models^{-} \neg q$ impossible

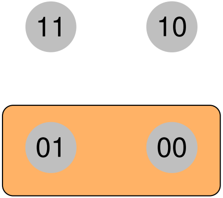
$s \models^{\circ} p \to ?q$ iff $s \cap |p| = \emptyset$



(a) support



(b) reject



(c) dismiss

Desired predictions

(1)
a. If Alf goes to the party, will Bea go too? $p \rightarrow ?q$
b. If Alf goes, then Bea will go as well. $p \rightarrow q$
c. If Alf goes, then Bea will not go. $p \rightarrow \neg q$
d. Alf won't go. $\neg p$

- Both (1b) and (1c) support the conditional question in (1a):

$$p \rightarrow q \models^{+} p \rightarrow ?q$$

$$p \rightarrow \neg q \models^{+} p \rightarrow ?q$$

- (1b) and (1c) are contradictory, they reject each other:

$$p \rightarrow q \models^{-} p \rightarrow \neg q$$

$$p \rightarrow \neg q \models^{-} p \rightarrow q$$

Desired predictions

(1)	a.	If Alf goes to the party, will Bea go too?	$p \to ?q$
	b.	If Alf goes, then Bea will go as well.	$p \to q$
	c.	If Alf goes, then Bea will not go.	$p \to \neg q$
	d.	Alf won't go.	$\neg p$

- Finally, (1d) suppositionally dismisses (1a)-(1c) :

$$\neg p \quad \models^{\otimes} \quad p \to ?q$$

$$\neg p \quad \models^{\otimes} \quad p \to q$$

$$\neg p \quad \models^{\otimes} \quad p \to \neg q$$

- In particular:

$$\neg p \quad \not\models^{+} \quad p \to q$$

Additional prediction, whether desired or not

(3)
a. If Alf goes to the party, will Bea go too? $p \to ?q$
b. ?Bea will go to the party. q
c. Whether Alf goes or not, Bea will go. $(p \vee \neg p) \to q$
d. If Alf goes, Bea will not go. $p \to \neg q$

- The response in (2b) needs marking, (2c) is fine.
- (2c) and (2d) are contradictory responses to (2a).
- We will return to the example later. For now we note:

$$q \quad \not\models^{+} \quad p \to ?q$$

$$q \quad \not\models^{+} \quad p \to q$$

- Reason: $|q \wedge \neg p|$ is a state that supports q, but it suppositionally dismisses, and therefore does not support $p \to q$ and $p \to ?q$.

Three more complex examples

We will consider three more complex examples:

(1) Inquisitive antecedent: $(p \vee q) \to r$

(2) Suppositional consequent: $p \to (q \to r)$

(3) Suppositional antecedent: $(p \to q) \to r$

Case 1: inquisitive antecedent: $(p \vee q) \rightarrow r$

- Both antecedent and consequent are non-suppositional
- There are two support-alternatives for the antecedent:

$$\textsc{alt}[p \vee q]^{+} = \{|p|, |q|\}$$

- So we have:

$$s \models^{+} (p \vee q) \rightarrow r \quad \text{iff} \quad \forall u \in \{|p|, |q|\} : u \cap s \models^{+} r$$

$$s \models^{-} (p \vee q) \rightarrow r \quad \text{iff} \quad \exists u \in \{|p|, |q|\} : u \cap s \models^{-} r$$

$$s \models^{\circ} (p \vee q) \rightarrow r \quad \text{iff} \quad \exists u \in \{|p|, |q|\} : u \cap s = \emptyset$$

Case 1: inquisitive antecedent: $(p \vee q) \rightarrow r$

$$s \models^{+} (p \vee q) \rightarrow r \quad \text{iff} \quad \forall u \in \{|p|, |q|\} : u \cap s \models^{+} r$$

$$s \models^{-} (p \vee q) \rightarrow r \quad \text{iff} \quad \exists u \in \{|p|, |q|\} : u \cap s \models^{-} r$$

$$s \models^{\circ} (p \vee q) \rightarrow r \quad \text{iff} \quad \exists u \in \{|p|, |q|\} : u \cap s = \emptyset$$

- Some (non-)supporting responses:

$$\begin{array}{lll} (p \rightarrow r) \wedge (q \rightarrow r) & \models^{+} & (p \vee q) \rightarrow r \\ \neg p \wedge \neg q & \not\models^{+} & (p \vee q) \rightarrow r \end{array}$$

Case 1: inquisitive antecedent: $(p \vee q) \to r$

$$s \models^+ (p \vee q) \to r \quad \text{iff} \quad \forall u \in \{|p|, |q|\} : u \cap s \models^+ r$$

$$s \models^- (p \vee q) \to r \quad \text{iff} \quad \exists u \in \{|p|, |q|\} : u \cap s \models^- r$$

$$s \models^\circ (p \vee q) \to r \quad \text{iff} \quad \exists u \in \{|p|, |q|\} : u \cap s = \emptyset$$

- Some rejecting responses:

$$\begin{array}{lll} p \to \neg r & \models^- & (p \vee q) \to r \\ q \to \neg r & \models^- & (p \vee q) \to r \\ (p \to \neg r) \vee (q \to \neg r) & \models^- & (p \vee q) \to r \end{array}$$

Case 1: inquisitive antecedent: $(p \vee q) \to r$

$$s \models^{+} (p \vee q) \to r \quad \text{iff} \quad \forall u \in \{|p|, |q|\}: u \cap s \models^{+} r$$

$$s \models^{-} (p \vee q) \to r \quad \text{iff} \quad \exists u \in \{|p|, |q|\}: u \cap s \models^{-} r$$

$$s \models^{\circ} (p \vee q) \to r \quad \text{iff} \quad \exists u \in \{|p|, |q|\}: u \cap s = \emptyset$$

- Some responses that dismiss a supposition:

$$\neg p \quad \models^{\oplus} \quad (p \vee q) \to r$$

$$\neg q \quad \models^{\oplus} \quad (p \vee q) \to r$$

$$\neg p \vee \neg q \quad \models^{\oplus} \quad (p \vee q) \to r$$

Affirming the consequent again

(3) a. If Alf goes to the party, will Bea go too? $p \rightarrow ?q$
b. Whether Alf goes or not, Bea will go. $(p \vee \neg p) \rightarrow q$
c. If Alf goes, Bea will not go. $p \rightarrow \neg q$

- (3b) is a felicitous, supporting response to (3a).
- (3b) and (3c) are contradictory responses.

$$
\begin{array}{lll}
(p \vee \neg p) \rightarrow q & \models^{+} & p \rightarrow ?q \\
p \rightarrow \neg q & \models^{-} & (p \vee \neg p) \rightarrow q \\
(p \vee \neg p) \rightarrow q & \models^{-} & p \rightarrow \neg q \\
(p \vee \neg p) \rightarrow q & \models^{+} & q
\end{array}
$$

Case 2: suppositional consequent: $p \rightarrow (q \rightarrow r)$

- The antecedent is still non-suppositional, so the persistent support condition does not come into play
- Moreover, there is a single support-alternative for the antecedent:

$$\text{ALT}[p]^{+} = \{|p|\}$$

- So we have:

$$s \models^{+} p \rightarrow (q \rightarrow r) \quad \text{iff} \quad s \cap |p| \models^{+} q \rightarrow r$$

$$s \models^{-} p \rightarrow (q \rightarrow r) \quad \text{iff} \quad s \cap |p| \models^{-} q \rightarrow r$$

$$s \models^{\circ} p \rightarrow (q \rightarrow r) \quad \text{iff} \quad s \cap |p| \models^{\circ} q \rightarrow r$$

Case 2: suppositional consequent: $p \to (q \to r)$

$$s \models^+ p \to (q \to r) \quad \text{iff} \quad s \cap |p| \models^+ q \to r$$

$$s \models^- p \to (q \to r) \quad \text{iff} \quad s \cap |p| \models^- q \to r$$

$$s \models^\circ p \to (q \to r) \quad \text{iff} \quad s \cap |p| \models^\circ q \to r$$

- Since the consequent is a simple conditional, this can be further reduced to:

$$s \models^+ p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^+ r$$

$$s \models^- p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^- r$$

$$s \models^\circ p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| = \emptyset$$

Case 2: suppositional consequent: $p \to (q \to r)$

$$s \models^+ p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^+ r$$

$$s \models^- p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^- r$$

$$s \models^\circ p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| = \emptyset$$

- Some (non-)supporting responses:

$$\begin{array}{lll} (p \wedge q) \to r & \models^+ & p \to (q \to r) \\ \neg p & \not\models^+ & p \to (q \to r) \\ \neg q & \not\models^+ & p \to (q \to r) \end{array}$$

Case 2: suppositional consequent: $p \to (q \to r)$

$$s \models^{+} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^{+} r$$

$$s \models^{-} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^{-} r$$

$$s \models^{\circ} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| = \emptyset$$

- Some (non-)rejecting responses:

$$\begin{array}{lll}
(p \wedge q) \to \neg r & \models^{-} & p \to (q \to r) \\
p \to \neg r & \not\models^{-} & p \to (q \to r) \\
p \to ((q \vee \neg q) \to \neg r) & \models^{-} & p \to (q \to r) \\
q \to \neg r & \not\models^{-} & p \to (q \to r) \\
(p \vee \neg p) \to (q \to \neg r) & \models^{-} & p \to (q \to r)
\end{array}$$

Case 2: suppositional consequent: $p \to (q \to r)$

$$s \models^{+} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^{+} r$$

$$s \models^{-} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^{-} r$$

$$s \models^{\circ} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| = \emptyset$$

- Some responses that dismiss a supposition:

$$\begin{array}{lll} \neg p & \models^{\otimes} & p \to (q \to r) \\ \neg q & \models^{\otimes} & p \to (q \to r) \\ \neg p \vee \neg q & \models^{\otimes} & p \to (q \to r) \end{array}$$

Case 3: suppositional antecedent: $(p \to q) \to r$

- Now the antecedent is suppositional, so the persistent support condition finally comes into play
- There is a single support-alternative u for the antecedent:

$$u = |p \to q|$$

- So we have:

$$s \models^+ (p \to q) \to r \;\text{ iff }\; \forall t \text{ from } u \text{ to } s \cap u\colon\ t \models^+ p \to q \text{ and } s \cap u \models^+ r$$

$$s \models^- (p \to q) \to r \;\text{ iff }\; \forall t \text{ from } u \text{ to } s \cap u\colon\ t \models^+ p \to q \text{ and } s \cap u \models^- r$$

$$s \models^\circ (p \to q) \to r \;\text{ iff }\; \exists t \text{ from } u \text{ to } s \cap u\colon\ t \not\models^+ p \to q \text{ or } s \cap u \models^\circ r$$

Case 3: suppositional antecedent: $(p \to q) \to r$

$s \models^+ (p \to q) \to r$ iff $\forall t$ from u to $s \cap u$: $t \models^+ p \to q$
and $s \cap u \models^+ r$

$s \models^- (p \to q) \to r$ iff $\forall t$ from u to $s \cap u$: $t \models^+ p \to q$
and $s \cap u \models^- r$

$s \models^\circ (p \to q) \to r$ iff $\exists t$ from u to $s \cap u$: $t \not\models^+ p \to q$
or $s \cap u \models^\circ r$

- Some non-supporting responses:

$$
\begin{array}{lll}
r & \not\models^+ & (p \to q) \to r \\
\neg p & \not\models^+ & (p \to q) \to r \\
p \wedge \neg q & \not\models^+ & (p \to q) \to r \\
p \to \neg q & \not\models^+ & (p \to q) \to r
\end{array}
$$

Case 3: suppositional antecedent: $(p \to q) \to r$

$$s \models^{+} (p \to q) \to r \text{ iff } \forall t \text{ from } u \text{ to } s \cap u\text{: } t \models^{+} p \to q \text{ and } s \cap u \models^{+} r$$

$$s \models^{-} (p \to q) \to r \text{ iff } \forall t \text{ from } u \text{ to } s \cap u\text{: } t \models^{+} p \to q \text{ and } s \cap u \models^{-} r$$

$$s \models^{\circ} (p \to q) \to r \text{ iff } \exists t \text{ from } u \text{ to } s \cap u\text{: } t \not\models^{+} p \to q \text{ or } s \cap u \models^{\circ} r$$

- Some rejecting responses:

$$(p \to q) \to \neg r \quad \models^{-} \quad (p \to q) \to r$$

$$p \wedge (q \to \neg r) \quad \models^{-} \quad (p \to q) \to r$$

Case 3: suppositional antecedent: $(p \to q) \to r$

$s \models^{+} (p \to q) \to r$ iff $\forall t$ from u to $s \cap u$: $t \models^{+} p \to q$
and $s \cap u \models^{+} r$

$s \models^{-} (p \to q) \to r$ iff $\forall t$ from u to $s \cap u$: $t \models^{+} p \to q$
and $s \cap u \models^{-} r$

$s \models^{\circ} (p \to q) \to r$ iff $\exists t$ from u to $s \cap u$: $t \not\models^{+} p \to q$
or $s \cap u \models^{\circ} r$

- Some responses that dismiss a supposition:

$$\neg p \models^{\otimes} (p \to q) \to r$$

$$p \to \neg q \models^{\otimes} (p \to q) \to r$$

$$p \wedge \neg q \models^{\otimes} (p \to q) \to r$$

Conclusion first part

- The general perspective on meaning in inquisitive semantics is that sentences express proposals to update the CG in one or more ways
- There are several ways one may respond to such proposals, depending on one's information state
- InqB characterizes which states support a given proposal
- InqR also characterizes which states reject a given proposal
- InqS further distinguishes states that dismiss a supposition of a given proposal
- We thus arrive at a more and more fine-grained formal characterization of proposals, and thereby a more and more fine-grained characterization of meaning

Conclusion first part

- This in turn leads to a better account of the behavior of certain types of sentences in conversation
- InqS especially improves on InqB and InqR in its treatment of conditional statements and questions
- Paradigm example:

$$p \rightarrow q \quad \text{evaluated in the state} \quad |\neg p|$$

- InqB: support
- InqR: both support and reject
- InqS: suppositional dismissal

2. Suppositional epistemic *might* and *must*

2.1. Epistemic *might* as a supposability check

Suppositional epistemic *might*

Might as a supposability check

- In InqS, $\Diamond\varphi$ can be treated as inducing a supposability check.
- In the most basic cases, checking supposability amounts to checking consistency.
- Thus, in these basic cases, our analysis of $\Diamond\varphi$ comes down to Veltman's analysis of *might* in update semantics (US).
- However, for more involved cases, the two analyses diverge.

Persistence

- For Veltman, $\Diamond\varphi$ is a basic example of a non-persistent update.
- In InqS, both $\Diamond\varphi$ and $\Box\varphi$ are support / reject-persistent modulo suppositional dismissal.

What does a supposability check amount to?

In order to answer this question, we first state some facts about suppositionally dismissing supportability

Suppositionally dismissing supportability

- $s \models^{\oplus} \varphi$ iff $s \models^{\circ} \varphi$ and $\forall t \subseteq s \colon t \not\models^{+} \varphi$.

For non-suppositional φ

- $s \models^{\oplus} \varphi$ iff $s = \emptyset$.

Generally

- If $s \models^{\oplus} \varphi$, then no support-alternative for φ is supposable in s.

Suppositional *might*: the intuitive idea

$\Diamond\varphi$ expresses a proposal to check the supposability of φ in s

- s supports $\Diamond\varphi$ iff
  - (a) there is at least one support-alternative for φ and
  - (b) every support-alternative for φ is supposable in s
- s rejects $\Diamond\varphi$ iff
  - (a) s does not suppositionally dismiss supportability of φ and
  - (b) every support-alternative for φ is not supposable in s
- s dismisses a supposition of $\Diamond\varphi$ iff
  - (a) there is no support-alternative for φ or
  - (b) some support-alternative for φ is not supposable in s

Suppositional *might*: support and dismissal

Support and dismissing a supposition contradict each other

- s supports $\Diamond\varphi$ iff
  - (a) there is at least one support-alternative for φ and
  - (b) every support-alternative for φ is supposable in s
- s dismisses a supposition of $\Diamond\varphi$ iff
  - (a) there is no support-alternative for φ or
  - (b) some support-alternative for φ is not supposable in s

Suppositional *might*: rejection and dismissal

Rejection implies suppositional dismissal

- s rejects $\Diamond\varphi$ iff
  - (a) s does not suppositionally dismiss supportability of φ and
  - (b) every support-alternative for φ is not supposable in s
- s dismisses a supposition of $\Diamond\varphi$ iff
  - (a) there is no support-alternative for φ or
  - (b) some support-alternative for φ is not supposable in s

Suppositional *might*: persistence

Two essential features of the clauses for $\Diamond\varphi$

- Support and dismissing a supposition contradict each other
- Rejection implies dismissal

Support of *might* can turn into reject + dismissal

- It can be the case that $s \models^{+} \Diamond\varphi$ and that it holds for some more informed state $t \subset s$ that $t \not\models^{+} \Diamond\varphi$, or even $t \models^{-} \Diamond\varphi$, but then it will also be the case that $t \models^{\circ} \Diamond\varphi$.
- Suppositional *might* is support-persistent, modulo suppositional dismissal.

Details of the rejection clauses

- s rejects $\Diamond\varphi$ iff
  - (a) s does not suppositionally dismiss supportability of φ and
  - (b) every support-alternative for φ is not supposable in s

- Clause (a) restricts clause (b), filtering out cases where not rejection, but only suppositional dismissal is at stake.
- Consider $\Diamond(p \to q)$. Let $s = |\neg p|$.
- The one support-alternative for $p \to q$ is not supposable in s.
- So, s dismisses a supposition of $\Diamond(p \to q)$.
- But s does not reject $\Diamond(p \to q)$, because s also suppositionally dismisses (supportability of) $p \to q$:
- After all, s dismisses a supposition of $p \to q$, and no substate of s supports $p \to q$.

Details of the rejection clauses

- s rejects $\Diamond\varphi$ iff
  (a) s does not suppositionally dismiss supportability of φ and
  (b) every support-alternative for φ is not supposable in s
- Consider $\Diamond((p \rightarrow q) \vee r)$. Let $s = |\neg p \wedge \neg r|$.
- The two support-alternatives for $(p \rightarrow q) \vee r$ are not supposable in s.
- So, s dismisses a supposition of $\Diamond((p \rightarrow q) \vee r)$.
- But s does not reject $\Diamond((p \rightarrow q) \vee r)$, because s also suppositionally dismisses (supportability of) $(p \rightarrow q) \vee r$:
- After all, s dismisses a supposition of $(p \rightarrow q) \vee r$, and no substate of s supports $(p \rightarrow q) \vee r$.

Suppositional *might* fully spelled out

$$s \models^{+} \Diamond\varphi \quad \text{iff} \quad \textsc{alt}[\varphi]^{+} \neq \emptyset \ \text{ and}$$
$$\forall u \in \textsc{alt}[\varphi]^{+}: \ \forall t \text{ from } u \text{ to } u \cap s: \ t \models^{+} \varphi$$

$$s \models^{-} \Diamond\varphi \quad \text{iff} \quad s \not\models^{\oplus} \varphi \ \text{ and}$$
$$\forall u \in \textsc{alt}[\varphi]^{+}: \ \exists t \text{ from } u \text{ to } u \cap s: \ t \not\models^{+} \varphi$$

$$s \models^{\circ} \Diamond\varphi \quad \text{iff} \quad \textsc{alt}[\varphi]^{+} = \emptyset \ \text{ or}$$
$$\exists u \in \textsc{alt}[\varphi]^{+}: \ \exists t \text{ from } u \text{ to } u \cap s: \ t \not\models^{+} \varphi$$

Reduction for non-suppositional φ

$$s \models^{+} \Diamond\varphi \quad \text{iff} \quad \textsc{alt}[\varphi]^{+} \neq \emptyset \ \text{ and } \ \forall u \in \textsc{alt}[\varphi]^{+}: u \cap s \neq \emptyset$$

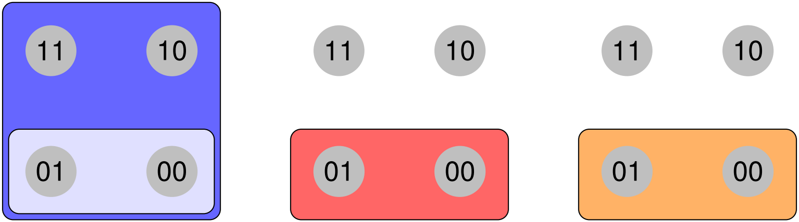
$$s \models^{-} \Diamond\varphi \quad \text{iff} \quad s \neq \emptyset \ \text{ and } \ \forall u \in \textsc{alt}[\varphi]^{+}: u \cap s = \emptyset$$

$$s \models^{\circ} \Diamond\varphi \quad \text{iff} \quad \textsc{alt}[\varphi]^{+} = \emptyset \ \text{ or } \ \exists u \in \textsc{alt}[\varphi]^{+}: u \cap s = \emptyset$$

Picture of meaning *might*

Reduced clauses for *might*

- $s \models^{+} \Diamond\varphi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^{+} : u \cap s \neq \emptyset$
- $s \models^{-} \Diamond\varphi$ iff $s \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^{+} : u \cap s = \emptyset$
- $s \models^{\circ} \Diamond\varphi$ iff $\textsc{alt}[\varphi]^{+} = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^{+} : u \cap s = \emptyset$



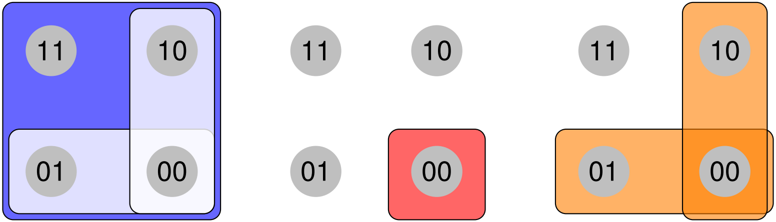
(a) support (b) reject (c) dismissal

$\Diamond p$

Epistemic free choice

Reduced clauses for *might*

- $s \models^{+} \Diamond\varphi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^{+} : u \cap s \neq \emptyset$
- $s \models^{-} \Diamond\varphi$ iff $s \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^{+} : u \cap s = \emptyset$
- $s \models^{\circ} \Diamond\varphi$ iff $\textsc{alt}[\varphi]^{+} = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^{+} : u \cap s = \emptyset$

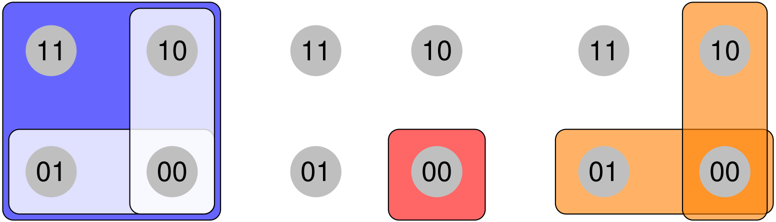


(a) support (b) reject (c) dismiss

$\Diamond(p \vee q)$

Epistemic free choice

- $\Diamond(p \vee q) \models^{+} \Diamond p \wedge \Diamond q$
- $\Diamond(p \vee q) \not\models^{+} \Diamond(p \wedge q)$



(a) support (b) reject (c) dismiss

$\Diamond(p \vee q)$

2.2. Epistemic *must* as a non-supposability check

Derived suppositional *must*

Must as a non-supposability check

- We standardly define *must* as the dual of *might*: $\Box\varphi := \neg\Diamond\neg\varphi$.
- So, $\Box\varphi$ is supported in s, when $\Diamond\neg\varphi$ is rejected in s
- $\Diamond\neg\varphi$ is a proposal to check for supposability of $\neg\varphi$ in s.
- When the check for supposability of $\neg\varphi$ fails in s, $\Diamond\neg\varphi$ is rejected in s and $\Box\varphi$ is supported in s.
- In InqS, then, $\Box\varphi$ induces a *non*-supposability check of $\neg\varphi$.
- Conversationally, a speaker uttering $\Box\varphi$, invites a responder to suppose that $\neg\varphi$, in the hope that in her state $\neg\varphi$ is (also) not supposable.

Reminder

Suppositionally dismissing rejectability

- $s \models^{\ominus} \varphi$ iff $s \models^{\circ} \varphi$ and $\forall t \subseteq s : t \not\models^{-} \varphi$.

For non-suppositional φ:

- $s \models^{\ominus} \varphi$ iff $s = \emptyset$.

Generally:

- If $s \models^{\ominus} \varphi$, then no reject-alternative for φ is supposable in s.

Suppositional *must*: intuitive idea derived from *might*

$\Box\varphi$ is a proposal to check the non-supposability of $\neg\varphi$ in s

- s supports $\Box\varphi$ iff
  - (a) s does not suppositionally dismiss rejectability of φ and
  - (b) every rejection-alternative for φ is not supposable in s
- s rejects $\Box\varphi$ iff
  - (a) there is at least one rejection-alternative for φ and
  - (b) every rejection-alternative for φ is supposable in s
- s dismisses a supposition of $\Box\varphi$ iff
  - (a) there is no rejection-alternative for φ or
  - (b) some rejection-alternative for φ is not supposable in s

Suppositional *must*: support and dismissal

Support implies suppositional dismissal

- s supports $\Box\varphi$ iff
  (a) s does not suppositionally dismiss rejectability of φ and
  (b) every rejection-alternative for φ is not supposable in s
- s dismisses a supposition of $\Box\varphi$ iff
  (a) there is no rejection-alternative for φ or
  (b) some rejection-alternative for φ is not supposable in s

Suppositional *must*: rejection and dismissal

Rejection and dismissing a supposition contradict each other

- s rejects $\Box\varphi$ iff
  (a) there is at least one rejection-alternative for φ and
  (b) every rejection-alternative for φ is supposable in s
- s dismisses a supposition of $\Box\varphi$ iff
  (a) there is no rejection-alternative for φ or
  (b) some rejection-alternative for φ is not supposable in s

Suppositional *must*: persistence

Two essential features of the clauses for $\Box\varphi$

- Rejection and dismissing a supposition contradict each other
- Support implies dismissal

Rejection of *must* can turn into support + dismissal

- It can be the case that $s \models^{-} \Box\varphi$ and that it holds for some more informed $t \subset s$ that $t \not\models^{-} \Box\varphi$, or even $t \models^{+} \Box\varphi$, but then it will also be the case that $t \models^{\circ} \Box\varphi$.
- Suppositional *must* is rejection-persistent, modulo suppositional dismissal.

Details of the support clause

- s supports $\Box\varphi$ iff

  (a) s does not suppositionally dismiss rejectability of φ and

  (b) every rejection-alternative for φ is not supposable in s

- Clause (a) restricts clause (b), filtering out cases where not support, but only suppositional dismissal is at stake.
- Consider $\Box(p \rightarrow q)$. Let $s = |\neg p|$.
- The single rejection-alternative for $p \rightarrow q$, i.e., $|p \rightarrow \neg q|$, is not supposable in s.
- So, s dismisses a supposition of $\Box(p \rightarrow q)$.
- But s does not support $\Box(p \rightarrow q)$, because s also suppositionally dismisses (rejectability of) $p \rightarrow q$.
- After all, s dismisses a supposition of $p \rightarrow q$, and no substate of s rejects $p \rightarrow q$.

Details of the support clause

- s supports $\Box\varphi$ iff
  - (a) s does not suppositionally dismiss rejectability of φ and
  - (b) every rejection-alternative for φ is not supposable in s

- Consider $\Box((p \to q) \wedge r)$. Let $s = |\neg p \wedge r|$.
- The two rejection-alternatives for $(p \to q) \wedge r$, i.e., $|p \to \neg q|$ and $|\neg r|$, are not supposable in s.
- So, s dismisses a supposition of $\Box((p \to q) \wedge r)$.
- But s does not support $\Box((p \to q) \wedge r)$, because s also suppositionally dismisses (rejectability of) $(p \to q) \wedge r$.
- After all, s dismisses a supposition of $(p \to q) \wedge r$, and no substate of s rejects $(p \to q) \wedge r$.

Suppositional epistemic *must* fully spelled out

$$s \models^{+} \Box\varphi \quad \text{iff} \quad s \not\models^{\ominus} \varphi \;\text{ and }\; \forall u \in \textsc{alt}[\varphi]^{-}\colon\; \exists t \text{ from } u \text{ to } u \cap s\colon t \not\models^{-} \varphi$$

$$s \models^{-} \Box\varphi \quad \text{iff} \quad \textsc{alt}[\varphi]^{-} \neq \emptyset \;\text{ and }\; \forall u \in \textsc{alt}[\varphi]^{-}\colon\; \forall t \text{ from } u \text{ to } u \cap s\colon t \models^{-} \varphi$$

$$s \models^{\circ} \Box\varphi \quad \text{iff} \quad \textsc{alt}[\varphi]^{-} = \emptyset \;\text{ or }\; \exists u \in \textsc{alt}[\varphi]^{-}\colon\; \exists t \text{ from } u \text{ to } u \cap s\colon t \not\models^{-} \varphi$$

Reduction for non-suppositional φ

$$s \models^{+} \Box\varphi \quad \text{iff} \quad s \neq \emptyset \quad \text{and} \quad \forall u \in \textsc{alt}[\varphi]^{-}\colon u \cap s = \emptyset$$

$$s \models^{-} \Box\varphi \quad \text{iff} \quad \textsc{alt}[\varphi]^{-} \neq \emptyset \quad \text{and} \quad \forall u \in \textsc{alt}[\varphi]^{-}\colon u \cap s \neq \emptyset$$

$$s \models^{\circ} \Box\varphi \quad \text{iff} \quad \textsc{alt}[\varphi]^{-} = \emptyset \quad \text{or} \quad \exists u \in \textsc{alt}[\varphi]^{-}\colon u \cap s = \emptyset$$

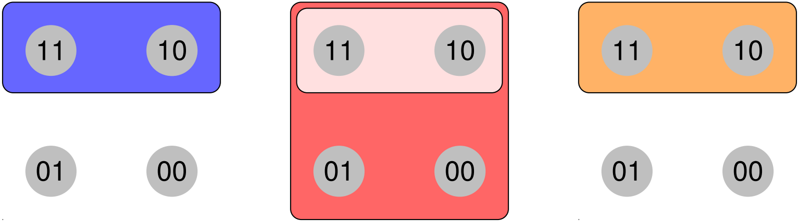
Picture of meaning *must*

Reduced clauses for *must*

$$s \models^{+} \Box\varphi \quad \text{iff} \quad s \neq \emptyset \quad \text{and} \quad \forall u \in \textsc{alt}[\varphi]^{-}: u \cap s = \emptyset$$

$$s \models^{-} \Box\varphi \quad \text{iff} \quad \textsc{alt}[\varphi]^{-} \neq \emptyset \quad \text{and} \quad \forall u \in \textsc{alt}[\varphi]^{-}: u \cap s \neq \emptyset$$

$$s \models^{\circ} \Box\varphi \quad \text{iff} \quad \textsc{alt}[\varphi]^{-} = \emptyset \quad \text{or} \quad \exists u \in \textsc{alt}[\varphi]^{-}: u \cap s = \emptyset$$



(a) support (b) reject (c) dismiss

$\Box p$

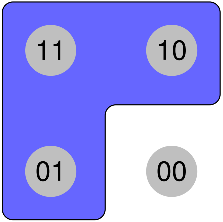
Picture of meaning *must*

Reduced clauses for *must*

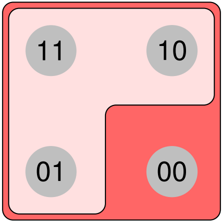
$$s \models^+ \Box\varphi \quad \text{iff} \quad s \neq \emptyset \quad \text{and} \quad \forall u \in \text{ALT}[\varphi]^- : u \cap s = \emptyset$$

$$s \models^- \Box\varphi \quad \text{iff} \quad \text{ALT}[\varphi]^- \neq \emptyset \quad \text{and} \quad \forall u \in \text{ALT}[\varphi]^- : u \cap s \neq \emptyset$$

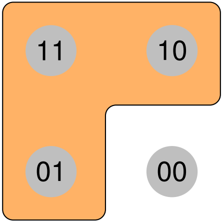
$$s \models^\circ \Box\varphi \quad \text{iff} \quad \text{ALT}[\varphi]^- = \emptyset \quad \text{or} \quad \exists u \in \text{ALT}[\varphi]^- : u \cap s = \emptyset$$



(a) support



(b) reject



(c) dismiss

$\Box(p \vee q)$

2.3. Non-inquisitive closure by *might* and *must*

Suppositional *must* and non-inquisitive closure

- The reject-informative content of $\Box\varphi$ is nil:

$$\bigcup[\Box\varphi]^{-} = \omega$$

- The support-informative content of $\Box\varphi$ equals that of φ:

$$\bigcup[\Box\varphi]^{+} = \bigcup[\varphi]^{+}$$

- But it does not hold generally that $[\Box\varphi]^{+} = [\varphi]^{+}$.

$$\textsc{alt}[p \vee q]^{+} = \{|p|, |q|\} \quad \neq \quad \textsc{alt}[\Box(p \vee q)]^{+} = \{|p| \cup |q|\}$$

- $p \vee q$ is support-inquisitive, but $\Box(p \vee q)$ is not.
- $\Box(p \vee \neg p)$ is supported in every state, support of $p \vee \neg p$ requires support of p or support of $\neg p$.

Suppositional *might* and non-inquisitive closure

- The support-informative content of $\Diamond\varphi$ is nil:

$$\bigcup[\Diamond\varphi]^+ = \omega$$

- The reject-informative content of $\Diamond\varphi$ equals that of φ:

$$\bigcup[\Diamond\varphi]^- = \bigcup[\varphi]^-$$

- But it does not hold generally that $[\Diamond\varphi]^- = [\varphi]^-$.

$$\textsc{alt}[p \wedge q]^- = \{|\neg p|, |\neg q|\} \quad \neq \quad \textsc{alt}[\Diamond(p \wedge q)]^- = \{|\neg p| \cup |\neg q|\}$$

- $p \wedge q$ is reject-inquisitive, but $\Diamond(p \wedge q)$ is not.
- $\Diamond(p \wedge \neg p)$ is rejected in every state, rejection of $p \wedge \neg p$ requires rejection of p or rejection of $\neg p$.

Suppositional inquisitiveness of *might* and *must*

Suppositional inquisitiveness

- Neither $\Diamond\varphi$ nor $\Box\varphi$ are ever support- or rejection-inquisitive.
- But both $\Diamond\varphi$ and $\Box\varphi$ can be suppositionally inquisitive.
- $\textsc{alt}[\Diamond(p \vee q)]^\circ = \{|\neg p|, |\neg q|\}$, and $\textsc{alt}[\Diamond(p \vee q)]^- = \{|\neg p| \cap |\neg q|\}$
- $\textsc{alt}[\Box(p \wedge q)]^\circ = \{|p|, |q|\}$, where $\textsc{alt}[\Box(p \wedge q)]^+ = \{|p| \cap |q|\}$

Partial support and rejection

- Dismissing a supposition of $\Diamond(p \vee q)$ can be thought of as partially rejecting $p \vee q$.
- Dismissing a supposition of $\Box(p \wedge q)$ can be thought of as partially supporting $p \wedge q$.

2.4. Modal and non-modal implications

Modal and non-modal implications

Rejecting implication

- In InqS, not just $p \wedge \neg q$, but also $p \rightarrow \neg q$ rejects $p \rightarrow q$.
- Some may feel this is still asking too much, and that $p \rightarrow \Diamond\neg q$ or $\Diamond(p \wedge \neg q)$ should already suffice to reject $p \rightarrow q$.
- But neither of these responses is support-informative, they are already supported by the ignorant state ω.
- But sheer ignorance about p and q should not suffice to reject the proposal to update the CG with the information that $p \rightarrow q$.
- Responding with $p \rightarrow \Diamond\neg q$ or $\Diamond(p \wedge \neg q)$ to $p \rightarrow q$, signals unwillingness and not unability to accept the proposal.

Modal and non-modal implications

Rejecting implication continued

- Both $p \to \Diamond\neg q$ and $\Diamond(p \wedge \neg q)$ do suffice to reject $p \to \Box q$.
- By proposing $p \to \Box q$ instead of $p \to q$, one signals that ignorance about p and q suffices to reject the proposal.
- One only intends an update of the CG with $p \to q$, in case the other participants also already support that $p \to q$ or $p \to \Box q$.

Implication in natural language

- InqS as such is neutral as to whether NL-conditionals should generally be analyzed as modal or non-modal implications.
- What matters to us here are the inquisitive and suppositional features of the semantics.

2.5. Discussion

Discussion

- Disagreement dialogue from Yanovich (2013), p.33:

(4) a. *Sarah:* Bill might be in Boston.
    b. *George:* No, that's not true. I just saw him ten minutes ago here in Berkeley.
    c. *Sarah:* Oh. Then I guess I was wrong.

- "There are several issues raised by (4) that any reasonable theory of the semantics and pragmatics of the epistemic modal *might* needs to explain:

  Assertion: Sarah is not wrong about (3a), though she may later retract it.

  Disagreement: George's disagreement in (3b) is (or at least may be) about where Bill is, not about what Sarah thinks.

  Retraction: It is reasonable for Sarah to retract her earlier assertion in (3c) after she learns Bill is in Berkeley."

Discussion

- “These explananda may seem to be trivial. The reason we need to discuss them at all is that many standard contextualist theories fail to account for all three: they either explain **Assertion** well, but fail with **Disagreement** and **Retraction**, or vice versa.”

- Yanovich develops his own detailed theory of “Practical Contextualism”.
- Such contextualist (relativist) theories are truth-conditional semantic-pragmatic analyses of epistemic *might*.
- *Might*-sentences such as (2a) are seen as epistemic claims relative to “some body of knowledge determined by the evaluation world and the context.”
- The problem is, of course, who’s knowledge is at stake.

Discussion

- In the analysis proposed here within InqS, the behavior of *might* in dialogues like (4) is at the heart of the semantics.
- The semantic content of the epistemic modalities is fully determined by their conversational function in the process of information exchange.
- There is no need to determine a specific single “body of knowledge” relative to which “epistemic claims” are evaluated as being “true or false” or “right or wrong”.
- That Sarah and George use such qualifications in their utterances in (4) does not imply that our semantic analysis needs to use such notions.

Discussion

(4)
a. *Sarah:* Bill might be in Boston.
b. *George:* No, that's not true. I just saw him ten minutes ago here in Berkeley.
c. *Sarah:* Oh. Then I guess I was wrong.

- The essence of the conversation in (4) is that, apparently, at the outset it is supposable relative to Sarah's information state, and to the CG, that Bill is in Boston.
- This is not so relative to George's state, he therefore rejects Sarah's utterance, and he tells her why that is.
- After Sarah's final response, the "disagreement" is resolved. She accepts George's rejection, and thereby it belongs to the CG that Bill cannot be in Boston, but must be in Berkeley instead.

Discussion

- In the contextualist/relativist dicussion, there are many interesting case studies that deserve our detailed attention.
- One thing we believe InqS can shed light on is epistemic *might* in the antecedent of an implication.
- Consider the contrast between (5) and (6):

(5) If John might go to the party, then I will not go.

(6) If John might go to the party, then Mary will not go.

- In InqS we can explain that (5) is quite alright, and that when no participant in the conversation rejects the antecedent in (5), then the speaker has committed himself to not go to the party.
- We can also explain that the acceptability of (6) depends on whether anyone involved in the conversation (might be Mary herself) can bring about whether Mary will go or not.

3. Accommodating presuppositions in InqS

Accommodating presuppositions

- The semantic apparatus of InqS might be rich enough to be able to deal with *certain* presuppositional phenomena.
- We could take it that:
  - φ presupposes ψ iff $\forall s$: if $s \models^{+} ?\varphi$, then $s \models^{+} \psi$
- Under this definition: $p \rightarrow q$ presupposes $\Diamond p$.
- Since InqS formulates conditions for dismissing a supposition, one should focus on presupposition failure rather than satisfaction.
- In turn this means that from an InqS perspective what matters most is whether presuppositions can be accommodated in a state, not whether they are already supported by it.

Presupposition failure as suppositional dismissal

- A natural candidate for a notion of presupposition failure is the InqS-notion of suppositional dismissal:

$$s \models^{\otimes} \varphi \text{ iff } s \models^{\circ} \varphi \text{ and } \forall t \subseteq s\colon\ t \not\models^{+} \varphi \text{ and } t \not\models^{-} \varphi$$

- Then in analogy with our characterization of when a sentence is suppositional, we could define when a sentence is presuppositional:

$$\varphi \text{ is suppositional iff } \exists s\colon\ s \neq \emptyset \text{ and } s \models^{\circ} \varphi$$

$$\varphi \text{ is presuppositional iff } \exists s\colon\ s \neq \emptyset \text{ and } s \models^{\otimes} \varphi$$

- Whereas suppositional content is cumulative, presuppositional content is not cumulative:
    - $p \to q$ is presuppositional
    - $p \wedge (p \to q)$ is suppositional but not presuppositional
    - $\neg p \vee (p \to q)$ is suppositional but not presuppositional

Suppositional atomic sentences

- Let a world w now be a partial valuation function such that for some atomic sentences, $w(p) \neq 1$ and $w(p) \neq 0$.
- Adapt the atomic clause in the following way:
  - $s \models^{+} p$ iff $s \neq \emptyset$ and $\forall w \in s: w(p) = 1$
    $s \models^{-} p$ iff $s \neq \emptyset$ and $\forall w \in s: w(p) = 0$
    $s \models^{\circ} p$ iff $\neg \exists w \in s: w(p) = 1$ or $w(p) = 0$
- Now, unlike before, some atomic sentences are (pre)suppositional.
- Assume, for the sake of the argument, that for any suppositional atomic sentence p, there is another non-suppositional atomic sentence, call it $\pi(p)$, such that for any world w:
  - $w(\pi(p)) = 1$ iff $w(p) = 1$ or $w(p) = 0$
    $w(\pi(p)) = 0$ iff $w(p) \neq 1$ and $w(p) \neq 0$

Presupposition cancellation

- With no further changes to the other semantic clauses, we obtain the following basic presupposition cancellation results for any suppositional atomic sentence p:
  - p is presuppositional and presupposes $(\pi)p$
  - $\pi(p) \wedge p$ is suppositional but not presuppositional
  - $\neg\pi(p) \vee p$ idem
  - $\pi(p) \rightarrow p$ is presuppositional, but only presupposes $\Diamond(\pi)p$
- Crucially, no additional features are needed beyond what is independently motivated in InqS for a general account of suppositions, in order to accomodate presuppositions as well.
- One thing the semantics as it is does not account for, to the extent that the phenomenon exists, is the directionality of presupposition cancellation.

Final remark

- One obvious question to ask is whether the semantics of epistemic modalities presented here can be extended to, e.g., deontic modalities.
- The latter have been studied by Martin Aher in his PhD-thesis within the framework of radical inquisitive semantics.
- He proposes a “modified Andersonian analysis” of deontic modalities, in which they are intimately linked with implication.
- In a joint talk we have ‘lifted’ this analysis to InqS, accounting simultaneously for both types of modalities, showing the structural similarities between the semantics of both types of modalities and the semantics of implication in InqS.
- The combined forces of both types of modalities shed new light on several of the “deontic puzzles” that have been discussed in the literature.

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