Negation as Lack of Sufficient Evidence

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Overview

- The need for negation as lack of sufficient evidence
  - The problem of expressive adequacy
  - Williamson's argument
- Some strategies
  - First-order languages
  - Stable warrant and temporal operators
  - The DeVidi-Solomon approach
- Negation as lack of sufficient evidence
  - A labeled tableaux system
  - A sequent calculus
  - Philosophical remarks

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Expressive adequacy I

- Suppose ♣ is a connective expressed by ordinary language expressions whose use is governed by a coherent set of rules.[1] Then if **L** is a logic in the language $\mathcal{L}$ for ordinary discourse, **L** is **expressively adequate w.r.t. ♣** if there is a connective $\otimes$ in **L** which **conservatively** extends the $\otimes$-less fragment of **L** and which corresponds to ♣. E.g. (a connective for) truth-functional conjunction may be added to the implicational fragment of classical logic by adding the usual intelim rules.
- If classical negation $\sim$ is a coherent connective then neither intuitionistic nor classical logic is expressively adequate, for adding classical negation to (the positive fragment of) either logic yields a nonconservative extension. E.g. Peirce's law $((A \to B) \to B) \to B)$ becomes provable. (Indeed we get $\vdash \neg A \leftrightarrow \sim A$ for $\neg$ intuitionistic negation.)

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Expressive adequacy II

- Here are a few things to note.
  - Only the trivial logic is adequate w.r.t. an incoherent connnective, since the addition of such a connective trivializes any logic.
  - Without conservativeness it is trivial that any language is expressively adequate w.r.t. any connective. We could weaken the definition of adequacy by replacing conservativeness with consistency, but then adequacy so revised becomes of little interest. E.g. it does not require logics (or languages in a broader sense) be "strongly molecular" (or "analytic") in the sense that the meaning of a sentence is determined completely by its constituents.[2]
- Intuitionists may (and do!) reject the charge of expressive inadequacy by denying that classical negation is a coherent connective. Compare this to the strategy taken up by truth theorists attempting to avoid paradox. (E.g. Kripke denies that "strong" negation, which takes the intermediate value to truth, is expressible in a language for the truth theory **KF**, for the existence of fixpoints can no longer be proved.)

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Expressive adequacy III

- If the intuitionist denies the coherence of classical negation, they must do so "all the way up". They cannot appeal to classical reasoning (unless of course it is intuitionistically acceptable, as in the case of decidable domains) even in some metalinguistic sense.
- But worse, it appears they must reject the notion of **failing to be proved** w.r.t. some given moment of time. Surely they do **not** reject such a notion since doing so implies that each proposition is either provable or refutable—i.e. the law of excluded middle (LEM) would hold! So in order to satisfy expressive adequacy, this kind of negation must be accommodated in an intuitionistic framework.

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Williamson's argument I

- So far we have seen one need for negation as lack of proof. Another reason comes from avoiding certain Fitch-like paradoxes involving knowledge and (neo-)verificationist principles. These sorts of paradoxes matter for intuitionists (e.g. Dummett) who wish to extend intuitionistic semantics for mathematical discourse to empirical discourse.
- Here is one such paradox given by Williamson [2]. Suppose intuitionistic semantics has been extended to empirical discourse, so that provability is replaced by warranted assertability, and proof by warrant. (We can still maintain that warrant for a mathematical proposition just is proof.) Then Williamson argues that any[3] sentence of the following form is inconsistent:

(*) A may never be decided.

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Williamson's argument II

- We may simplify matters by eliminating the epistemic noise of (*), for if we can show the inconsistency of a non-epistemic claim '*A* **will** never be decided', then we **know** it to be inconsistent, and so the (weaker) epistemic claim would be shown inconsistent. Clearly the converse is true (the weaker implies the stronger). All we need to show then is that '*A* will never be decided' is inconsistent.
- Let KA mean 'at some past, present or future time someone possesses a warrant to assert A'. We may read KA simply as A is warranted. Then the claim that A will never be decided may be formalized as $\neg KA \wedge \neg K\neg A$. But $\neg KA$ implies $\neg A$ which, together with $\neg KA \wedge \neg K\neg A$, yields the contradiction $\neg A \wedge \neg\neg A$.

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Williamson's argument III

(1)	$A \wedge \neg KA$	supposition
(2)	$K(A \wedge \neg KA)$	1 BHK interpretation
(3)	$KA \wedge K\neg KA$	2 Distribution
(4)	$KA \wedge \neg KA$	3 axiom T
(5)	$\neg(A \wedge \neg KA)$	$1, 4 \neg I$
(6)	$\neg KA \rightarrow \neg A$	5 **IPC**

The DeVidi-Solomon approach I

- An approach of David DeVidi and Graham Solomon ([1]) is to weaken the negation sufficiently so that $\neg KA \wedge \neg K \neg A$ turns out consistent. The approach the matter semantically by defining, w.r.t. Kripke models, a "negation"[4] $\sim$ as follows.
- Let $(W, \leq, V)$ be a usual model for the language of **IPC**. We extend the language to include an "empirical negation" $\sim$ and let $M = (W, E, \leq, V)$, $E \subseteq W$, be a model for the language so obtained. Then the semantic clause for $\sim$ is

$$M, a \models \sim A \text{ iff } \forall b (a \leq b \wedge b \in E \Rightarrow M, b \not\models A).$$

- Truth in a model is truth at every point, validity on a frame is truth in every model based on the frame, and validity *simpliciter* is validity on every frame.

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The DeVidi-Solomon approach II

- Some facts:
  - $\models \neg A \rightarrow \sim A$
  - Not valid: $A \vee \sim A$; $\sim A \rightarrow \neg A$; any mixed DNE or De Morgan principles.
- **Main problems**: While $\sim K \wedge \sim K \sim A$ turns out satisfiable, and hence consistent, in the extended logic, $\sim$ is far too weak to be considered a genuine negation. One consequence of being too weak is that "vacuous" negations (i.e. when $b \notin E$ or $a \not\leq b$) make sets such as $\{p \wedge \sim p : p \in Prop\}$ consistent—highly (dialetheic and) counterintuitive!

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Negation as lack of sufficient evidence

- The idea is to model the idea of lacking evidence **at the present moment** or **in the actual state of information**. Clearly certain principles need be satisfied by this notion, a number of which are classical—e.g. LEM (either there is presently evidence for A or there isn't), DNE (if there is evidence for A then that A lacks evidence lacks evidence), the De Morgan principles, etc.
- Again we let $(W, \leq, V)$ be a usual **IPC**-model and $M = (W, @, \leq, V)$, $@ \in W$, a model for our extended language which includes a negation $\sim$ expressing lack of sufficient evidence. It's semantic clause is

$$M, a \models \sim A \text{ iff } M, @ \not\models A.$$

- Heredity is easily verified, as are the following validities, where validity is defined as above, except that truth in a model is now truth at @ (rather than every point).

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Validities and invalidities for $\mathbf{IPC}^{\sim}$

- Validities:

$$
\begin{array}{ll}
A \vee {\sim}A; & {\sim}{\sim}A \rightarrow A \\
({\sim}A \rightarrow A) \rightarrow A; & \neg A \rightarrow {\sim}A \\
{\sim}(A \wedge B) \leftrightarrow ({\sim}A \vee {\sim}B); & {\sim}(A \vee B) \leftrightarrow ({\sim}A \wedge {\sim}B) \\
{\sim}(A \rightarrow B) \rightarrow {\sim}B; & (A \wedge {\sim}B) \rightarrow {\sim}(A \rightarrow B) \\
\neg{\sim}A \rightarrow A; & {\sim}(A \wedge {\sim}A) \\
A \wedge {\sim}A \models B & A \rightarrow B \models {\sim}B \rightarrow {\sim}A
\end{array}
$$

- Invalidities:

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({\sim}A \rightarrow {\sim}B) \rightarrow (B \rightarrow A); & (A \rightarrow B) \rightarrow ({\sim}B \rightarrow {\sim}A) \\
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\sim(A \wedge B) \leftrightarrow (\sim A \vee \sim B); & \sim(A \vee B) \leftrightarrow (\sim A \wedge \sim B) \\
\sim(A \rightarrow B) \rightarrow \sim B; & (A \wedge \sim B) \rightarrow \sim(A \rightarrow B) \\
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A labeled tableaux system for $\mathbf{IPC}^{\sim}$

$$
\begin{array}{c} \mathsf{T}{\sim}A, i \\ | \\ \mathsf{F}A, @ \end{array}
\qquad
\begin{array}{c} \mathsf{F}{\sim}A, i \\ | \\ \mathsf{T}A, @ \end{array}
\qquad
\begin{array}{c} \mathsf{T}\neg A, i \\ irj \\ | \\ \mathsf{F}A, j \end{array}
\qquad
\begin{array}{c} \mathsf{F}\neg A, i \\ | \\ irj \\ \mathsf{T}A, j \\ (\text{new } j) \end{array}
\qquad
\begin{array}{c} \mathsf{T}A \rightarrow B, i \\ irj \\ \swarrow \quad \searrow \\ \mathsf{F}A, j \quad \mathsf{T}B, j \end{array}
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$$

In addition there are two rules (reflexivity, transitivity) for the labeled formulas. A tree for $A_1, \ldots, A_n \vdash B$ starts by listing the $\mathsf{T}A_i, @$ and $\mathsf{F}B, @$. A branch closes if for some A, i, both $\mathsf{T}A, i$ and $\mathsf{F}A, i$ occur on the branch.

A sequent calculus for **IPC**$^{\sim}$

- Take as initial sequents (i.e. zero-premise rules) all those valid for **IPC**, together with

$$A, \sim A \vdash B \text{ (EFQ)} \qquad \frac{\Gamma, A \vdash B \qquad \Gamma, \sim A \vdash B}{\Gamma \vdash B} \text{ (LEM)}$$

$$\frac{\Gamma, \sim A \vdash A}{\Gamma \vdash A} \text{ (RAA)} \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B} \;(\rightarrow \mathrm{I})$$

- In ($\rightarrow$I), (i) the double-line warrants inference in both upward and downward directions, and (ii) there is the proviso that each letter in A and B occurs either positively or negatively. A letter p occurs positively (negatively) in A iff every occurrence of p in A is preceded by an even (odd) number of occurrences of $\sim$.
- DNE for $\sim$ follows from LEM and EFQ, and $A \vee \sim A$ follows from LEM.

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Remarks on the sequent calculus I

- Notice that we had to take as initial sequents all those valid for **IPC**. The reason is that we do not have in general a deduction theorem (or $\rightarrow$I in a natural deduction setting), which is verified by the fact that $A \wedge \sim A \models B$ for any A and B, yet $\not\models (A \wedge \sim A) \rightarrow B$. The former is vacuously satisfied, since $A \wedge \sim A$ is never true in a model (it always fails at @). The "problem" with the latter is that $\rightarrow$ takes us to successors of @ which may satisfy $A \wedge \sim A$ while not satisfying B, for some B (e.g. let B be $\perp$).
- As such, **IPC**$^{\sim}$ is not an axiomatic extension of **IPC** in the sense that one may obtain **IPC**$^{\sim}$ simply by adding axioms to **IPC**. For any axiomatic extension of a system satisfying the deduction theorem satisfies the deduction theorem.
- However, **IPC**$^{\sim}$ is a conservative extension of **IPC** since for any formula in the language of the latter, if provable in the former, was clearly already provable in the latter. Our choice of initial sequents secures this, and it is easy to see semantically.

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Remarks on $\mathbf{IPC}^{\sim}$ I

- There is also a failure of the disjunction property. For $A \vee \sim A$ is provable for every A while neither disjunct may be (e.g. let A be an atom).
- Given our intended interpretation of the language it is not hard to justify the failure of the deduction theorem and disjunction properties. In the case of the deduction theorem, $A \longrightarrow B$ is no longer read (in general) 'there is a construction which transforms any proof of A into a proof of B' since we may be dealing with empirical A and B, and replacing 'proof' by 'warrant' or 'evidence' would not be satisfactory for the reason that it is not clear what such a construction would consist in.

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Remarks on **IPC**$^{\sim}$ II

- More reasonably we could read $A \rightarrow B$ as 'any evidence for A supports B **to at least the same degree**'. Proof is all-or-nothing, whereas evidential support permits of degree. But then any evidence for $A \wedge \sim A$, that A may be warranted even though it presently lacks warrant, is not equally strong evidence for any sentence whatever. For there is at least some evidence of e.g. Goldbach's conjecture (we have never found a counterexample!), though we lack sufficient evidence to assert it, i.e. there is some evidence for $A \wedge \sim A$. But there is absolutely no evidence for an absurdity. Whence $A \wedge \sim A \rightarrow \bot$ should fail.
- But then how are we to read $A \vdash B$ if not as $\vdash A \rightarrow B$? In fact it says something weaker[5], viz. that if A is warranted then so is B. But $A \wedge \sim A$ could never be warranted, *even though there may be some evidence supporting it.*

[5] It is weaker in the sense that if $\vdash A \rightarrow B$ holds, then $A \vdash B$ holds. If it is warranted to assert that any evidence for A supports B to at least the same degree, it follows that if A is warranted so is B.

Remarks on $\mathbf{IPC}^{\sim}$ II

- More reasonably we could read $A \rightarrow B$ as 'any evidence for A supports B **to at least the same degree**'. Proof is all-or-nothing, whereas evidential support permits of degree. But then any evidence for $A \wedge \sim A$, that A may be warranted even though it presently lacks warrant, is not equally strong evidence for any sentence whatever. For there is at least some evidence of e.g. Goldbach's conjecture (we have never found a counterexample!), though we lack sufficient evidence to assert it, i.e. there is some evidence for $A \wedge \sim A$. But there is absolutely no evidence for an absurdity. Whence $A \wedge \sim A \rightarrow \bot$ should fail.
- But then how are we to read $A \vdash B$ if not as $\vdash A \rightarrow B$? In fact it says something weaker[5], viz. that if A is warranted then so is B. But $A \wedge \sim A$ could never be warranted, *even though there may be some evidence supporting it*.

[5] It is weaker in the sense that if $\vdash A \rightarrow B$ holds, then $A \vdash B$ holds. If it is warranted to assert that any evidence for A supports B to at least the same degree, it follows that if A is warranted so is B.

Concluding remarks

- The neoverficiation program demands expressive adequacy. One of the challenges of this demand is conservatively adding a negation as lack of evidence to intuitionistic logic.
- **IPC**$^{\sim}$ gives a nice characterization of this negation, but the proof theory lacks certain niceties one might expect. In this case I think the tradeoff is essential.
- There are a lot of other negations that one might explore, by starting either proof-theoretically or semantically (as I have done). I think both approaches are fruitful.

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References

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