Games for Learning
A Sabotage Approach

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Outline

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AIM

To highlight the *interactive* nature of the learning process by showing its relation with games.

Our perspective on learning

- High-level analysis of inductive inference.
- Singling out a correct hypothesis from a range of possibilities.
- Many steps of "update" before the conclusion is reached.

Successful Learning

Definition

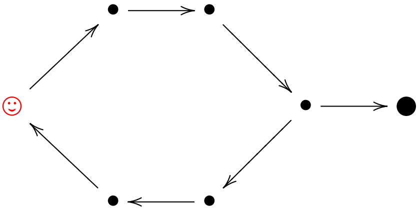
Learner identifies Teacher's choice *in the limit* if after some finite number of guesses his choices stabilize on a correct hypothesis.

Definition

Learner *finitely* identifies Teacher's choice if after some finite number of guesses he makes the right choice.

Outline

Sabotage Game – Example



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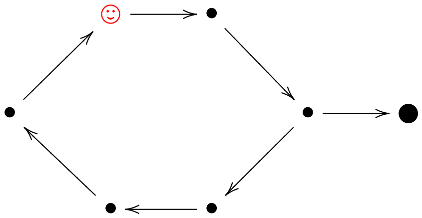

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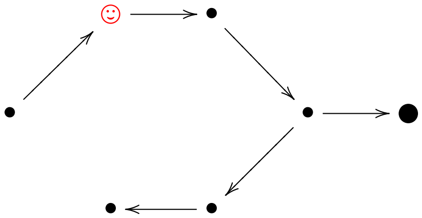

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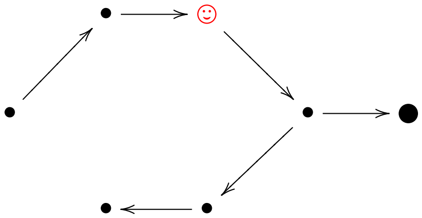

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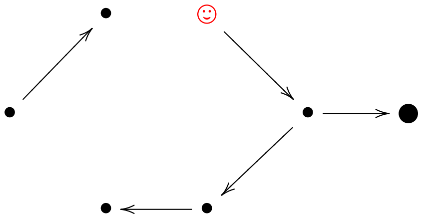

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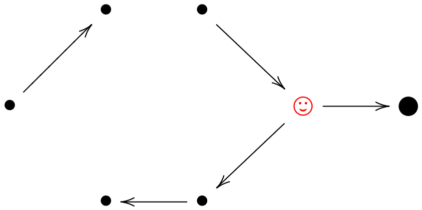

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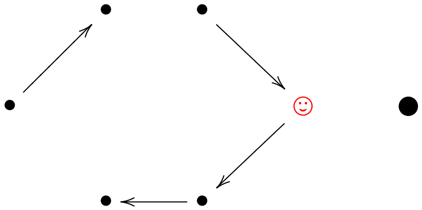

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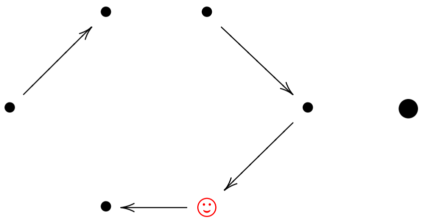

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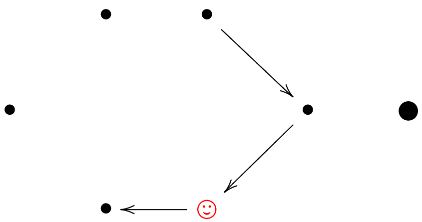

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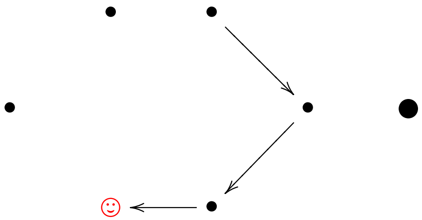

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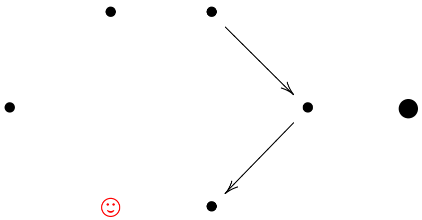

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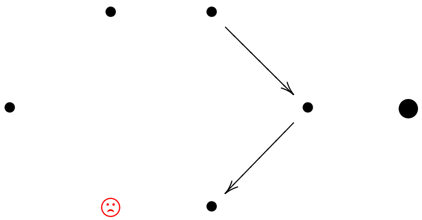


Sabotage Modal Logic

Definition (Sabotage Modal Language)

$$\phi ::= p \mid \neg\phi \mid \phi \vee \phi \mid \Diamond_a \phi \mid \diamondminus_a \phi$$

with $p \in \mathrm{PROP}$ and $a \in \Sigma$ (finite).

$$\Diamond\phi := \bigvee_{a\in\Sigma} \Diamond_a \phi \qquad \diamondminus\,\phi := \bigvee_{a\in\Sigma} \diamondminus_a \phi$$

Definition (Sabotage Model)

$M = \langle W, \{R_a \mid a \in \Sigma\}, Val\rangle$ where

$$W \neq \varnothing, \qquad R_a \subseteq W \times W, \qquad Val : \mathrm{PROP} \to \mathcal{P}(W)$$

Sabotage Modal Logic

Definition (Removal Operation)

Let $M = \langle W, \{R_a \mid a \in \Sigma\}, Val \rangle$ be a Sabotage Model.

$$M^a_{(u,v)} := \langle W, \{R_b \mid b \in \Sigma \setminus \{a\}\} \cup R_a \setminus \{(u,v)\}, Val \rangle$$

Definition (Semantics)

$$M, w \models \diamondminus_a \phi \quad \text{iff} \quad \text{there is } (u,v) \in R_a \text{ s. t. } M^a_{(u,v)}, w \models \phi$$

Theorem

Model checking of SML is PSPACE-complete.

Sabotage Learning Game

Definition

A *Sabotage Learning Game* is a Sabotage Game played between *Learner* and *Teacher* on a directed multi-graph with an initial vertex and a "goal" vertex.

Various scenarios

Game	Winning Condition
SLG_{UE}	Learner wins iff he reaches the goal state, Teacher wins otherwise.
SLG_{HU}	Teacher wins iff Learner reaches the goal state, Learner wins otherwise.
SLG_{HE}	Both players win iff Learner reaches the goal state, Both lose otherwise.

Characterization Results

Theorem

Game	Existence of winning strategy	Winner
SLG_{UE}	$\gamma_0^{UE} := goal$, $\gamma_{n+1}^{UE} := goal \vee \Diamond \boxminus \gamma_n^{UE}$	*Learner*
SLG_{HU}	$\gamma_0^{HU} := goal$, $\gamma_{n+1}^{HU} := goal \vee (\Diamond\top \wedge (\Box \diamondminus \gamma_n^{HU}))$	*Teacher*
SLG_{HE}	$\gamma_0^{HE} := goal$, $\gamma_{n+1}^{HE} := goal \vee \Diamond \diamondminus \gamma_n^{HE}$	*Both*

Complexity of Sabotage-Type Learning

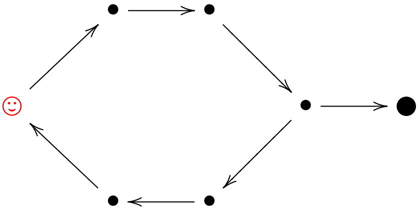
Game	Winning Condition	Complexity
SLG_{UE}	Learner wins iff he reaches the goal state, Teacher wins otherwise	PSPACE-complete.
SLG_{HU}	Teacher wins iff Learner reaches the goal state, Learner wins otherwise.	PSPACE
SLG_{HE}	Both players win iff Learner reaches the goal state. Both lose otherwise.	NL-complete.

Outline

Local vs global moves

- Players moves are of a different nature:
  - Learner moves by *local* transitions.
  - Teacher moves by *globally* removing an edge.
- Teacher only needs to act when necessary.

Non-strict alternation – Example



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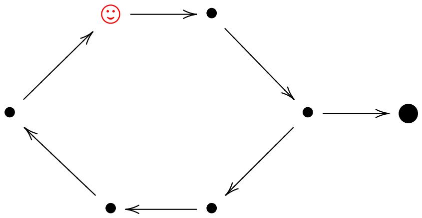

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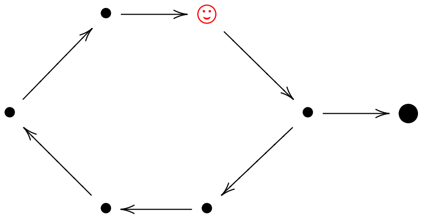

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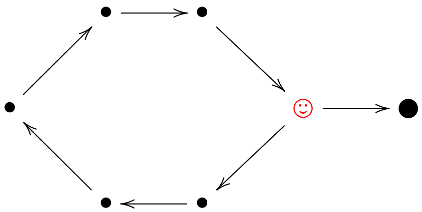

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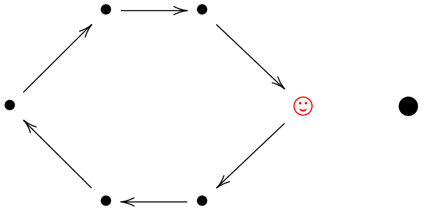

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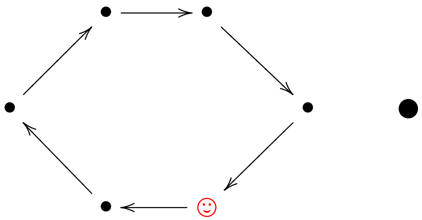

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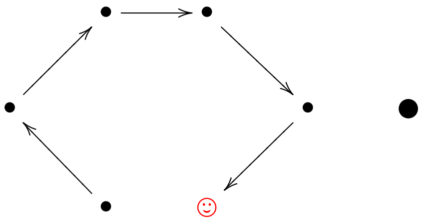

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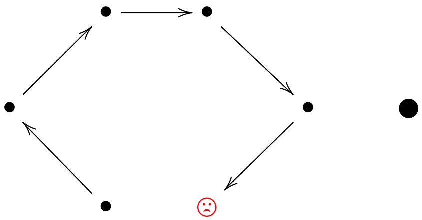


Different game, same scenarios

Game	Winning Condition
SLG^*_{UE}	Learner wins iff he reaches the goal state, Teacher wins otherwise.
SLG^*_{HU}	Teacher wins iff Learner reaches the goal state, Learner wins otherwise.
SLG^*_{HE}	Both players win iff Learner reaches the goal state, Both lose otherwise.

Strict vs non-strict alternation

Theorem

1. *Learner has a w.s. in* SLG^*_{UE} *iff he has a w.s. in* SLG_{UE}.
2. *Teacher has a w.s. in* SLG^*_{HU} *iff she has a w.s. in* SLG_{HU}.
3. *Teacher and Learner have a joint w.s. in* SLG^*_{HE} *iff they have a joint w.s. in* SLG_{HE}.

Corollary

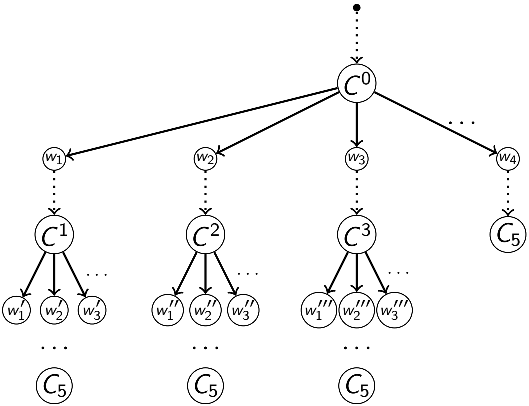
Formulas provided before characterize existence of a winning strategy in SLG^*_{UE}, SLG^*_{HU} and SLG^*_{HE}, resp.

Outline

Game Theory and Learning Theory

- The use of GT in LT.
- The use of LT in Games.

The Game of Queries and Counterexamples



GAME THEORETIC APPROACH TO INDUCTIVE INFERENCE

- Epistemic status of the players, imperfect information, payoff characteristics.
- Choice for Learner
  1. at each step the learner can choose from one or more procedures which are part of one algorithm;
  2. in the beginning Learner can decide with which of the available algorithms he is going to proceed.

Learning Theory in Games

- Inductive inference games.
- Eleusis (identification in the limit), Zendo (queries and counterexamples).
- Complexity analysis of corresponding algorithms.
- Empirical work 1: difficulty vs. complexity of hidden rules.
- Empirical work 2: knowledge reports.

Outline

Summary and Conclusion

Aim

Provide a high-level game-theoretical perspective on formal learning theory.
In particular, study strategic abilities, information flow and interaction.

Summary

Game-theoretical approach to learning that accounts for different levels of cooperativeness between Learner and Teacher.

Further Work

- Identification in the limit (stable positions).
  - Epistemic and doxastic interpretation: operational, non-introspective knowledge.
  - Fixed-point logics for identification in the limit.
- GT approach to learning algorithms.
- Inductive inference games - theoretical and empirical account.

Dziękuję, Danke and Gracias