Dynamic Epistemic Questions

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Classical question semantics

Hamblin (1973), Karttunen (1977):

- The meaning of questions is reduced to propositions by assuming that questions denote the set of all possible or true answers.

Higginbotham & May (1981), Groenendijk & Stokhof (1984):

- Questions partition the logical space into the mutually exclusive possibilities that can serve as an answer.

The inquisitive turn

Groenendijk (2008):

- The meaning of a sentence is not primarily identified with its informative content or how it changes information. Rather, the focus is on exchanging information, i.e. the role that uttered sentences play in raising and resolving issues.

Goal

We want to formulate the idea of partitions in Dynamic Epistemic Logic, which allows us to relate it to the knowledge of discourse participants.

We will consider:

- direct questions (raising an issue) and announcement of answers (resolving an issue),
- interpretation of embedded questions, e.g. `John knows whether it snows` (talking about knowledge and issues)

Outline

E-PDL: Syntax

E-PDL (Propositional Dynamic Logic under an epistemic interpretation) is an extension of propositional logic with programs which represent epistemic accessibilities of (groups of) agents.

Definition

$$\phi ::= \top \mid p \mid \neg\,\phi \mid \phi \wedge \phi \mid [\pi]\phi$$
$$\pi ::= i \mid \pi\,;\pi \mid \pi \cup \pi \mid \pi^{*} \mid \mathrm{TEST}\,\phi$$

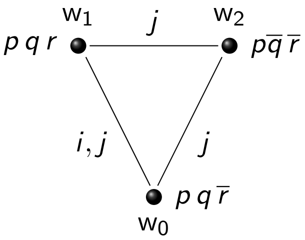
E-PDL: Semantics

The state of knowledge of a group of agents is modeled as a multimodal S5 Kripke model $M = (W, V, R)$ where

- W is a non-empty set of possible worlds
- V is a valuation function that assigns to every basic proposition a set of possible worlds (the worlds where it is true)
- R is a function that assigns to every agent i an equivalence relation $\sim_i$ over W, where $w \sim_i w'$ expresses that i cannot distinguish between w and w'

Epistemic alternatives: example

Connections between worlds with label i represent $\sim_i$, i.e. indicate that agent i confuses these worlds.



- agent i knows q ($[i]q$) but does not know r ($\neg[i]r$)
- whereas j is ignorant both about q and r
- it is common knowledge among i and j that p ($[(i \cup j)^*]p$) and that $r \rightarrow q$

Semantics

The formula $[\pi]\,\phi$ expresses that ϕ is true in all π-accessible worlds:

$$M, w \models [\pi]\,\phi \text{ iff for all } w' \text{ with } (w, w') \in [\![\pi]\!]^M\text{: } M, w' \models \phi$$

Interpretation of epistemic constructs π:

- i is interpreted by $\sim_i$
- ; is relational composition, $\cup$ is union, * is the reflexive transitive closure
- TEST is the usual test of dynamic logics: $[\![\text{TEST}\,\phi]\!]^M = \{(w, w) \mid w \in W \text{ and } M, w \models \phi\}$

Partition

To formulate a partition of the logical space, we define an additional program (where $G = W \times W$):

Definition

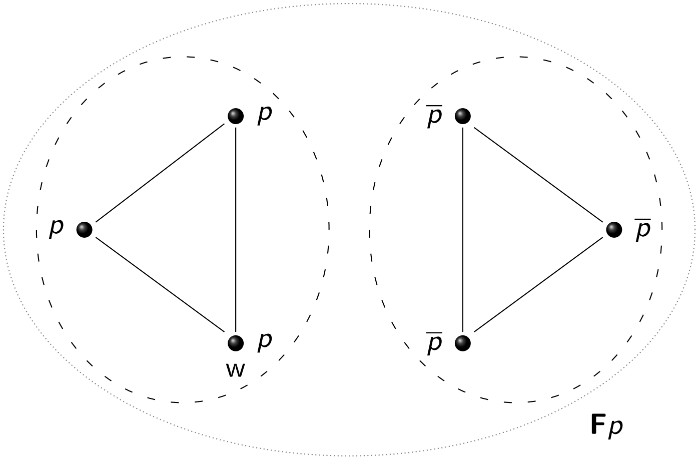
$$\mathbf{F}\,\phi = (\text{TEST}\,\phi\,;G\,;\text{TEST}\,\phi) \cup (\text{TEST}\,\neg\,\phi\,;G\,;\text{TEST}\,\neg\,\phi)$$

With respect to the semantics, $\mathbf{F}\,\phi$ denotes an equivalence relation:

$$[\![\mathbf{F}\,\phi]\!]^M = \{(w, w') \mid M, w \models \phi \text{ iff } M, w' \models \phi\}$$

I.e., the process $\mathbf{F}\,\phi$ partitions the model with respect to ϕ.

Example



Possible and true answers

Intuitively, a possible answer is a formula with a denotation that spans exactly one of the partition cells induced by the question. It is a true answer if this partition cell contains the actual world.

- A formula ψ is a true answer to the question whether ϕ w.r.t. a model M and a world w (the actual world) if for all worlds $w' \in W$: $M, w' \models \psi$ iff $(w, w') \in [\![\mathbf{F}\,\phi]\!]^M$.
- A formula ψ is a possible answer to the question whether ϕ w.r.t. a model M if there is some world $w \in W$, such that ψ is a true answer to the question whether ϕ w.r.t. M and w.

Questioning and answering in Dynamic Epistemic Logic

Now we also want to talk about what it means to pose a question in a conversation, which effects it has on the epistemic states of agents, and what it means to answer it.

We extend our models with an accessibility relation $\textsc{focus}$ that represents the focus of a conversation. We initialize it as $W \times W$.

Extending the language: public announcement

Definition

$$\phi ::= \ldots \mid [!\phi]\,\phi \mid [?\phi]\,\phi \mid [\text{WH}\,\pi]\,\phi$$

! is a modal operator taken from Public Announcement Logic. It models the event of all agents being told simultaneously and transparently that a certain formula holds.

The effect of a public announcement is an update of the knowledge state:

$$M, w \models [!\phi]\,\psi \text{ iff } M, w \models \phi \text{ implies } M|\phi, w \models \psi$$

where $|$ is a restriction operation on epistemic models.

Extending the language: direct public questions

Definition

$$\phi ::= \ldots \mid [!\phi]\,\phi \mid [?\phi]\,\phi \mid [\textsc{wh}\,\pi]\,\phi$$

Analogously, we add another modal operator ?, which shifts the focus of a conversation by setting it to the partition w.r.t. a particular formula.

$$M, w \models [?\phi]\,\psi \text{ iff } M[\textsc{focus} := [\![\mathbf{F}\,\phi]\!]^M], w \models \psi$$

Answering a question

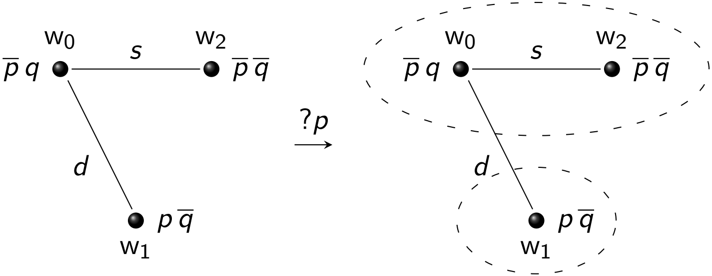
Answering a question can simply be seen as the announcement of the true answer.

- If ψ is an answer to the question whether ϕ, then either $M \models [?\phi][!\psi]\,\phi$ or $M \models [?\phi][!\psi]\,\neg\,\phi$ holds.

Answering a question will reset the FOCUS to $W \times W$ again, because the update will eliminate all but one partition cell.

FOCUS thus allows us to talk about raising and resolving issues.

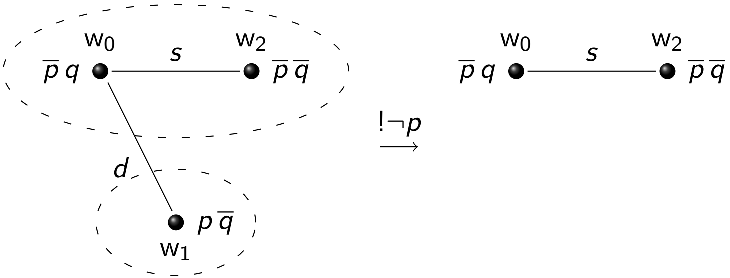
Example



d, s = agents, e.g. Data and Spock

p, q = propositions, e.g. `The Geiger counter detects radiation` `and Schrödinger's cat is alive`

Example



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Extending the language: embedded questions

Definition

$$\phi ::= \ldots \mid [!\phi]\,\phi \mid [?\phi]\,\phi \mid [\textsc{wh}\,\pi]\,\phi$$

A formula $[\textsc{wh}\,i]\,\phi$ expresses that agent i knows whether ϕ (in other words, i does not confuse worlds that belong to different partition cells).

Interpretation:

$$M, w \models [\textsc{wh}\,\pi]\,\phi \text{ iff } [\![\pi]\!]^M \subseteq [\![\mathbf{F}\,\phi]\!]^M$$

Extending the language: embedded questions

It follows that:

- $[\text{WH } i]\,\phi \models [i]\,\phi \vee [i]\,\neg\,\phi$
- $[\text{WH } i]\,\phi \not\models \phi$
  $[\text{WH } i]\,\phi \not\models \neg\,\phi$

That is, a statement of the form *i* `knows whether` *p* implies that *i* knows either *p* or $\neg\,p$, but does not provide the information which one is the case.

Constituent questions

We now extend this approach to the predicate logic case to also account for direct and embedded constituent questions.

Extending the language: predicate logic

We start with first order dynamic logic.

Definition

$$t ::= c \mid x$$
$$\phi ::= \top \mid t \mid P\,t \ldots t \mid \neg\,\phi \mid \phi \wedge \phi \mid \exists x\,\phi \mid [\pi]\,\phi$$
$$\pi ::= i \mid \pi\,;\pi \mid \pi \cup \pi \mid \pi^* \mid \mathrm{TEST}\ \phi$$

This language is interpreted with respect to a first order model M with fixed domain D, a world w, and a variable assignment g, as usual.

Partition

Partitioning takes place w.r.t. the free variables in a formula.

$$[\![\mathbf{F}\,\phi]\!]^{M} = \{(w, w') \mid \text{ for all } g : [\![\phi]\!]^{M,w,g} = [\![\phi]\!]^{M,w',g}\}$$

Extending the language: constituent questions

We extend the language with two modal operators: one for direct and one for embedded constituent questions.

Definition

$$\phi ::= \ldots \mid [?\phi]\,\phi \mid [\textsc{wh}\,\pi]\,\phi$$

The interpretation is completely parallel to the interpretation of yes/no questions:

- $M, w \models [?\phi]\,\psi$ iff $M[\textsc{focus} := [\![\mathbf{F}\,\phi]\!]^M], w \models \psi$
- $M, w \models [\textsc{wh}\,\pi]\,\phi$ iff $[\![\pi]\!]^M \subseteq [\![\mathbf{F}\,\phi]\!]^M$

True, possible and partial answers

- A formula ψ is a true answer to the question ϕ w.r.t. a model M and a world w (the actual world) if for all worlds $w' \in W$: $M, w' \models \psi$ iff $(w, w') \in [\![\mathbf{F}\,\phi]\!]^M$.
- A formula ψ is a partial answer to the question ϕ w.r.t. a model M and a world w (the actual world) if for all worlds $w' \in W$: if $(w, w') \in [\![\mathbf{F}\,\phi]\!]^M$, then $M, w' \models \psi$.
- A formula ψ is a possible answer to the question ϕ w.r.t. a model M if there is some world $w \in W$, such that ψ is a true answer to the question whether ϕ w.r.t. M and w.

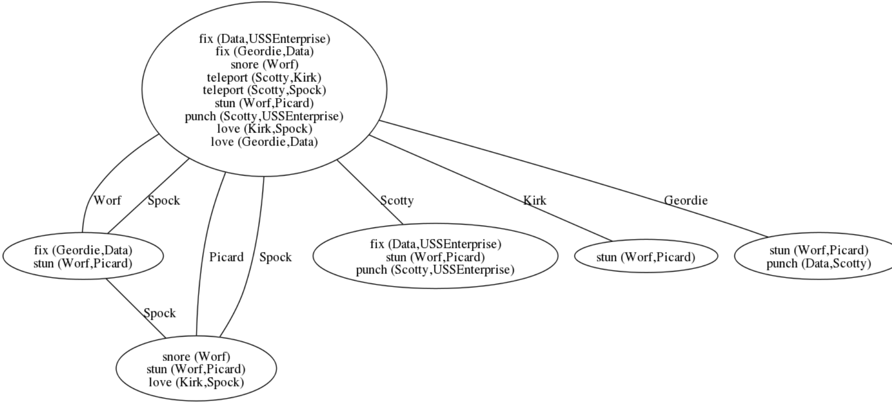
Conclusions

We extended DEL with an equivalence relation over possible worlds that models a partition of the logical space.

- Direct questions and their answers were raising and resolving issues.
- Embedded questions were interpreted w.r.t. a partition, but don't raise or resolve issues.

What can we do with it? For example, analyse felicitousness of question/answer pairs.

Example


fix (Data,USSEnterprise)
fix (Geordie,Data)
snore (Worf)
teleport (Scotty,Kirk)
teleport (Scotty,Spock)
stun (Worf,Picard)
punch (Scotty,USSEnterprise)
love (Kirk,Spock)
love (Geordie,Data)
Worf
Spock
Scotty
Kirk
Geordie
fix (Geordie,Data)
stun (Worf,Picard)
Picard
Spock
fix (Data,USSEnterprise)
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punch (Data,Scotty)
Spock
snore (Worf)
stun (Worf,Picard)
love (Kirk,Spock)

Example

$$[\![\text{Who loves whom?}]\!] = \mathbf{love}(x, y)$$

- "Kirk loves Spock" would be a partial answer
- "Kirk loves Spock and Picard loves Data" would be the true answer
- "Someone loves Spock and someone loves Data" would be also the true answer
- "Scotty fixes the Enterprise" would not be a possible answer

Another example

$$[\![\text{Who stuns whom?}]\!] = \textbf{stun}(x, y)$$

This does not raise an issue as it is common knowledge already.

Outlook

Further work:

- interaction with other pragmatic phenomena (e.g. presuppositions)
- extending the logic with different epistemic modalities (e.g. belief)
- examining the means of language to talk about those modalities (`believe`) and about issue managment (`wonder`)