A Dynamic Analysis of Interactive Rationality

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(Joint work with Olivier Roy)

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"*The* fundamental insight of game theory [is] that a rational player must take into account that the players reason about each other in deciding how to play" [pg. 81]

R. Aumann and J. Dreze. *Rational expectations in games*. American Economic Review, Vol. 98, pgs. 72 – 86 (2008).

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Exactly *how* the players incorporate the fact that they are interacting with other (actively reasoning) rational agents is the subject of much debate.

Reasoning in Games

- Brian Skyrms' models of "dynamic deliberation"
- Ken Binmore's analysis of "eductive reasoning"
- Robin Cubitt and Robert Sugden's "common modes of reasoning"

Different framework, common thought: *the "rational solutions" of a game are the result of individual (rational) decisions in specific informational "contexts".*

Two Faces of Rationality

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"Neither theme alone exhausts our notion of rationality. Reasons without reliability seem emtpy, reliability without reasons seems blind. In tandem these make a powerful unit, but how exactly are they related and why?" (pg. 64)

R. Nozick. *The Nature of Rationality*. Princeton University Press, 1993.

Two Faces of Rationality

1. Rationality is a matter of *reasons*
2. Rationality is a matter of *reliability*

"It is important to understand that we have two forms of irrationality in this paper...For us, a player is rational if he optimizes and also rules nothing out. So irrationality might mean not optimizing. But it can also mean optimizing while not considering everything possible."

(pg. 314)

A. Brandenburger, A. Friedenberg and H. J. Keisler. *Admissibility in Games*. Econometrica, 76:2, 2008, pgs. 307 - 352.

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Plan for Today

- Describing the "informational context" of a game
- A puzzle about admissibility
- Flat vs. dynamic analysis

Describing the "Informational Context"

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  - imperfect information about the play of the game
  - incomplete information about the structure of the game
  - strategic information (what will the other players do?)
  - higher-order information (what are the other players thinking?)

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  - hard (“knowledge”)
  - soft (“beliefs”)

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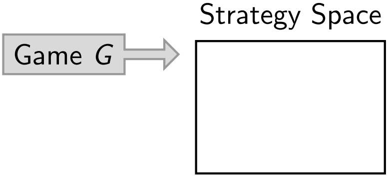
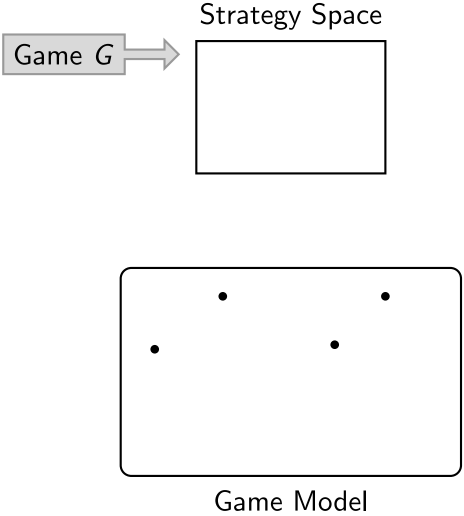
Game Models

Game G

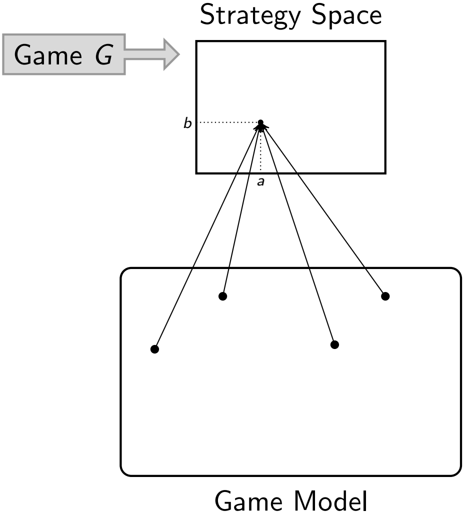
Game Models



Game Models



Game Models



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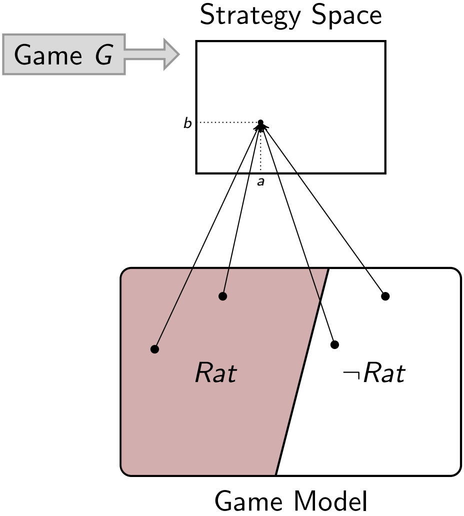
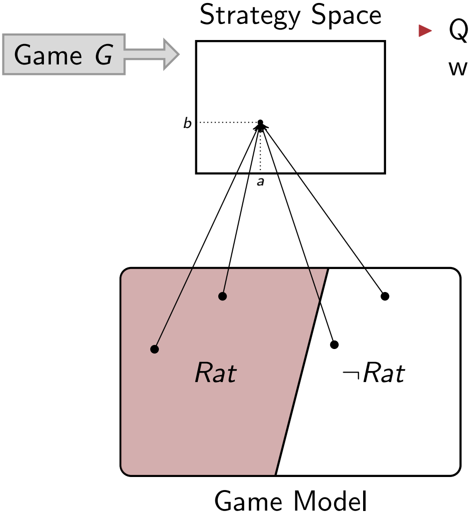
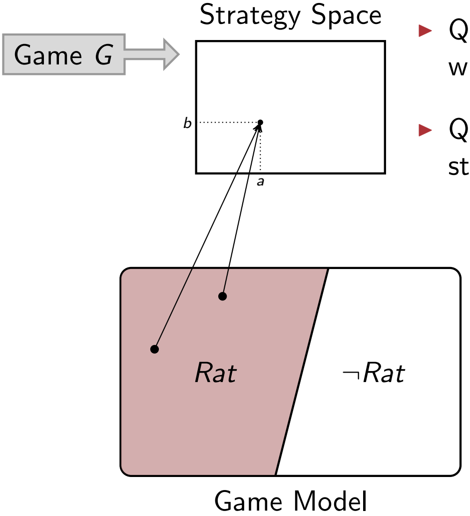


Game Models



- Q1: Can we always find a model where $Rat \neq \emptyset$?

Game Models



- Q1: Can we always find a model where $Rat \neq \emptyset$?
- Q2: Can we *characterize* the strategies that are *always* in Rat?

Some Key Properties

- Knowledge of own choice
  the agents choices are uniform in their information sets

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- *Common Knowledge* of "rational choice"
  there is no "Ann-Bob path" that leads outside of Rat

Other Natural Properties...

- Only play *admissible* strategies
- If two strategies are rational for an opponent, then neither can be "ruled out"
- Do not *initially* rule out any *types* of the other players

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- Only play *admissible* strategies

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...Lead to Puzzles and Paradoxes

L. Samuelson. *Dominated Strategies and Common Knowledge*. Games and Economic Behavior (1992).

R. Cubitt and R. Sugden. *Rationally Justiable Play and the Theory of Non-cooperative games*. Economic Journal, 104, pgs. 798 - 803, 1994.

R. Cubitt and R. Sugden. *Common reasoning in games: A Lewisian analysis of common knowledge of rationality*. Manuscript, 2011.

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Admissibility

The condition that the players incorporate admissibility *into their rationality calculations seems to conflict with the condition that the players think the other players are rational (there is a tension between admissibility and strategic reasoning)*

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Does assuming that it is commonly known that players play only admissible strategies lead to a process of iterated removal of weakly dominated strategies?

Admissibility

The condition that the players incorporate admissibility *into their rationality calculations seems to conflict with the condition that the players think the other players are rational (there is a tension between admissibility and strategic reasoning)*

Does assuming that it is commonly known that players play only admissible strategies lead to a process of iterated removal of weakly dominated strategies? No!

L. Samuelson. *Dominated Strategies and Common Knowledge*. Games and Economic Behavior (1992).

Iterated Admissibility

		Bob	
		L	*R*
Ann	*T*	1,1	1,0
	B	1,0	0,1

Iterated Admissibility

		Bob	
		L	R
Ann	T	1,1	1,0
	B	1,0	0,1

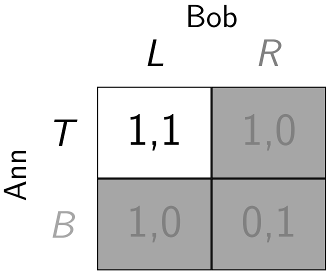
T weakly dominates B

Iterated Admissibility

		Bob	
		L	R
Ann	T	1,1	1,0
	B	1,0	0,1

Then L strictly dominates R.

Iterated Admissibility



		Bob	
		L	*R*
Ann	*T*	1,1	1,0
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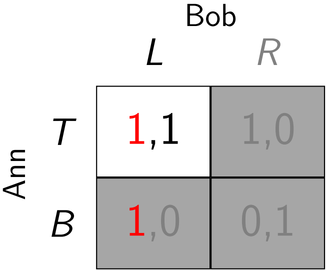
The IA set

Iterated Admissibility



		Bob	
		L	R
Ann	T	1,1	1,0
	B	1,0	0,1

But, now what is the reason for not playing B?

Common Knowledge of Admissibility

Bob

		L	*R*
Ann	*T*	1,1	1,0
	B	1,0	0,1

There is no model of this game with *common knowledge* of admissibility.

Common Knowledge of Admissibility

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		L	*R*
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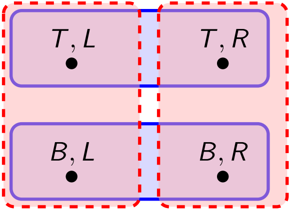

The "full" model of the game

Common Knowledge of Admissibility

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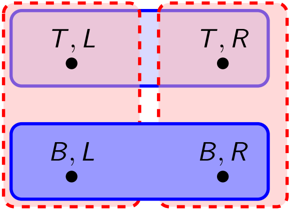
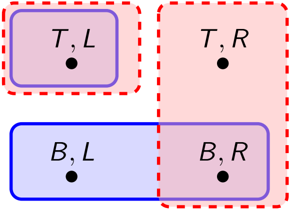


The "full" model of the game: *B is not admissible given Ann's information*

Common Knowledge of Admissibility

		Bob	
		L	R
Ann	T	1,1	1,0
	B	1,0	0,1



What is wrong with this model?

Common Knowledge of Admissibility

		Bob	
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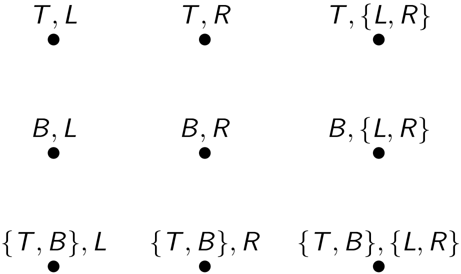
Ann	T	1,1	1,0
	B	1,0	0,1



Moving to choice *sets*.

Common Knowledge of Admissibility

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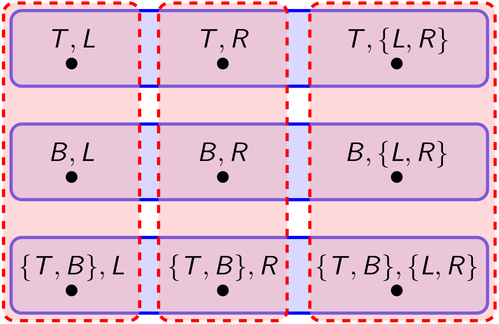

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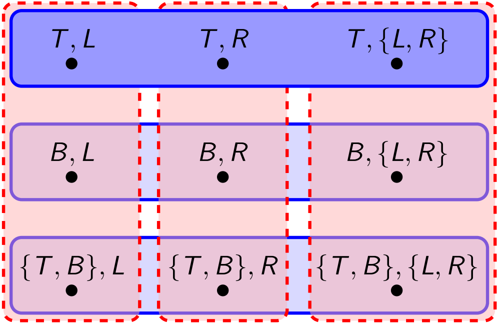
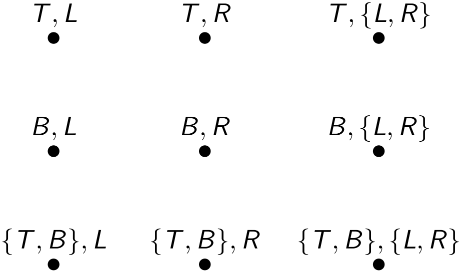


Ann thinks: Bob *has a reason to play L* OR Bob *has a reason to play R* OR Bob *has not yet settled on a choice*

Common Knowledge of Admissibility

		Bob	
		L	R
Ann	T	1,1	1,0
	B	1,0	0,1



T, L T, R $T, \{L, R\}$

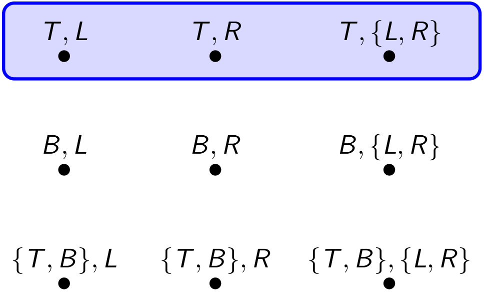
B, L B, R $B, \{L, R\}$

$\{T, B\}, L$ $\{T, B\}, R$ $\{T, B\}, \{L, R\}$

Still there is no model with common knowledge that players have *admissibility-based reasons*

Common Knowledge of Admissibility

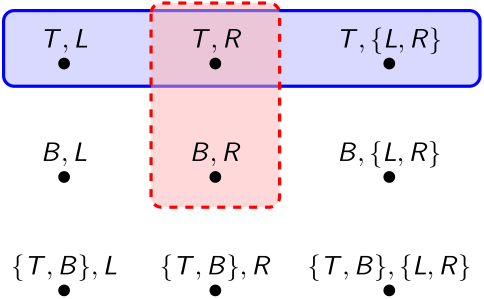
		Bob	
		L	R
Ann	T	1,1	1,0
	B	1,0	0,1



there is a reason to play T provided Ann considers it possible that Bob might play R (actually three cases to consider here)

Common Knowledge of Admissibility

		Bob	
		L	R
Ann	T	1,1	1,0
	B	1,0	0,1

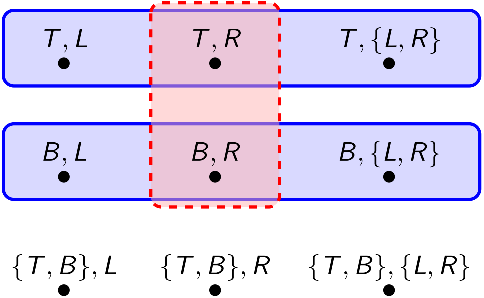


T, L	T, R	$T, \{L, R\}$
B, L	B, R	$B, \{L, R\}$
$\{T, B\}, L$	$\{T, B\}, R$	$\{T, B\}, \{L, R\}$

But there is a reason to play R provided it is possible that Ann has a reason to play B

Common Knowledge of Admissibility

		Bob	
		L	R
Ann	T	1,1	1,0
	B	1,0	0,1



But, there is no reason to play B if there is a reason for Bob to play R.

Common Knowledge of Admissibility

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		L	R
Ann	T	1,1	1,0
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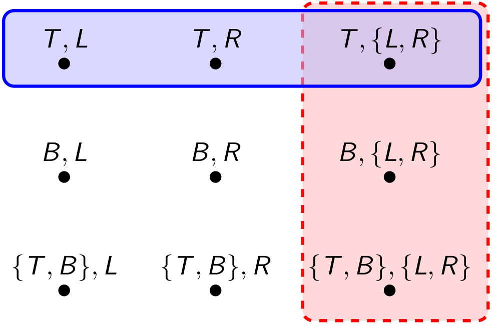

T, L	T, R	$T, \{L, R\}$
B, L	B, R	$B, \{L, R\}$
$\{T, B\}, L$	$\{T, B\}, R$	$\{T, B\}, \{L, R\}$

R can be ruled out unless there is a possibility that B will be played.

Common Knowledge of Admissibility

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		L	R
Ann	T	1,1	1,0
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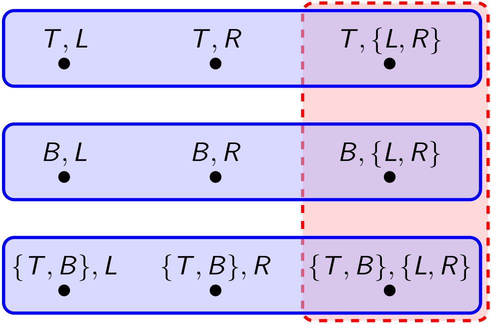

T, L T, R $T, \{L, R\}$

B, L B, R $B, \{L, R\}$

$\{T, B\}, L$ $\{T, B\}, R$ $\{T, B\}, \{L, R\}$

there is no reason to play B if R is a *possible play for Bob*.

Common Knowledge of Admissibility

		Bob	
		L	R
Ann	T	1,1	1,0
	B	1,0	0,1

T, L ●	T, R ●	$T, \{L, R\}$ ●
B, L ●	B, R ●	$B, \{L, R\}$ ●
$\{T, B\}, L$ ●	$\{T, B\}, R$ ●	$\{T, B\}, \{L, R\}$ ●

We can check all the possibilities and see we cannot find a model...

More Puzzles

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Another Puzzle

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	in_2	out_2
in_1	$1, 1, 1$	$1, 1, 1$
out_1	$1, 1, 1$	$0, 1, 1$

in_3

	in_2	out_2
in_1	$1, 1, 1$	$1, 0, 1$
out_1	$1, 1, 0$	$0, 0, 0$

out_3

There is no Bayesian model of the above game satisfying privacy of tie-breaking.

Another Puzzle

	in_2	out_2
in_1	$1, 1, 1$	$1, 1, 1$
out_1	$1, 1, 1$	$0, 1, 1$

in_3

	in_2	out_2
in_1	$1, 1, 1$	$1, 0, 1$
out_1	$1, 1, 0$	$0, 0, 0$

out_3

1. If 1 considers out_2 possible, then it is common knowledge that out_1 is not possible

Another Puzzle

	in_2	out_2
in_1	1, 1, 1	1, 1, 1
out_1	1, 1, 1	0, 1, 1

in_3

	in_2	out_2
in_1	1, 1, 1	1, 0, 1
out_1	1, 1, 0	0, 0, 0

out_3

1. If 1 considers out_2 possible, then it is common knowledge that out_1 is not possible
2. If 2 considers out_3 possible, then it is common knowledge that out_2 is not possible

Another Puzzle

	in_2	out_2
in_1	$1, 1, 1$	$1, 1, 1$
out_1	$1, 1, 1$	$0, 1, 1$

in_3

	in_2	out_2
in_1	$1, 1, 1$	$1, 0, 1$
out_1	$1, 1, 0$	$0, 0, 0$

out_3

1. If 1 considers out_2 possible, then it is common knowledge that out_1 is not possible
2. If 2 considers out_3 possible, then it is common knowledge that out_2 is not possible
3. If 3 considers out_1 possible, then it is common knowledge that out_3 is not possible

Another Puzzle

	in_2	out_2
in_1	1, 1, 1	1, 1, 1
out_1	1, 1, 1	0, 1, 1

in_3

	in_2	out_2
in_1	1, 1, 1	1, 0, 1
out_1	1, 1, 0	0, 0, 0

out_3

4. If 1 does not consider out_2 possible, then 2 & 3 must consider in_1 & out_1 possible

Another Puzzle

	in_2	out_2
in_1	$1, 1, 1$	$1, 1, 1$
out_1	$1, 1, 1$	$0, 1, 1$

in_3

	in_2	out_2
in_1	$1, 1, 1$	$1, 0, 1$
out_1	$1, 1, 0$	$0, 0, 0$

out_3

4. If 1 does not consider out_2 possible, then 2 & 3 must consider in_1 & out_1 possible
5. If 2 does not consider out_3 possible, then 1 & 3 must consider in_2 & out_2 possible

Another Puzzle

	in_2	out_2
in_1	$1, 1, 1$	$1, 1, 1$
out_1	$1, 1, 1$	$0, 1, 1$

in_3

	in_2	out_2
in_1	$1, 1, 1$	$1, 0, 1$
out_1	$1, 1, 0$	$0, 0, 0$

out_3

4. If 1 does not consider out_2 possible, then 2 & 3 must consider in_1 & out_1 possible
5. If 2 does not consider out_3 possible, then 1 & 3 must consider in_2 & out_2 possible
6. If 3 does not consider out_1 possible, then 1 & 2 must consider in_3 & out_3 possible

Another Puzzle

	in_2	out_2
in_1	$1, 1, 1$	$1, 1, 1$
out_1	$1, 1, 1$	$0, 1, 1$

in_3

	in_2	out_2
in_1	$1, 1, 1$	$1, 0, 1$
out_1	$1, 1, 0$	$0, 0, 0$

out_3

- If i considers out_{i+1} possible, then it is common knowledge that out_i is not possible
- If i does not consider out_{i+1} possible, then $i + 1$ & $i + 2$ must consider in_i & out_i possible

Another Puzzle

	in_2	out_2
in_1	$1, 1, 1$	$1, 1, 1$
out_1	$1, 1, 1$	$0, 1, 1$

in_3

	in_2	out_2
in_1	$1, 1, 1$	$1, 0, 1$
out_1	$1, 1, 0$	$0, 0, 0$

out_3

- If i considers out_{i+1} possible, then it is common knowledge that out_i is not possible
- If i does not consider out_{i+1} possible, then $i+1$ & $i+2$ must consider in_i & out_i possible
- 1 does consider out_2 possible $\implies$ 3 does not consider out_1 possible $\implies$ 2 considers out_3 possible $\implies$ 1 does not consider out_2 possible

Diagnosing the Issues

Moving away from a “flat” model....

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Diagnosing the Issues

Moving away from a “flat” model....

- *Describing* ideally rational agents vs. *explaining* how ideally rational agents will interact. (where do the models come from?)
- Rationality as a property of the players’ *choice* vs. rationality as a property of the players’ *reasoning*
- We want “optimal choice” to be a parameter (maximize expected utility, minmax, minregret, heuristics, etc.).

Diagnosing the Issues

Moving away from a “flat” model....

- *Describing* ideally rational agents vs. *explaining* how ideally rational agents will interact. (where do the models come from?)
- Rationality as a property of the players' *choice* vs. rationality as a property of the players' *reasoning*
- We want “optimal choice” to be a parameter (maximize expected utility, minmax, minregret, heuristics, etc.).
- Dynamic logics are just better...

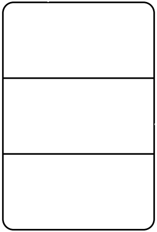
Ingredients of a Dynamic Analysis of Common Knowledge of Rationality

- Dynamic analysis of informational attitudes

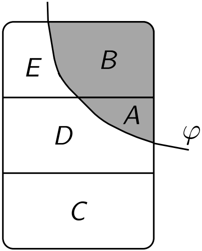
- Incorporating practical reasoning

- Integrating the two aspects of rational strategic reasoning

Informative Actions

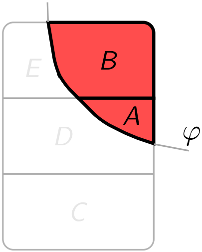


Informative Actions



Incorporate the new information φ

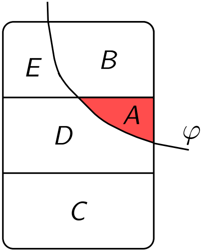
Informative Actions



Public Announcement: Information from an infallible source

$(!\varphi)$: $A \prec_i B$

Informative Actions



Public Announcement: Information from an infallible source
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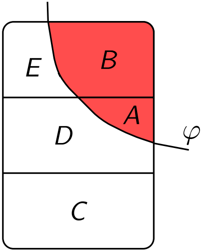
Conservative Upgrade: Information from a trusted source
$(\uparrow\varphi)$: $A \prec_i C \prec_i D \prec_i B \cup E$

Informative Actions



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Radical Upgrade: Information from a strongly trusted source
$(\Uparrow\varphi)$: $A \prec_i B \prec_i C \prec_i D \prec_i E$

Dynamic Characterization of Informational Attitudes

$!\varphi_1, !\varphi_2, !\varphi_3, \ldots, !\varphi_n$
always reaches a fixed-point

$\Uparrow p \Uparrow \neg p \Uparrow p \cdots$
Contradictory beliefs leads to oscillations

$\uparrow\varphi, \uparrow\varphi, \ldots$
Simple beliefs may never stabilize

$\Uparrow\varphi, \Uparrow\varphi, \ldots$
Simple beliefs stabilize, but conditional beliefs do not

A. Baltag and S. Smets. *Group Belief Dynamics under Iterated Revision: Fixed Points and Cycles of Joint Upgrades*. TARK, 2009.

Ingredients of a Dynamic Analysis of Common Knowledge of Rationality

✓ Dynamic analysis of informational attitudes

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R. Cubitt and R. Sugden. *The reasoning-based expected utility procedure*. Games and Economic Behavior, 2010.

- Integrating the two aspects of rational strategic reasoning

Reasoning-Based Solution Concepts

A **categorization** is a *ternary* partition of the players choices (rather than a binary partition of what is in and what is out):

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Example: RBEU (reasoning based expected utility):

- accumulate strategies that maximize expected utility for every possibly probability distribution
- delete strategies that do not maximize probability against any probability distribution
- accumulated strategies must receive positive probability, deleted strategies must receive zero probability

RBEU: Example

	L	R
T	1,1	1,1
M_1	0,0	1,0
M_2	2,0	0,0
B	0,2	0,0

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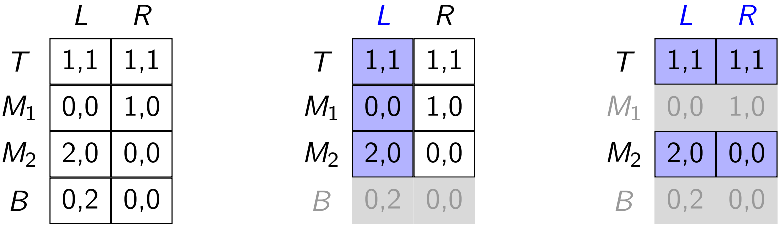

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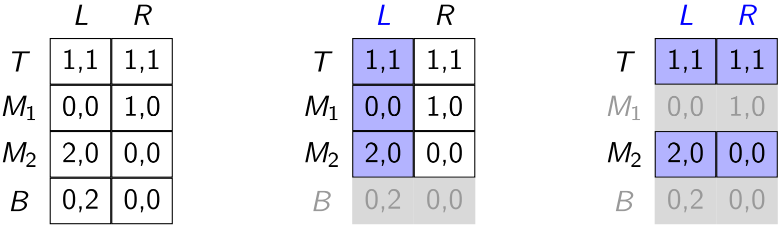

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RBEU: Another Example

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U	1,1	1,0
D	1,0	0,1

RBEU: Another Example

	L	R
U	1,1	1,0
D	1,0	0,1

$$S^+ = \{U\}$$
$$S^- = \emptyset$$

RBEU: Another Example

	L	R
U	1,1	1,0
D	1,0	0,1

	L	R
U	1,1	1,0
D	1,0	0,1

$$S^{+} = \{U\}$$
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RBEU: Another Example

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U	1,1	1,0
D	1,0	0,1

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U	1,1	1,0
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$$S^+ = \{U\}$$
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$$\begin{tikzcd}
\mathcal{M}_0 \arrow[r, Rightarrow, "!\varphi_1"] & \mathcal{M}_1 \arrow[r, Rightarrow, "!\varphi_2"] & \mathcal{M}_2 \arrow[r, Rightarrow, "!\varphi_3"] & \cdots \arrow[r, Rightarrow, "!\varphi_n"] & \mathcal{M}_f
\end{tikzcd}$$

initial
model

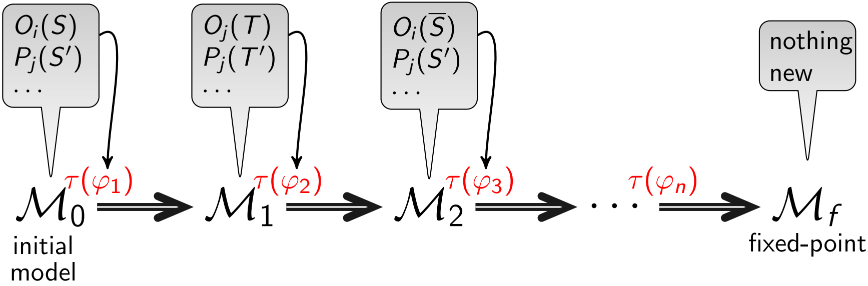
fixed-point

$$\begin{tikzcd}
\mathcal{M}_0 \arrow[r, Rightarrow, "\Uparrow\varphi_1"] & \mathcal{M}_1 \arrow[r, Rightarrow, "\Uparrow\varphi_2"] & \mathcal{M}_2 \arrow[r, Rightarrow, "\Uparrow\varphi_3"] & \cdots \arrow[r, Rightarrow, "\Uparrow\varphi_n"] & \mathcal{M}_f \\
\text{initial model} & & & & \text{fixed-point}
\end{tikzcd}$$

$$\begin{tikzcd}
\underset{\substack{\text{initial}\\\text{model}}}{\mathcal{M}_0} \arrow[r, Rightarrow, "!\varphi_1"] & \mathcal{M}_1 \arrow[r, Rightarrow, "\Uparrow\varphi_2"] & \mathcal{M}_2 \arrow[r, Rightarrow, "\uparrow\varphi_3"] & \cdots \arrow[r, Rightarrow, "\Uparrow\varphi_n"] & \underset{\text{fixed-point}}{\mathcal{M}_f}
\end{tikzcd}$$

$$\begin{tikzcd}
\mathcal{M}_0 \arrow[r, Rightarrow, "\tau(\varphi_1)"] & \mathcal{M}_1 \arrow[r, Rightarrow, "\tau(\varphi_2)"] & \mathcal{M}_2 \arrow[r, Rightarrow, "\tau(\varphi_3)"] & \cdots \arrow[r, Rightarrow, "\tau(\varphi_n)"] & \mathcal{M}_f \\
\text{initial model} & & & & \text{fixed-point}
\end{tikzcd}$$

Where do the φ_k come from?



$O_i(S)$
$P_j(S')$
$\ldots$

$O_j(T)$
$P_j(T')$
$\ldots$

$O_i(\overline{S})$
$P_j(S')$
$\ldots$

nothing new

$$\mathcal{M}_0 \xRightarrow{\tau(\varphi_1)} \mathcal{M}_1 \xRightarrow{\tau(\varphi_2)} \mathcal{M}_2 \xRightarrow{\tau(\varphi_3)} \cdots \xRightarrow{\tau(\varphi_n)} \mathcal{M}_f$$

initial model

fixed-point

Where do the φ_k come from? from the players practical reasoning/rational requirements

Our Framework

Strategic game: $G = \langle N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle$

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Strategies in Play

$S_{-i}(\mathcal{M}_G) = \{s_{-i} \in \Pi_{j\neq i} S_j \mid \exists w \in Min_{\preceq}(W) \text{ such that } \sigma_{-i}(w) = s_{-i}\}$

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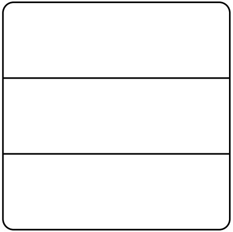
Categorization

$\mathbf{S}_i(\mathcal{M}_G) = (S_i^+, S_i^-)$ where $S_i^+ \cup S_i^- \subseteq S_i$ and

for each $a \in S_i$, if there is no $v \in W$ with $\sigma_i(v) = a$ then $a \in S_i^-$

Responding to a Categorization



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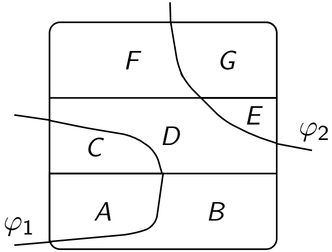

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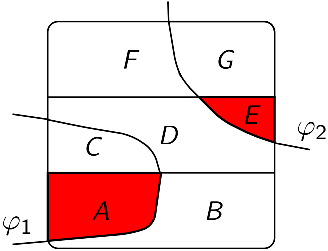

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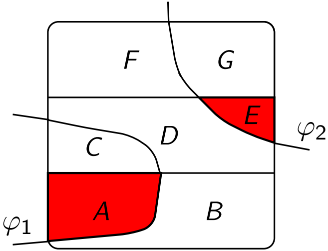
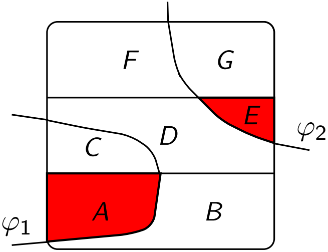


$$\uparrow\{\varphi_1, \varphi_2\} : A \cup E \prec B \prec C \cup D \prec F \cup G$$

Responding to a Categorization



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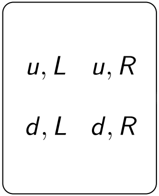
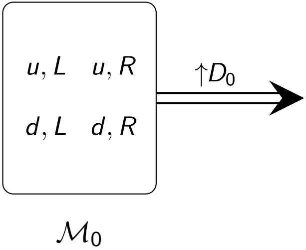


u, L u, R

d, L d, R

$\mathcal{M}_0$

	L	R
U	1,1	1,0
D	1,0	0,1



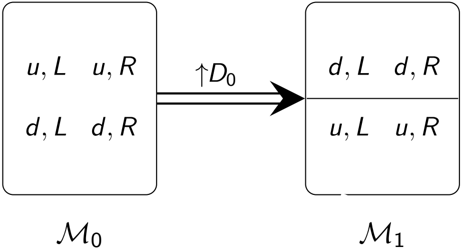
$u, L \quad u, R$

$d, L \quad d, R$

$\uparrow D_0$

$\mathcal{M}_0$

	L	R
U	1,1	1,0
D	1,0	0,1



u, L u, R

d, L d, R

$\uparrow D_0$

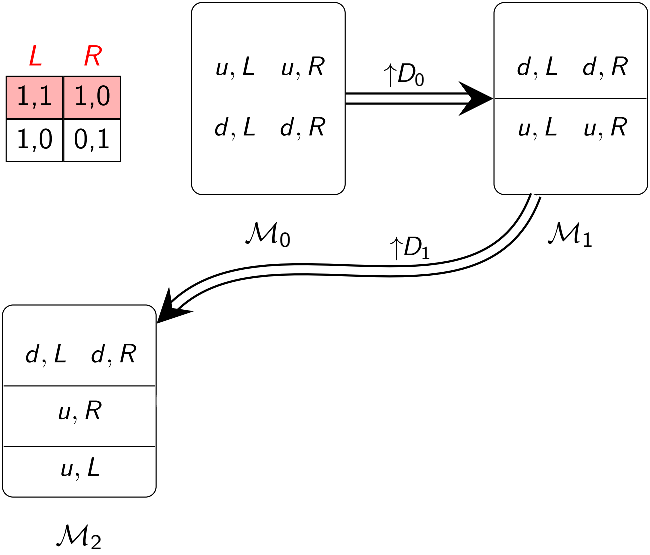
d, L d, R

u, L u, R

$\mathcal{M}_0$

$\mathcal{M}_1$

	L	R
U	1,1	1,0
D	1,0	0,1



u, L u, R

d, L d, R

$\mathcal{M}_0$

$\Uparrow D_0$

d, L d, R

u, L u, R

$\mathcal{M}_1$

$\Uparrow D_1$

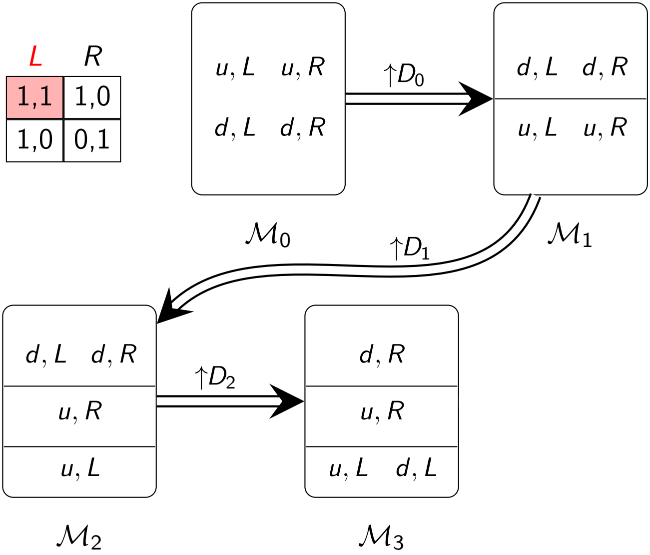
d, L d, R

u, R

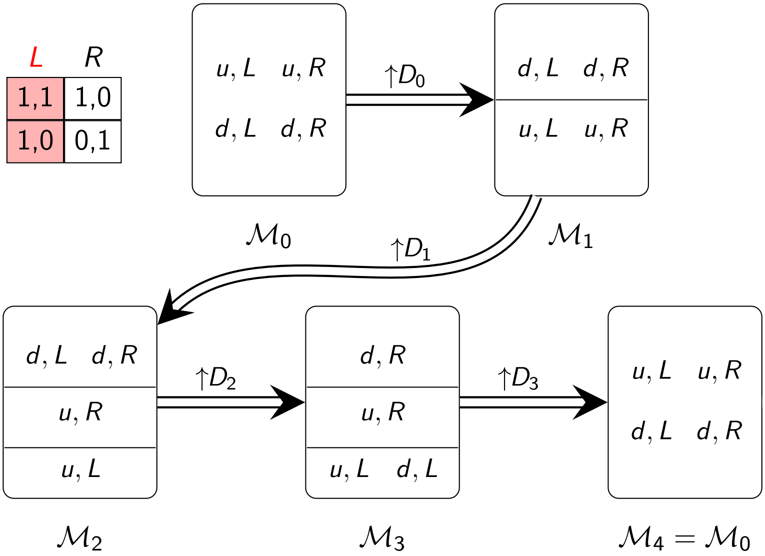
u, L

$\mathcal{M}_2$

	L	R
U	1,1	1,0
D	1,0	0,1



	L	R
U	1,1	1,0
D	1,0	0,1



$u, L \quad u, R$

$d, L \quad d, R$

$\mathcal{M}_0$

$\Uparrow D_0$

$d, L \quad d, R$

$u, L \quad u, R$

$\mathcal{M}_1$

$\Uparrow D_1$

$d, L \quad d, R$

u, R

u, L

$\mathcal{M}_2$

$\Uparrow D_2$

d, R

u, R

$u, L \quad d, L$

$\mathcal{M}_3$

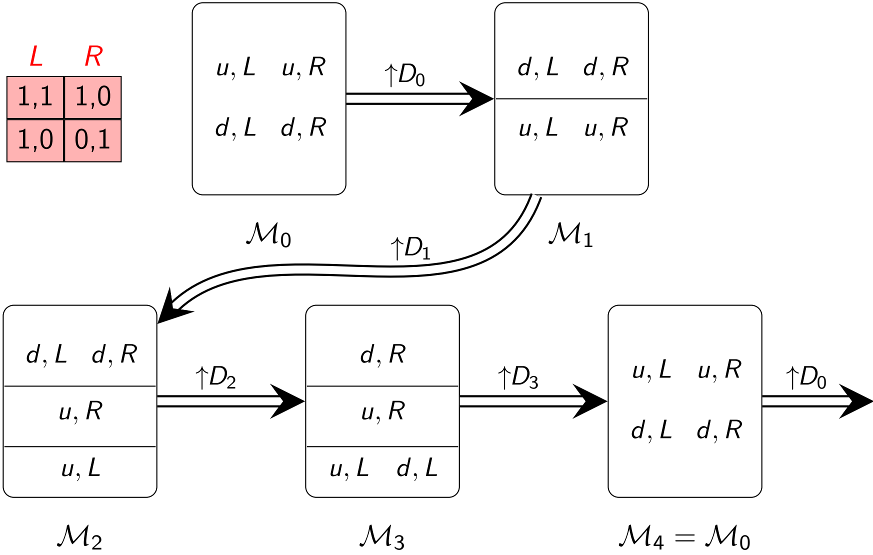
$\Uparrow D_3$

$u, L \quad u, R$

$d, L \quad d, R$

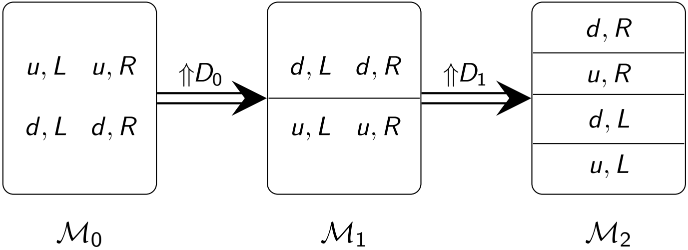
$\mathcal{M}_4 = \mathcal{M}_0$

	L	R
U	1,1	1,0
D	1,0	0,1



Remembering Reasons

	L	R
U	1,1	1,0
D	1,0	0,1



u, L u, R

d, L d, R

$\Uparrow D_0$

d, L d, R

u, L u, R

$\Uparrow D_1$

d, R

u, R

d, L

u, L

$\mathcal{M}_0$ $\mathcal{M}_1$ $\mathcal{M}_2$

Discussion: Common Knowledge of Rationality

Common Knowledge of Rationality

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Common Knowledge of Rationality

Discussion: Common Knowledge of Rationality

Common Knowledge of Rationality

Baseline Result

$\tau : \mathbb{M} \times \wp(\mathcal{L}_G) \to \mathbb{M}$, write $\mathcal{M}^{\tau(\mathcal{X})}$ for $\tau(\mathcal{M}, \mathcal{X})$

Let $\mathcal{X}_{\mathcal{M}} = \{[\![\varphi]\!]_{\mathcal{M}} \mid \varphi \in \mathcal{X}\}$.

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If $a \in S_i^-(\mathcal{M})$ then $\mathcal{M}^{\tau(\mathcal{X})} \models B\neg P_i^a$

If $a \in S_i^+(\mathcal{M})$ then $\mathcal{M}^{\tau(\mathcal{X})} \models \neg B\neg P_i^a$

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$S_i^-(\mathcal{M}) \subseteq S_i^-(\mathcal{M}^{\tau(Do(\mathcal{M}))})$
$S_i^+(\mathcal{M}) \subseteq S_i^+(\mathcal{M}^{\tau(Do(\mathcal{M}))})$
$S_i^+(\mathcal{M}) \supseteq S_i^+(\mathcal{M}^{\tau(Do(\mathcal{M}))})$

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Theorem. Suppose that G is a finite game and $\mathcal{M}_G$ a (finite) initial model. If a categorization (method) is belief sensitive and monotonic on a upgrade sequence $(\mathcal{M}_m)_{m\in\mathbb{N}}$, then the upgrade stream stabilizes.

Related Ideas

Think of the choice rule as a predicate $\varphi(s, X, Y)$ expressing "s is 'optimal' in X given the other's choices Y"

K. Apt and J. Zvesper. *The Role of Monotonicity in the Epistemic Analysis of Strategic Games*. Games 1(4), 2010, pp. 381–394.

Look at general properties of choice rules

M. Trost. *On the Equivalence of Iterated Application of a Choice Rule and Common Belief of Applying that Rule*. Manuscript, 2010.

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- players should not respond to every *model change* (eg., even rational players should not play differently in bisimilar models).
- reasoning about what to do: choices may be accepted (there is a reason to play it), deleted (there is a reason to not play it) or neither (no reason either way)
- many parameters to play with: optimal choice, type of update/upgrade, what announcements are "admissible" (the *protocol*)

Thank You!

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L. Samuelson. *Dominated Strategies and Common Knowledge*. Games and Economic Behavior (1992).

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Results

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2. There exist games in which assuming that admissibility is common knowledge does not provide players with sufficient information to determine which strategies should be eliminated on admissibility grounds.
3. There exists games in which assuming that admissibility is common knowledge yields a contradiction (i.e., there is no model of a game where there is common knowledge of "admissible choice")

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Common Knowledge of Admissibility

Theorem Iterated admissibility is not equivalent to common knowledge of admissibility.

	Y_1	Y_2	Y_3
X_1	2,4	5,4	-1,0
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Where does common knowledge come from?

R. Cubitt and R. Sugden. *Common Knowledge, Salience and Convention: A Reconstruction of David Lewis' Game Theory*. Economics and Philosophy, 19, pgs. 175-210 , 2003..

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Reason to Believe

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- Anyone who accept the rules of arithmetic has a reason to believe $618 \times 377 = 232,986$, but most of us do not hold have firm beliefs about this.
- Definition: $R_i(\varphi)$ means φ is true within some logic of reasoning that is *endorsed* by (that is, accepted as a normative standard by) person i...φ must be either regarded as *self-evident* or derivable by rules of inference (deductive or inductive)

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A indicates to i that φ

A is a "state of affairs"

A *ind*$_i$ φ: i's reason to believe that A holds *provides* i's reason for believing that φ is true.

(A1)For all i, for all A, for all φ: $[R_i(A\ \textit{holds}) \wedge (A\ \textit{ind}_i\ \varphi)] \Rightarrow R_i(\varphi)$

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- $[(A\ \mathit{ind}_i\ R_j[A'\ \mathit{holds}]) \wedge R_i(A'\ \mathit{ind}_j \varphi)] \Rightarrow A\ \mathit{ind}_i R_j(\varphi)$

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Reflexive Common Indicator

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- $(A \textit{ ind}_i \; \psi) \Rightarrow R_i[A \textit{ ind}_j \; \psi]$

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Let $R^G(\varphi)$: $R_i\varphi, R_j\varphi, \ldots$, $R_i(R_j\varphi)$, $R_j(R_i(\varphi))$, $\ldots$
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iterated reason to believe φ.

Theorem. (Lewis) For all states of affairs A, for all propositions φ, and for all groups G: if A holds, and if A is a reflexive common indicator in G that φ, then $R^G(\varphi)$ is true.

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Lewis and Aumann

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Example. Suppose there is an agent $i \notin G$ that is *authoritative* for each member of G. So, for $j \in G$, "i states to j that φ is true" *indicates to j that φ*. Suppose that *separately and privately* to each member of G, i states that φ and $R^G(\varphi)$ are true.Then, we have $R^i\varphi$ and $R_i(R^G(\varphi))$ for each $i \in G$. *But there is no common indicator that φ is true.* The agents $j \in G$ may have no reason to believe that everyone heard the statement from i or that all agents in G treat i as authoritative.

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