Some Themes in the Logic of Norms

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Outline

1. Legal rules and legal concepts
2. Some methodological points
3. Modal logics

Part I

Tû-Tû

“**On the Noîsulli Islands** in the South Pacific lives the Noît-cif tribe, generally regarded as one of the more primitive peoples to be found in the world today [...]. This tribe [...] holds the belief that in the case of an infringement of certain taboos---for example, if a man encounters his mother-in-law, or if a totem animal is killed, or if someone has eaten of the food prepared for the chief---there arises what is called *tû-tû*. The members of the tribe also say that the person who committed the infringement has become *tû-tû*. It is very difficult to explain what is meant by this. [...] *tû-tû* is conceived as a kind of dangerous force [...] a person who has become *tû-tû* must be subjected to a special ceremony of purification.”

A. Ross, "Tû-Tû" Harward Law Review, 1958

- *Tû-Tû* works like our less mysterious

  legal concepts: right, violation, permission, etc.
- It makes normative inference work:
  1. "If a person has eaten of the chief's food then he/she is *tû-tû*."
  2. "If a person is *tû-tû* he/she shall be subjected to a ceremony of purification."
  3. "If a person has eaten of the chief's food he/she shall be subjected to a ceremony of purification."

... a "basic technology of presentation"

- For connecting **p** legal facts all to **n** normative consequences you need **p·n** rules.

$F_1 - C_1$	$F_2 - C_1$	$F_3 - C_1$	......	$F_p - C_1$
$F_1 - C_2$	$F_2 - C_2$	$F_3 - C_2$	......	$F_p - C_2$
$F_1 - C_3$	$F_2 - C_3$	$F_3 - C_3$	......	$F_p - C_3$
$\vdots$	$\vdots$	$\vdots$		$\vdots$
$F_1 - C_n$	$F_2 - C_n$	$F_3 - C_n$	......	$F_p - C_n$

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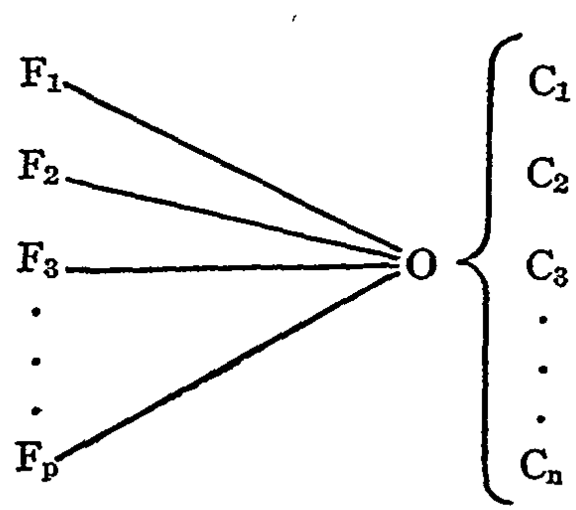
... a "basic technology of presentation"

- For connecting **p** legal facts all to **n** normative consequences you need **p·n** rules.

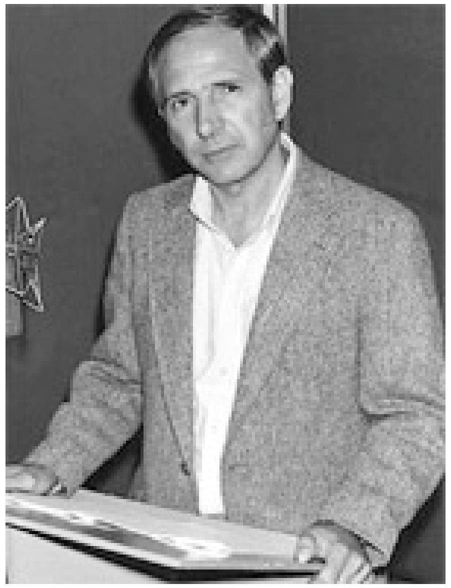


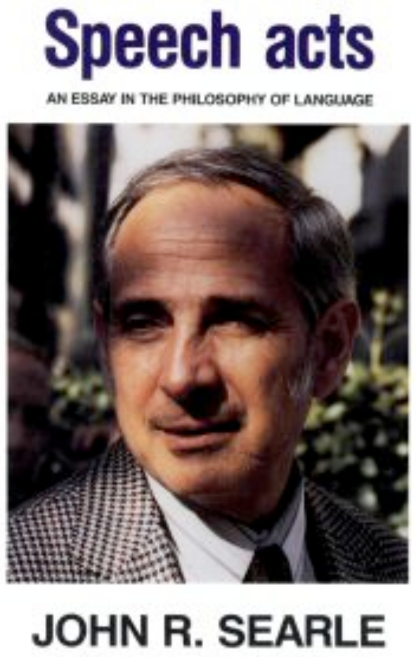
- Using the *tû-tû technique* you need **p+n** rules.

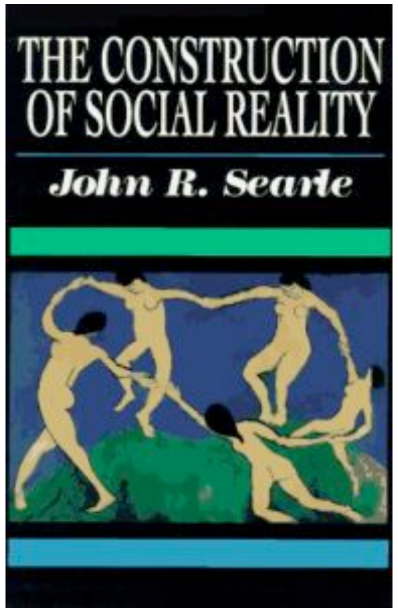
"***Our legal rules are in a wide measure couched in a "tû-tû" terminology***" (Ross,58)

What is the logic of Tû-Tû statements?

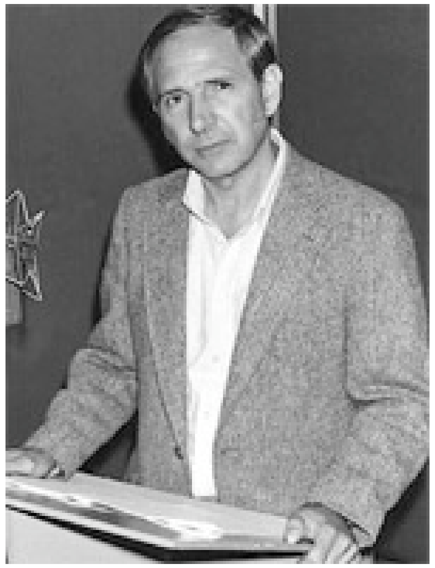
- *"Eating of the Chief's food counts as Tû-Tû, in the context of the Noisulli tribe"*
- These are the so-called *constitutive rules* [Searle 69, Searle 95]







Counts-as Conditionals



"Institutions are systems of constitutive rules. Every institutional fact is underlain by a (system of) rule(s) of the form **X counts as Y in context C**" [Searle,1969]

- ☐ Axiomatic approaches in conditional logic.
- ☐ Are counts-as conditionals reflexive? Are they transitive? Do they satisfy Cut? Antecedent strengthening?

Preliminaries

Method

Axioms for counts-as conditionals

A. Jones and M. Sergot, "A Formal Characterization of Institutionalized Power", Journal of the IGPL, 1996

$$\gamma_2 \leftrightarrow \gamma_3 \ / \ (\gamma_1 \Rightarrow_c \gamma_2) \leftrightarrow (\gamma_1 \Rightarrow_c \gamma_3)$$

$$\gamma_1 \leftrightarrow \gamma_3 \ / \ (\gamma_1 \Rightarrow_c \gamma_2) \leftrightarrow (\gamma_3 \Rightarrow_c \gamma_2)$$

$$((\gamma_1 \Rightarrow_c \gamma_2) \wedge (\gamma_1 \Rightarrow_c \gamma_3)) \rightarrow (\gamma_1 \Rightarrow_c (\gamma_2 \wedge \gamma_3))$$

$$((\gamma_1 \Rightarrow_c \gamma_2) \wedge (\gamma_3 \Rightarrow_c \gamma_2)) \rightarrow ((\gamma_1 \vee \gamma_3) \Rightarrow_c \gamma_2)$$

$$\boxed{(\gamma_1 \Rightarrow_c \gamma_2) \wedge (\gamma_2 \Rightarrow_c \gamma_3) \rightarrow (\gamma_1 \Rightarrow_c \gamma_3) \quad ?}$$

- **A good example of the pitfalls of axiomatics-driven approaches in the formal analysis of** *ambiguous, polysemic* **concepts!**

The semantic way to concept analysis (ii)

A. Tarski, "Grundlegung der Wissentschaflichen Semantik" (1936). Translated in "The Establishment of Scientific Semantics", Logic Semantics Metamathematics. Papers from 1923 to 1938, Clarendon Press1956

"[…] the choice of axioms always has rather accidental character, depending on inessential factors (such as e.g. the actual state of our knowledge).

[…] Apart from the problem of consistency, a method of constructing a theory does not seem to be very natural […] if in this method the role of primitive concepts---thus of *concepts whose meaning should appear evident*---is played by concepts which have led to various misunderstanding in the past."

The semantic way to concept analysis (i)

A. Tarski, "The Semantic Conception of Truth", Philosophy and Phenomenological Research, 1944

"[…] it seems to me obvious that the only rational approach to such problems would be the following:

[1] We should reconcile ourselves with the fact that we are confronted, not with one concept, but with *several different concepts which are denoted by one word*;

[2] we should try to make these concepts as clear as possible (by means of *definition*, or of an axiomatic procedure, or in some other way);

[3] to avoid further confusions, we should agree to use *different terms for different concepts*; and then we may proceed to a quiet and systematic study of all concepts involved, which will *exhibit their main properties and mutual relations*."

This is the plan

contextual classification	→	Cxt
proper contextual classification	→	Cxt^{U}
context definition	→	nCxt^{U}
language creation	→	UpTo
context dynamics	→	DCxt^{U}

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Part II

Contextual classification

with: J.-J. Ch. Meyer and F. Dignum

University of Utrecht

Classifying states

"Eating of the chief's food **counts as** being *tû-tû*."

$$\begin{aligned} \mathcal{I}(\texttt{Eating}) &\subseteq \mathcal{I}(\texttt{TûTû}) \\ (W, \mathcal{I}) &\models \texttt{Eating} \rightarrow \texttt{TûTû} \end{aligned}$$

"Eating of the chief's food **counts as** being *tû-tû*

in the context of the Noîsulli tribe."

$$\begin{aligned} C_{\mathrm{tribe}} \cap \mathcal{I}(\texttt{Eating}) &\subseteq \mathcal{I}(\texttt{TûTû}) \\ (W, C_{\mathrm{tribe}}, \mathcal{I}) &\models [\mathrm{tribe}](\texttt{Eating} \rightarrow \texttt{TûTû}) \end{aligned}$$

Logic of contextual classification (i)

Definition 1 *(Syntax)*

$$\mathcal{L}_{\mathbf{Cxt}} : \varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid [X]\varphi$$

Definition 2 *(Models) A* **Cxt***-model* $\mathcal{M} = (W, R, \mathcal{I})$ *is a tuple such that:*

- W *is a nonempty set of possible worlds;*
- $R : \mathcal{C} \longrightarrow 2^W$ *maps each context* X *to a subset of* W*;*
- $\mathcal{I} : \Phi \longrightarrow 2^W$ *is a valuation.*

We write R_X *for* $R(X)$ *and* $w \in \mathcal{M}$ *for* $w \in W$*.*

Definition 3 *(Semantics) Let* $\mathcal{M}$ *be a* **Cxt***-model, and let* $w \in \mathcal{M}$*.*

$\mathcal{M}, w \models p$ *iff* $w \in \mathcal{I}(p)$.
$\mathcal{M}, w \models [X]\varphi$ *iff for all* $w' \in R_X$, $\mathcal{M}, w' \models \varphi$.

and as usual for the Boolean operators.

Logic of contextual classification (ii)

$$
\begin{array}{rl}
(\mathtt{P}) & \text{all propositional schemata and rules} \\
(\mathtt{K}^{X}) & [X](\varphi \rightarrow \varphi') \rightarrow ([X]\varphi \rightarrow [X]\varphi') \\
(4^{XY}) & [X]\varphi \rightarrow [Y][X]\varphi \\
(5^{XY}) & \langle X \rangle\varphi \rightarrow [Y]\langle X \rangle\varphi \\
(\mathtt{N}^{X}) & \textsc{If} \vdash \varphi \ \textsc{then} \vdash [X]\varphi
\end{array}
$$

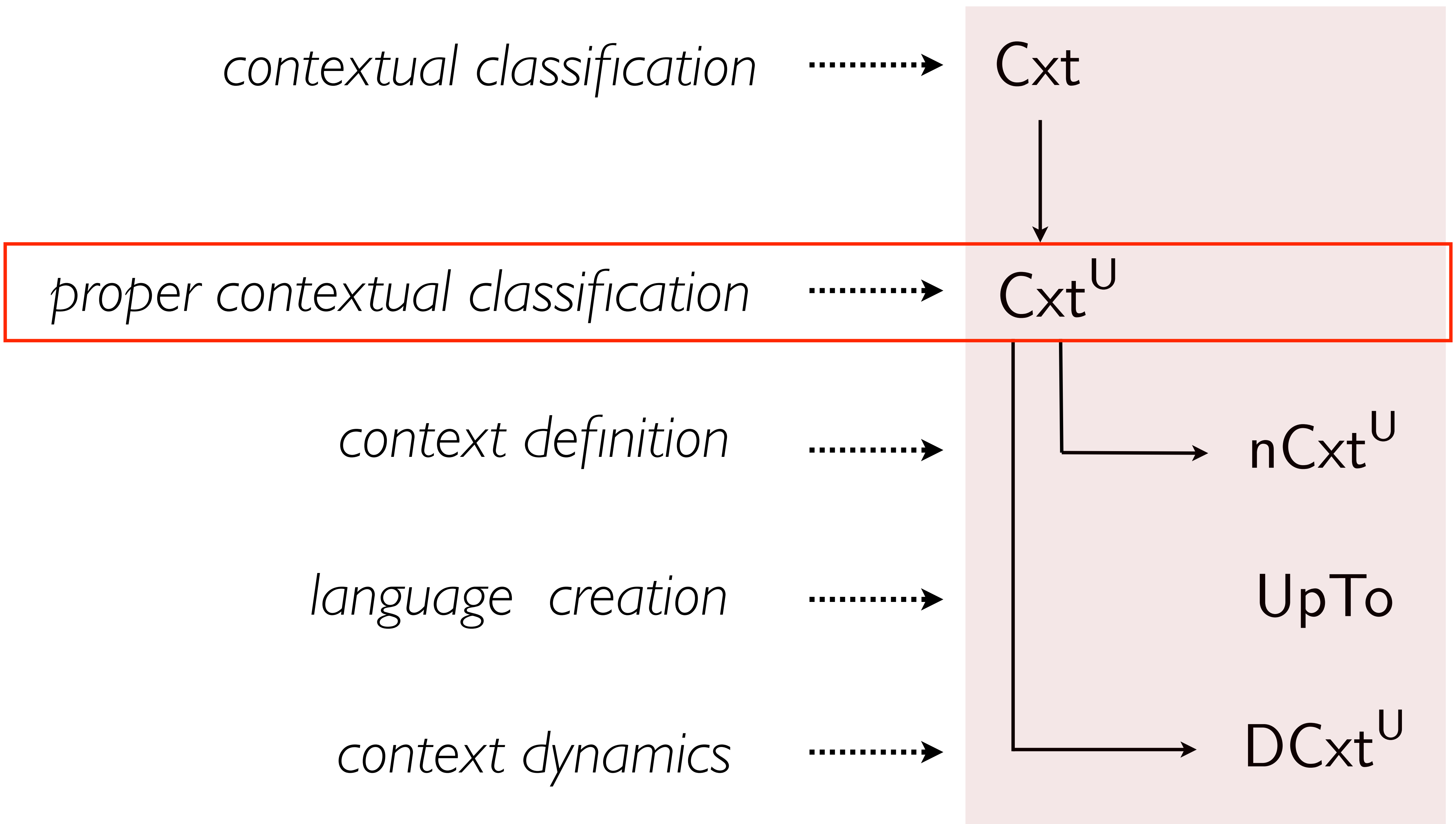
- This axiomatics is sound and strongly complete w.r.t. the semantics presented.

Counts-as = Contextual classification

$$\varphi \Rightarrow^{cl}_{X} \psi \quad := \quad [X](\varphi \rightarrow \psi)$$

- ☐ It satisfies all the structural properties isolated in [Jones & Sergot, 1996]
- ☐ It formalizes in Modal Logic the notion of "Pivotal Consequence Relation" [Makinson, 2005]
- ☐ It satisfies Transitivity, but now we know why! It satisfies much more (e.g. Reflexivity, Antecedent Strengthening, etc.)

Logics



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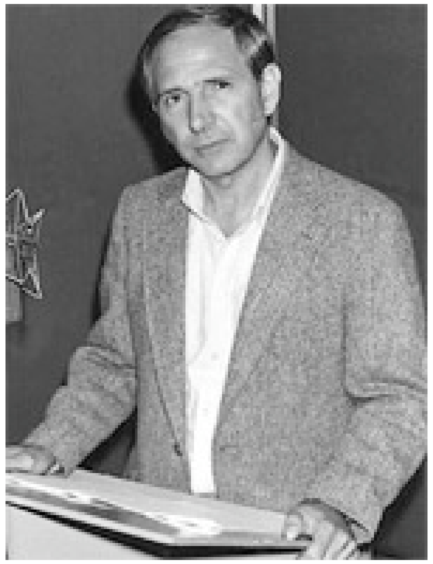
TûTû's first premise revisited



"Where the rule (or system of rules) is constitutive, behaviour which is in accordance with the rule can receive specifications or descriptions which *it could not receive if the rule did not exist* [Searle,1969]"

$$\begin{array}{rcl} C_{\text{tribe}} \cap \mathcal{I}(\texttt{Eating}) & \subseteq & \mathcal{I}(\texttt{TûTû}) \\ \& \quad \mathcal{I}(\texttt{Eating}) & \not\subseteq & \mathcal{I}(\texttt{TûTû}) \end{array}$$

$$\begin{array}{rcl} (W, C_{\text{tribe}}, \mathcal{I}) & \models & [\text{tribe}](\texttt{Eating} \rightarrow \texttt{TûTû}) \\ & & \wedge \neg [\text{U}](\texttt{Eating} \rightarrow \texttt{TûTû}) \end{array}$$

Logic of proper contextual classification (i)

Definition 1 *(Syntax)*

$$\mathcal{L}_{\mathbf{Cxt}^{\mathsf{U}}} : \varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid [\mathsf{U}]\varphi \mid [X]\varphi$$

Definition 2 *(Models) A* $\mathbf{Cxt}^{\mathsf{U}}$*-model* $\mathcal{M} = (W, R, \mathcal{I})$ *is a tuple such that:*

- W *is a nonempty set of possible worlds;*
- $R : \mathcal{C} \longrightarrow 2^W$ *maps each context* X *to a subset of* W *and* $\mathcal{I}(\mathsf{U}) = W$*;*
- $\mathcal{I} : \Phi \longrightarrow 2^W$ *is a valuation.*

We write R_X *for* $R(X)$ *and* $w \in \mathcal{M}$ *for* $w \in W$*.*

Definition 3 *(Semantics) Let* $\mathcal{M}$ *be a* $\mathbf{Cxt}^{\mathsf{U}}$*-model, and let* $w \in \mathcal{M}$*.*

$\mathcal{M}, w \models p$ *iff* $w \in \mathcal{I}(p)$.
$\mathcal{M}, w \models [X]\varphi$ *iff for all* $w' \in R_X$, $\mathcal{M}, w' \models \varphi$;
$\mathcal{M}, w \models [\mathsf{U}]\varphi$ *iff for all* $w' \in W$, $\mathcal{M}, w' \models \varphi$.

and as usual for the Boolean operators.

Logic of proper contextual classification (ii)

(P)	all propositional schemata and rules
$(\mathtt{K}^X)$	$[X](\varphi \rightarrow \varphi') \rightarrow ([X]\varphi \rightarrow [X]\varphi')$
$(\mathtt{T}^{\mathsf{U}})$	$[\mathsf{U}]\varphi \rightarrow \varphi$
(4^{XY})	$[X]\varphi \rightarrow [Y][X]\varphi$
(5^{XY})	$\langle X\rangle\varphi \rightarrow [Y]\langle X\rangle\varphi$
$(\mathtt{N}^X)$	IF $\vdash \varphi$ THEN $\vdash [X]\varphi$

- The following is a theorem: $[\mathsf{U}]\varphi \rightarrow [X]\varphi$
- This axiomatics is sound and strongly complete w.r.t. the semantics presented

Counts-as = Proper contextual classification

$$\varphi \Rightarrow^{cl+}_{X} \psi \quad := \quad [X](\varphi \rightarrow \psi) \wedge \neg[\mathsf{U}](\varphi \rightarrow \psi)$$

- ☐ It satisfies all the structural properties isolated in [Jones & Sergot, 1996] and falsifies the ones they reject!
- ☐ It falsifies Transitivity, but now we know why! It satisfies Cut.

Logic of proper contextual classification (iii)

Take the set of atoms to be $\Phi \cup \mathcal{C}$ and consider:

$$\mathcal{L}^{[\mathsf{U}]} : \varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid [\mathsf{U}]\varphi$$

and $f : \mathcal{L}_{\mathbf{Cxt}^{\mathsf{U}}} \longrightarrow \mathcal{L}^{[\mathsf{U}]}$ so defined:

$$\begin{array}{rcl} f(p) & = & p \\ f(\neg\varphi) & = & \neg f(\varphi) \\ f(\varphi \wedge \varphi') & = & f(\varphi) \wedge f(\varphi') \\ f([\mathsf{U}]\varphi) & = & [\mathsf{U}]f(\varphi) \\ f([X]\varphi) & = & [\mathsf{U}](X \rightarrow f(\varphi)) \end{array}$$

Function f is a truth-preserving poly-time reduction of $\mathcal{L}_{\mathbf{Cxt}^{\mathsf{U}}}$ to $\mathcal{L}^{[\mathsf{U}]}$.

- ☐ The satisfiability problem is NP-complete.

Part III

Context definition

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contextual classification → Cxt

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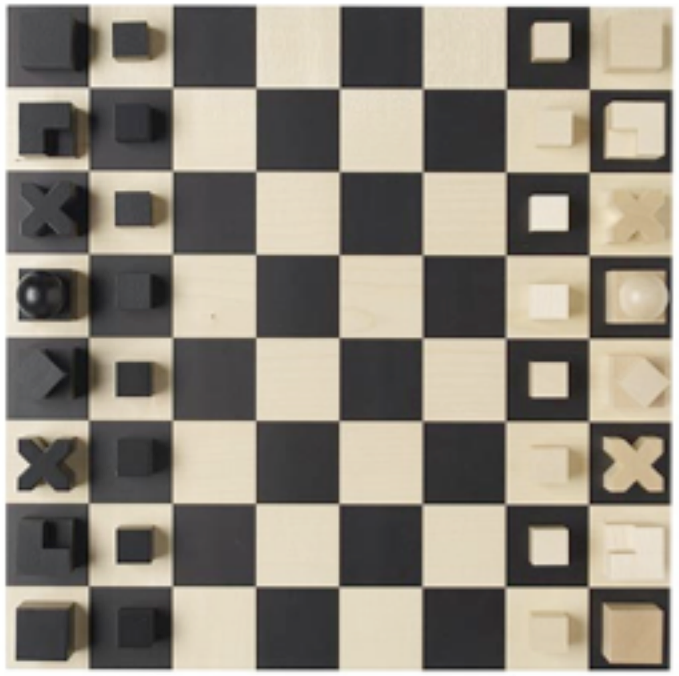
To "constitute" = to "define a logical space"



"Rules are constitutive if and only if they are part of a set of rules. Strictly speaking, there is no such thing as a rule that is constitutive in isolation. [...] A set of constitutive rules defines a logical space" [Ricciardi, 1997]

"No logic of norms without a system of which they form part" [Makinson, 1999]

- The context of a normative system is defined by the set of its rules.
- The "logical space" of chess is defined by its rules (Husserl, *Logische Untersuchungen*; Wittgenstein, *Philosophical Investigations*).

Context definition

Take a $\mathbf{Cxt}^{\mathsf{U}}$ model $\mathcal{M}$. A context R_X is defined by a finite set of propositional formulae Γ iff:

$$\begin{array}{lll} w \in R_X & \text{implies} & \mathcal{M}, w \models \bigwedge \Gamma \\ w \notin R_X & \text{implies} & \mathcal{M}, w \models \neg \bigwedge \Gamma \end{array}$$

- ☐ We need to be able to express context complementation
- ☐ “All I know” logics [Levesque, 1990]
- ☐ Nominals + Universal modality! (Sofia School of Modal Logic)
- ☐ Or *difference operator* [de Rijke, 1992]

Logic of context definition (i)

Definition 1 *(Syntax)*

$$\mathcal{C} : X ::= C \mid -C \mid \mathsf{U}$$
$$\mathcal{L}_{\mathbf{Cxt}_n^{\mathsf{U}}} : \varphi ::= p \mid i \mid \neg\varphi \mid \varphi \wedge \varphi \mid [X]\varphi$$

with $p \in \mathbf{P}$ *and* $i \in \mathbf{N}$

Definition 2 *(Models) A* $\mathbf{Cxt}_n^{\mathsf{U}}$*-model* $\mathcal{M} = (W, R, \mathcal{I})$ *is a tuple such that:*

- W *is a nonempty set of possible worlds;*
- $R : \mathcal{C} \longrightarrow 2^W$ *maps each context* X *to a subset of* W *so that:* $\mathcal{I}(\mathsf{U}) = W$ *and* $\mathcal{I}(-X) = W - \mathcal{I}(X)$*;*
- $\mathcal{I} : \mathbf{P} \cup \mathbf{N} \longrightarrow 2^W$ *is a valuation s.t.* $\mathbf{N}\rceil\mathcal{I}$ *is a surjection on the set of all singletons of* W*.*

Definition 3 *(Semantics) Let* $\mathcal{M}$ *be a* $\mathbf{Cxt}_n^{\mathsf{U}}$*-model, and let* $w \in \mathcal{M}$*.*

$\mathcal{M}, w \models i$ *iff* $\mathcal{I}(i) = w$.
$\mathcal{M}, w \models [C]\varphi$ *iff for all* $w' \in R_C$, $\mathcal{M}, w' \models \varphi$;
$\mathcal{M}, w \models [-C]\varphi$ *iff for all* $w' \notin R_C$, $\mathcal{M}, w' \models \varphi$;.

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Logic of context definition (ii)

(P)	all propositional axiom schemata and rules
$(\mathtt{K}^X)$	$[X](\varphi \rightarrow \varphi') \rightarrow ([X]\varphi \rightarrow [X]\varphi')$
$(\mathtt{T}^{\mathsf{U}})$	$[\mathsf{U}]\varphi \rightarrow \varphi$
(4^{XY})	$[X]\varphi \rightarrow [Y][X]\varphi$
(5^{XY})	$\langle X \rangle\varphi \rightarrow [Y]\langle X \rangle\varphi$
(Least)	$\langle \mathsf{U} \rangle\, i$
(Most)	$\langle \mathsf{U} \rangle\, (i \wedge \varphi) \rightarrow [\mathsf{U}](i \rightarrow \varphi)$
(Covering)	$[C]\varphi \wedge [-C]\varphi \rightarrow [\mathsf{U}]\varphi$
(Packing)	$\langle -C \rangle\, i \rightarrow \neg\, \langle C \rangle\, i$
$(\mathtt{N}^X)$	IF $\vdash \varphi$ THEN $\vdash [X]\varphi$
(Name)	IF $\vdash i \rightarrow \theta$ THEN $\vdash \theta$, for i not occurring in θ

- This axiomatics is sound and strongly complete w.r.t. the semantics

Context definition at work (i)

$$Def(C, \Gamma) \quad := \quad [C] \bigwedge \Gamma \wedge [-C] \neg \bigwedge \Gamma$$

$$@_i \varphi \quad := \quad [\mathsf{U}](i \rightarrow \varphi)$$

- We can now capture context definition modally
- We can represent the @ of Hybrid Logics
- Notice we are using only modalities of a global kind

Context definition at work (ii)

$$(\varphi \Rightarrow^{cl}_{C} \psi \wedge \mathit{Def}(C,\Gamma)) \rightarrow ((@_i \bigwedge \Gamma \wedge @_i\varphi) \rightarrow @_i\psi))$$

"[...] ethics [...] looks for [...] **universal criteria** which could be used, on the one hand, to read the ethic good or bad in the **single cases** [...], and on the other hand, to positively determine whether a practical decision is ethically correct or not. The analysis of each single case [...] can imply considerable difficulties; however, the fundamental thing that everything should in the end depend on a simple **subsumption**" (Husserl, 1908-14)

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Part IV

Equivalence ***up to*** a signature

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Logics

contextual classification → Cxt

proper contextual classification → Cxt^{U}

context definition → nCxt^{U}

language creation → UpTo

context dynamics → DCxt^{U}

$$\begin{array}{l} \mathrm{Cxt} \to \mathrm{Cxt}^{U} \\ \mathrm{Cxt}^{U} \to \mathrm{nCxt}^{U} \\ \mathrm{Cxt}^{U} \to \mathrm{DCxt}^{U} \end{array}$$

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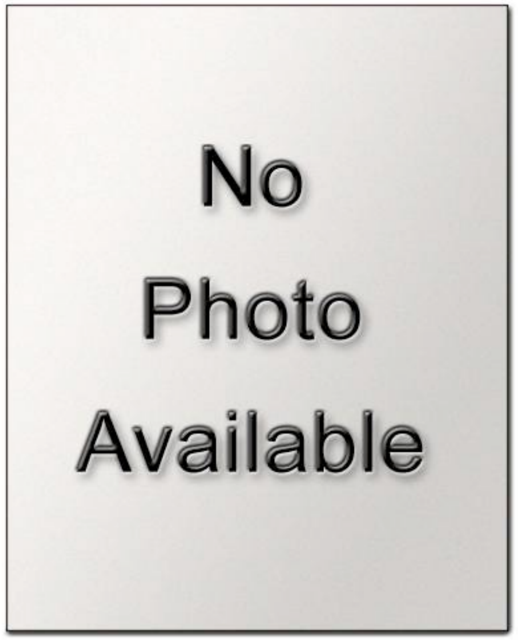
New vocabularies

"Now, as the original manner of producing physical entities is creation, there is hardly a better way to describe the production of moral entities than by the word *imposition*. For moral entities do not arise from the intrinsic substantial principles of things but are *superadded* to things already existent and physically complete" [Pufendorf, 1688]

- Tû-Tû is a somewhat *conventional* term
- It is invented for the purpose of regulating existing situations
- Brute vs. Institutional facts [Searle, 1995]

Norms as ascriptions



“How is a sentence of the form ‘Such and such is to be so and so’ to be verified? How is it for instance to be verified that all promises are to be kept? To this question I know of no other answer than the following: The phrase ‘is to be etc.’ describes not a property which an action or a state of affairs either has or not, but a kind of quasi-property which is *ascribed* to an action or a state of affairs when a person is willing or commanding the action to be performed, resp. the state of affairs to be produced” [Jørgensen, 1937]

- Good/Bad, Legal/Illegal are Tû-Tû terms extending the standard descriptions of things.

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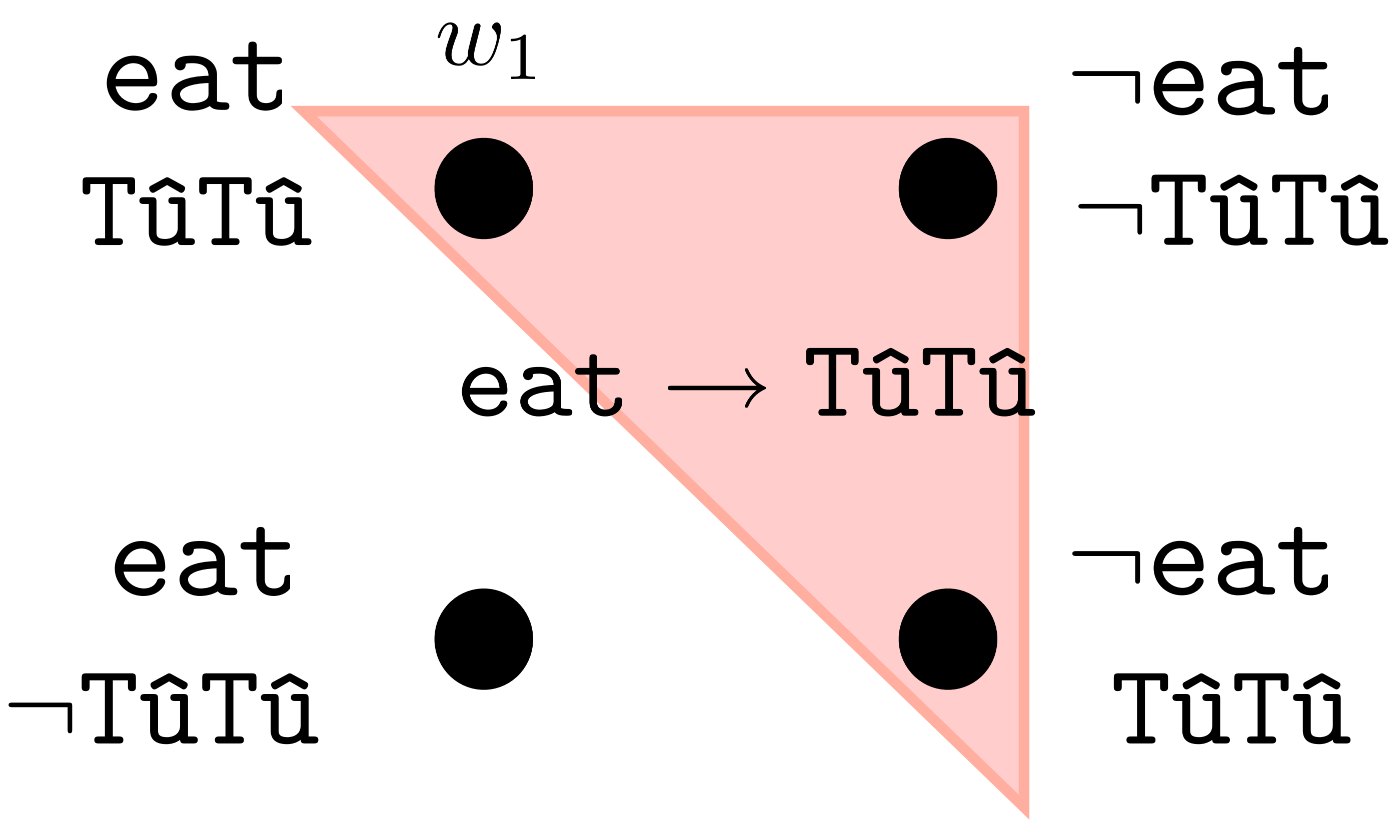
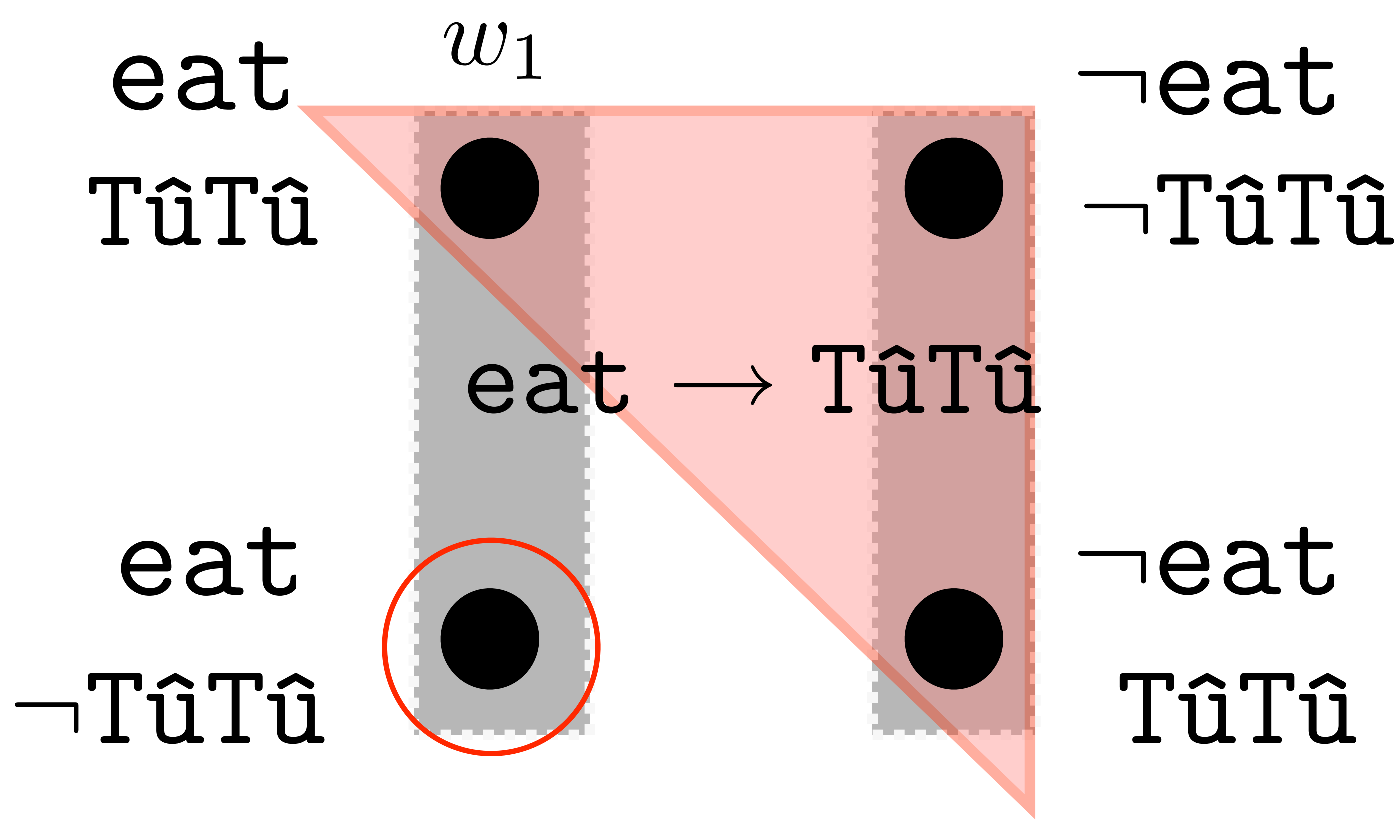
"Thou shall not eat of the chief's food!" (i)



$$\mathcal{M} \models [\mathrm{Tribe}](\mathtt{eat} \rightarrow \mathtt{T\hat{u}T\hat{u}})$$

"Thou shall not eat of the chief's food!" (ii)



$$\mathcal{M} \models [\mathrm{Tribe}](\mathtt{eat} \rightarrow \mathtt{T\hat{u}T\hat{u}})$$

$$\boxed{\mathcal{M}, w_1 \models \langle\{\mathtt{eat}\}\rangle\neg(\mathtt{eat} \rightarrow \mathtt{T\hat{u}T\hat{u}})}$$

Propositional equivalence *up to* a signature (ii)

Definition 1 *(Equivalence up to a signature) Two models w and w' for a propositional language $\mathcal{L}(\mathbf{P})$ are equivalent up to signature $P \in 2^{\mathbf{P}}$, or P-equivalent, if and only if for any $p \in P, w \models p$ iff $w' \models p$. If w and w' are P-equivalent we write $w \sim_P w'$.*

Fact 1 *(Properties of $\sim_P$) Let W be a set of models for the propositional language $\mathcal{L}(\mathbf{P})$. The following holds:*

1. *For all $P \in 2^{\mathbf{P}}$, the relation $\sim_P$ is an equivalence relation on W;*
2. *For all $P, Q \in 2^{\mathbf{P}}$, if $P \subseteq Q$ then $\sim_Q \subseteq \sim_P$;*
3. *For each atom $p \in \mathbf{P}$, the relation $\sim_{\{p\}}$ yields a bipartition of W;*
4. *$\sim_{\mathbf{P}} = \sim$ and $\sim_{\emptyset} = W^2$.*

Logic of equivalence *up to* a signature (i)

Definition 1 *(Syntax)*

$$\mathcal{L}_{\mathsf{UpTo}} : \varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid [P]\varphi$$

where $P \in 2^{\mathbf{P}}$.

Definition 2 *(Models) An* UpTo*-model* $\mathcal{M} = (W, \{\sim_P\}_{P \in 2^{\mathbf{P}}}, \mathcal{I})$ *is a tuple such that:*

- W *is a nonempty set of possible states;*
- *Each* $\sim_P$ *is the* P*-equivalence relation yielded by signature* $P \in 2^{\mathbf{P}}$*;*
- $\mathcal{I} : \mathbf{P} \longrightarrow 2^W$ *is a valuation.*

Definition 3 *(Semantics) Let* $\mathcal{M}$ *be a* UpTo*-model, and let* $w \in \mathcal{M}$.

$\mathcal{M}, w \models [P]\varphi$ *iff for all* $w' \in W, w \sim_P w' : \mathcal{M}, w' \models \varphi$.

and as usual for atoms and Boolean operators.

UpTo operators

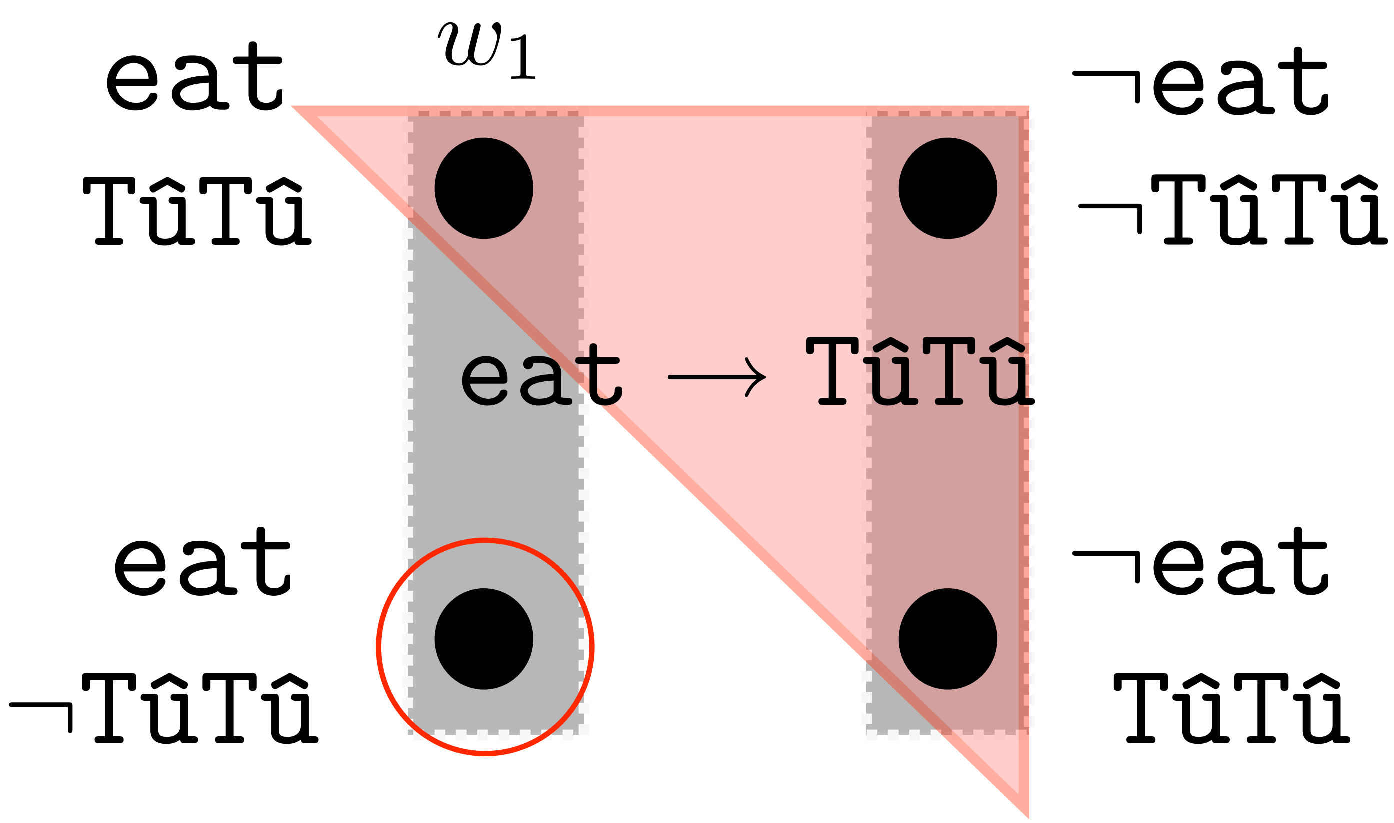
- They express truth in a state "modulo a given part of the vocabulary": Release Logics [Krabbendam & Meyer, 2000]
- They express truth in a state "everything else being equal, which you can express in the given signature": Ceteris Paribus Logics [van Benthem, Girard, Roy 2009] where the set Γ is the (infinite) set of all (propositional) formulae generated by the given (finite) signature.

Logic of equivalence ***up to*** a signature (ii)

(P)	all propositional axiom schemata and rules
(K)	$[P](\varphi \rightarrow \psi) \rightarrow ([P]\varphi \rightarrow [P]\psi)$
(T)	$[P]\varphi \rightarrow \varphi$
(4)	$[P]\varphi \rightarrow [P][P]\varphi$
(5)	$\langle P\rangle\varphi \rightarrow [P]\langle P\rangle\varphi$
(PO)	$[P]\varphi \rightarrow [Q]\varphi$ IF $P \subseteq Q$
(Bipart)	$[\{p\}]p \vee [\{p\}]\neg p$
($\mathtt{N}^P$)	IF $\vdash \varphi$ THEN $\vdash [P]\varphi$

- This schema is derivable: $[P]p \vee [P]\neg p$ IF $p \in P$
- The axiomatics is sound and strongly complete

"Thou shall not eat of the chief's food!" (iii)



$$\mathcal{M}, w_1 \models [\{\texttt{eat}, \texttt{TûTû}\}](\texttt{eat} \rightarrow \texttt{TûTû})$$

$$\mathcal{M}, w_1 \not\models [\{\texttt{eat}\}](\texttt{eat} \rightarrow \texttt{TûTû})$$

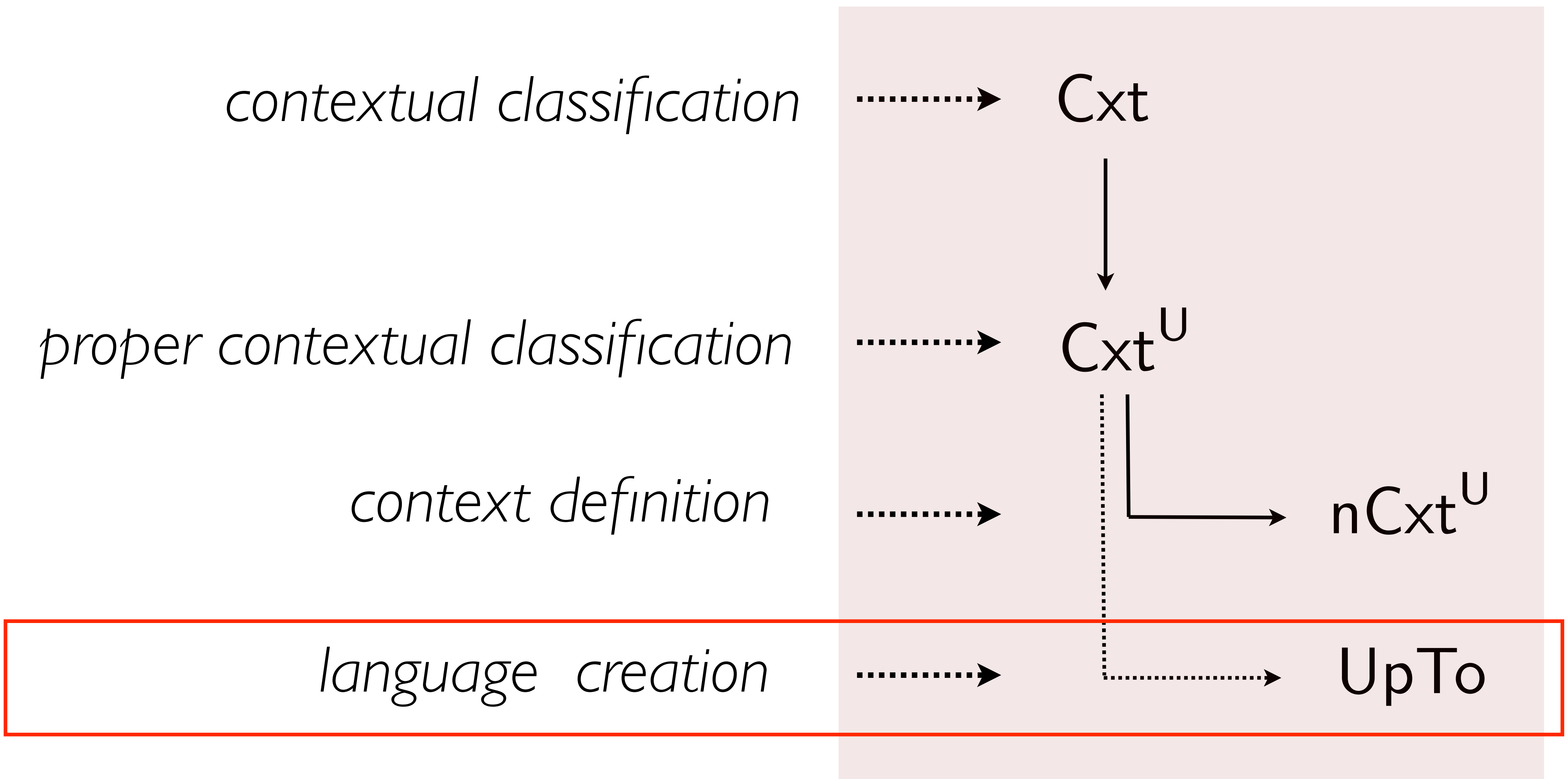
Logic of equivalence *up to* a signature (iii)

$$
\begin{array}{rcl}
f(p) & = & p \\
f(\neg\varphi) & = & \neg f(\varphi) \\
f(\varphi \wedge \psi) & = & f(\varphi) \wedge f(\psi) \\
f([\emptyset]\varphi) & = & \Box f(\varphi) \\
f([P]\varphi) & = & \bigwedge_{\pi_i \in 2^P} \left(\left(\bigwedge \pi_i^+ \wedge \bigwedge \pi_i^- \right) \rightarrow \Box \left(\left(\bigwedge \pi_i^+ \wedge \bigwedge \pi_i^- \right) \rightarrow f(\varphi) \right) \right)
\end{array}
$$

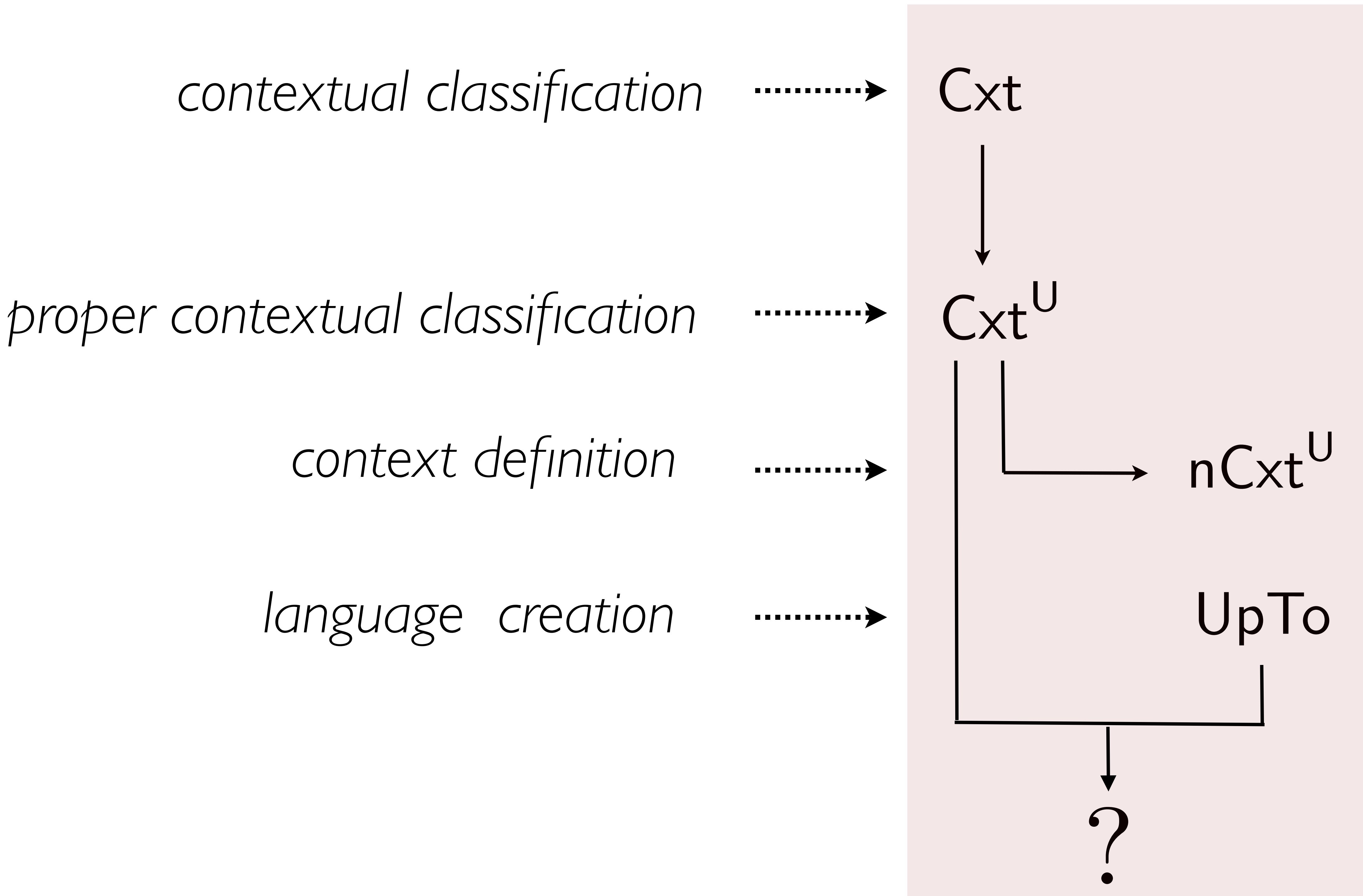
where $\pi_i^+ = \pi_i$ and $\pi_i^- = \{\neg p \mid p \in P \;\&\; p \notin \pi_i\}$.

- Exptime truth-preserving reduction of **UpTo** to **S5**
- Compact representation of complex **S5** formulae!

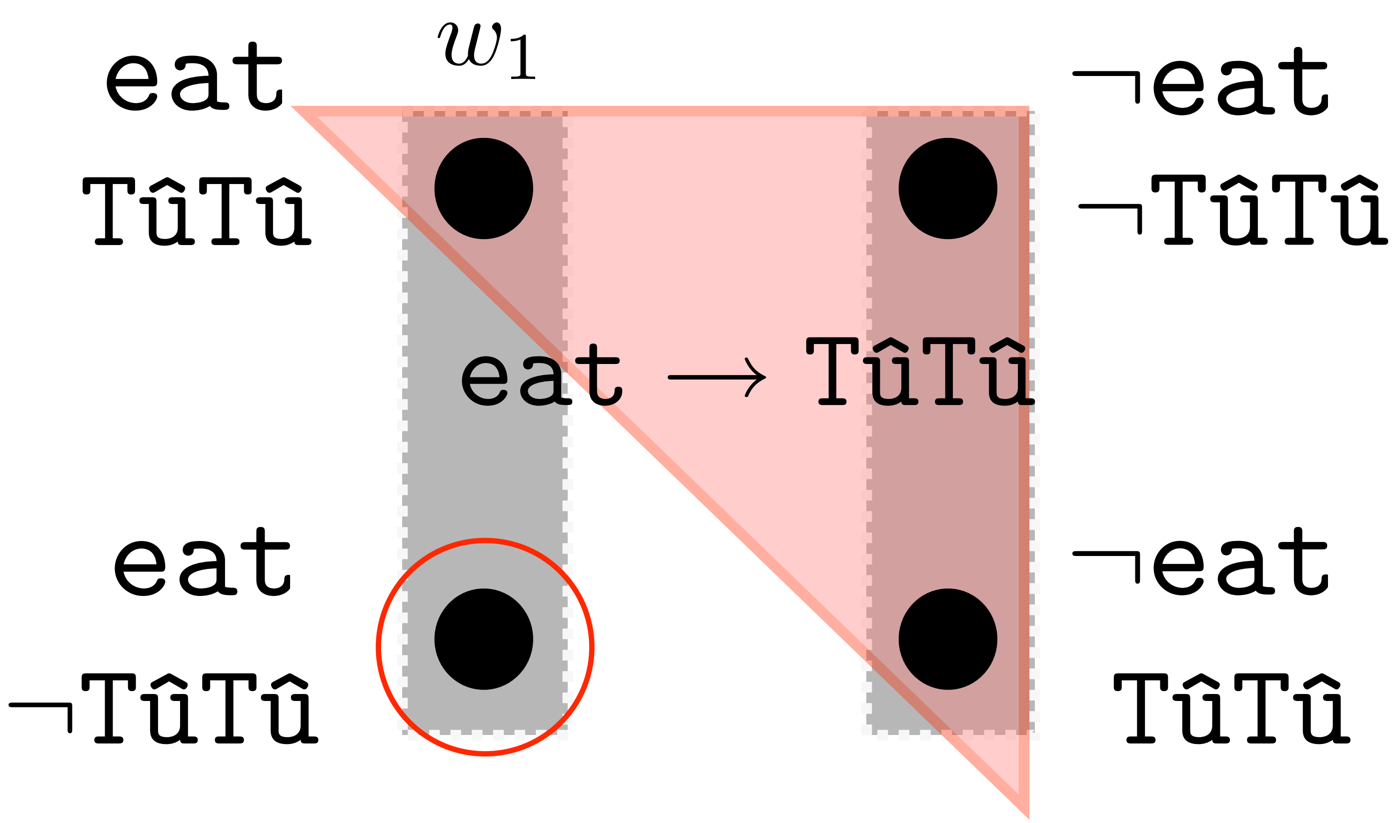
Recap ...


contextual classification
Cxt
proper contextual classification
Cxt^U
context definition
$nCxt^U$
language creation
UpTo

Contexts + UpTo operators? (i)



"Thou shall not eat of the chief's food!" (iv)



$$\mathcal{M}, w_1 \models [\text{Tribe}][\{\texttt{eat}, \texttt{TûTû}\}](\texttt{eat} \rightarrow \texttt{TûTû})$$

$$\mathcal{M}, w_1 \not\models [\text{Tribe}][\{\texttt{eat}\}](\texttt{eat} \rightarrow \texttt{TûTû})$$

Contexts + UpTo operators?

- Shortcut: use *Fusions* (Gabbay et. al., 2003)
- UpTo is not closed under taking disjoint unions in the given semantics, but it becomes so if we interpret it on the more general class of frames containing a set of partially-ordered equivalence relations.
- Same holds for $\mathsf{Cxt}^{\mathsf{U}}$
- Their fusion is sound and complete w.r.t. the fusion of the generalized classes of frames
- Of course we get a lot of "non-standard" models, and we miss important interactions, e.g.: $[\mathsf{U}]\varphi \leftrightarrow [\emptyset]\varphi$

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Part V

Context dynamics

with: G. Aucher, A. Herzig, E. Lorini

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The dynamics of Tû-Tû rulings

The chief's commandment does two things:

1. Increases the granularity of the tribe's view of the world by expanding their language (add the term Tû-Tû)

2. defines the "logical space" of the tribe's norms stating axioms of the kind: $\texttt{eat} \longrightarrow \texttt{TûTû}$

- ☐ Towards a logic of legislative rulings!
- ☐ We start with point 2, i.e., context dynamics

Rewind ...

$$
\begin{tikzcd}
\textit{contextual classification} \arrow[r, dotted] & \mathrm{Cxt} \arrow[d] \\
\textit{proper contextual classification} \arrow[r, dotted] & \mathrm{Cxt}^{\mathrm{U}}
\end{tikzcd}
$$

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Back to logic $\mathsf{Cxt}^{\mathsf{U}}$

$$\mathcal{L}_{\mathbf{Cxt}^{\mathsf{U}}} : \varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid [\mathsf{U}]\varphi \mid [X]\varphi$$

$\mathbf{Cxt}^{\mathsf{U}}$-models $\mathcal{M} = (W, R, \mathcal{I})$ s.t.:

- $W \neq \emptyset$;
- $R : \mathcal{C} \longrightarrow 2^W$ s.t. $\mathcal{I}(\mathsf{U}) = W$;
- $\mathcal{I} : \Phi \longrightarrow 2^W$.

(P)	all propositional schemata and rules
($\mathtt{K}^X$)	$[X](\varphi \rightarrow \varphi') \rightarrow ([X]\varphi \rightarrow [X]\varphi')$
($\mathtt{T}^{\mathsf{U}}$)	$[\mathsf{U}]\varphi \rightarrow \varphi$
(4^{XY})	$[X]\varphi \rightarrow [Y][X]\varphi$
(5^{XY})	$\langle X\rangle\varphi \rightarrow [Y]\langle X\rangle\varphi$
($\mathtt{N}^X$)	IF $\vdash \varphi$ THEN $\vdash [X]\varphi$

Contexts "know" about one another

- Every formula is equivalent to a formula of depth 1

Let α range over $\mathcal{L}_{Prop}$.

Base Straightforward.

Step If φ is of the form $[X]\psi$ with $X \in \mathcal{C} \cup \{\mathsf{U}\}$ then by IH there are $\alpha_k, \alpha^i_j, \beta^i \in \mathcal{L}_{Prop}$ such that

$$\varphi \leftrightarrow [X] \bigwedge_{1 \leq k \leq m} (\alpha_k \vee \bigvee_{1 \leq i \leq n_k} ([X_i]\alpha^i_1 \vee \ldots \vee [X_i]\alpha^i_{n_i} \vee \langle X_i \rangle \beta^i))).$$

However, using (4^{XY}) and (5^{XY}), one can easily show that:

$$\begin{aligned} \vdash_{\mathbf{Cxt}^{\mathsf{U}}} \quad & [X](\alpha_k \vee \bigvee_{1 \leq i \leq n_k} ([X_i]\alpha^i_1 \vee \ldots \vee [X_i]\alpha^i_{n_i} \vee \langle X_i \rangle \beta^i))) \\ & \leftrightarrow ([X]\alpha_k \vee \bigvee_{1 \leq i \leq n_k} ([X_i]\alpha^i_1 \vee \ldots \vee [X_i]\alpha^i_{n_i} \vee \langle X_i \rangle \beta^i))). \end{aligned}$$

$$\boxed{[X](\alpha \vee [Y]\beta) \leftrightarrow ([X]\alpha \vee [Y]\beta)}$$

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Two types of events

1. Context expansion (*norm promulgation*) $X + \varphi$
2. Context contraction (*norm derogation*) $X - \varphi$

- Recall that AGM models of belief revision sparked from the interest of Alchourrón and Makinson in the logical structure of derogation in legal codes.

Dynamic Context Logic (i)

Definition 1 *(Syntax)*

$$\mathcal{L}_{\mathbf{DCxt}^{\mathsf{U}}} : \varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid [X]\varphi \mid [\mathsf{U}]\varphi \mid [X{+}\psi]\varphi \mid [X{-}\psi]\varphi$$

where p ranges over $\mathbf{P}$, X over $\mathcal{C}$ and ψ over $\mathcal{L}_{\mathbf{Cxt}^{\mathsf{U}}}$

Definition 2 *(Models) Let $(\mathcal{M}, w) = (W, R, \mathcal{I}, w)$ and $(\mathcal{M}', w') = (W', R', \mathcal{I}', w')$ be two pointed $\mathbf{Cxt}^{\mathsf{U}}$-models, and let $\varphi \in \mathcal{L}_{\mathbf{Cxt}^{\mathsf{U}}}$ and $X \in \mathcal{C}$.*
We set $(\mathcal{M}, w) \xrightarrow{X+\psi} (\mathcal{M}', w')$ iff $W = W', w = w', \mathcal{I} = \mathcal{I}'$, and

- $R'_Y = R_Y$ *if* $Y \neq X$;
- $R'_X = R_X \cap ||\psi||_{\mathcal{M}}$.

We set $(\mathcal{M}, w) \xrightarrow{X-\psi} (\mathcal{M}', w')$ iff $W = W', w = w', \mathcal{I} = \mathcal{I}'$, and

- $R'_Y = R_Y$ *if* $Y \neq X$;
- $R'_X = \begin{cases} R_X & \text{if } \mathcal{M}, w \models \neg[X]\psi \vee [\mathsf{U}]\psi \\ R_X \cup S & \text{otherwise, for some } \emptyset \neq S \subseteq ||\neg\psi||_{\mathcal{M}} \end{cases}$

In case $(\mathcal{M}, w) \xrightarrow{X+\psi} (\mathcal{M}', w')$ (resp. $(\mathcal{M}, w) \xrightarrow{X-\psi} (\mathcal{M}', w')$), we say that $\mathcal{M}'$ is a (context) expansion *(resp.* contraction*) of $\mathcal{M}$.*

Dynamic Context Logic (ii)

Definition 3 *(Semantics) Let* $\mathcal{M}$ *be a* $\mathbf{Cxt}^{\mathsf{U}}$*-model,* $w \in W$*,* $\psi \in \mathcal{L}_{\mathbf{Cxt}^{\mathsf{U}}}$ *and* $\varphi \in \mathcal{L}_{\mathbf{DCxt}^{\mathsf{U}}}$

$$\begin{array}{lll} \mathcal{M}, w \models [X{+}\psi]\varphi & \textit{iff} & \mathcal{M}', w' \models \varphi \textit{ for all } \mathbf{Cxt}^{\mathsf{U}}\textit{-models } (\mathcal{M}', w') \\ & & \textit{such that } (\mathcal{M}, w) \xrightarrow{X+\psi} (\mathcal{M}', w'); \\ \mathcal{M}, w \models [X{-}\psi]\varphi & \textit{iff} & \mathcal{M}', w' \models \varphi \textit{ for all } \mathbf{Cxt}^{\mathsf{U}}\textit{-models } (\mathcal{M}', w') \\ & & \textit{such that } (\mathcal{M}, w) \xrightarrow{X-\psi} (\mathcal{M}', w'). \end{array}$$

- Context expansion is deterministic (functional)
- Context contraction is NOT! So, the $\Box$ of contraction talks about properties of the set of all possible contractions, and $\Diamond$ about the existence of a contraction with certain properties.

Dynamic Context Logic (iii)

(Cxt)	All axiom schemata and rules of $\mathbf{Cxt}^{\mathsf{U}}$
(RRE)	Rule of replacement of proven equivalence
(R+1)	$[X{+}\psi]\varphi_{\neq X} \leftrightarrow \varphi_{\neq X}$
(R+2)	$[X{+}\psi][X]\alpha \leftrightarrow [X](\psi \rightarrow \alpha)$
(R+3)	$[X{+}\psi]\neg\varphi \leftrightarrow \neg[X{+}\psi]\varphi$
(R–1)	$[X{-}\psi](\varphi_{\neq X} \vee \varphi_{=X}) \leftrightarrow (\varphi_{\neq X} \vee [X{-}\psi]\varphi_{=X})$
(R–2)	$\neg[X{-}\psi]\bot$
(R–3)	$[X{-}\psi]([X]\alpha_1 \vee \ldots \vee [X]\alpha_n \vee \langle X\rangle\alpha) \leftrightarrow$
	$((\neg[X]\psi \vee [\mathsf{U}]\psi) \wedge ([X]\alpha_1 \vee \ldots \vee [X]\alpha_n \vee \langle X\rangle\alpha))$
	$\vee\ (([X]\psi \wedge \neg[\mathsf{U}]\psi) \wedge ((\bigvee_{1\leq i\leq n} ([X]\alpha_i \wedge [\mathsf{U}](\psi \vee \alpha_i))) \vee \langle X\rangle\alpha \vee [\mathsf{U}](\psi \vee \alpha)))$
(K^{+})	$[X{+}\psi](\varphi \rightarrow \varphi') \rightarrow ([X{+}\psi]\varphi \rightarrow [X{+}\psi]\varphi')$
(K^{-})	$[X{-}\psi](\varphi \rightarrow \varphi') \rightarrow ([X{-}\psi]\varphi \rightarrow [X{-}\psi]\varphi')$

where $X \in \mathcal{C}$, $\varphi, \varphi' \in \mathcal{L}_{\mathbf{DCxt}^{\mathsf{U}}}$, $\psi \in \mathcal{L}_{\mathbf{Cxt}^{\mathsf{U}}}$, $\varphi_{=X} \in \mathcal{L}_{=X}$, $\varphi_{\neq X} \in \mathcal{L}_{\neq X}$, and $\alpha, \alpha_i \ldots \in \mathcal{L}_{Prop}$.

$\mathcal{L}_{=X} : \varphi ::= [X]\alpha \mid \neg\varphi \mid \varphi \wedge \varphi$
$\mathcal{L}_{\neq X} : \varphi ::= \alpha \mid [Y]\alpha \mid \neg\varphi \mid \varphi \wedge \varphi$
where $\alpha \in \mathcal{L}_{Prop}$, $Y \in (\mathcal{C} \cup \{\mathsf{U}\}) \backslash X$.

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Dynamic Context Logic (iv)

- Completeness can be proven via the reduction argument *à la Amsterdam,* by induction on the number of occurrences of dynamic operators.
- Decidability also follows, but the reduction is exponential, so it doesn't tell us anything about complexity.

Semantics of norm-change

$$\texttt{eat} \Rightarrow^{cl}_{X} \texttt{TûTû} \rightarrow [X - (\texttt{eat} \wedge \texttt{hungry} \rightarrow \texttt{TûTû})][X + (\texttt{eat} \wedge \texttt{hungry})]\langle X \rangle \neg \texttt{TûTû}$$

- Example of the introduction of exceptions to rules
- Expansion and contraction can capture both “legislative” and “juridical” aspects of normative reasoning!

Part V

Conclusions

A recap

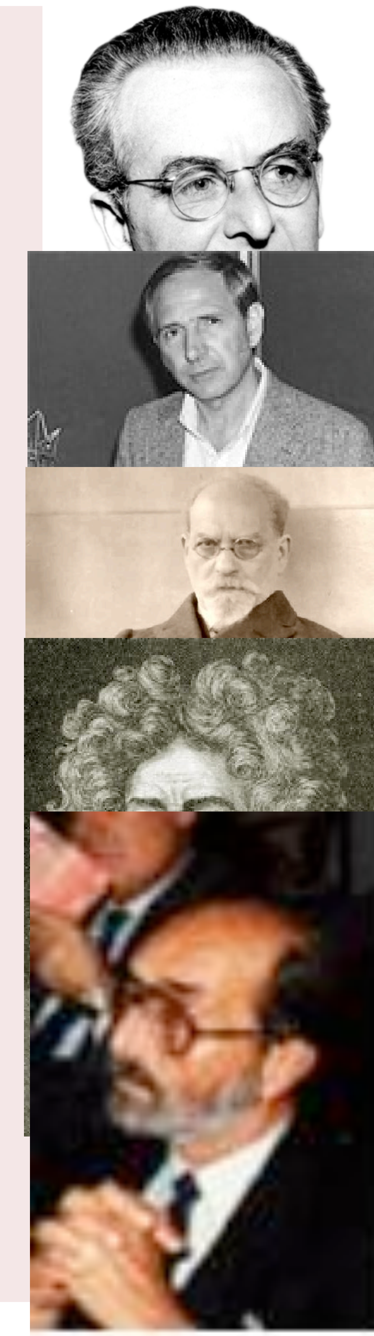
contextual classification → Cxt

proper contextual classification → Cxt^U

context definition → $nCxt^U$

language creation → UpTo

context dynamics → $DCxt^U$



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Future work (i)

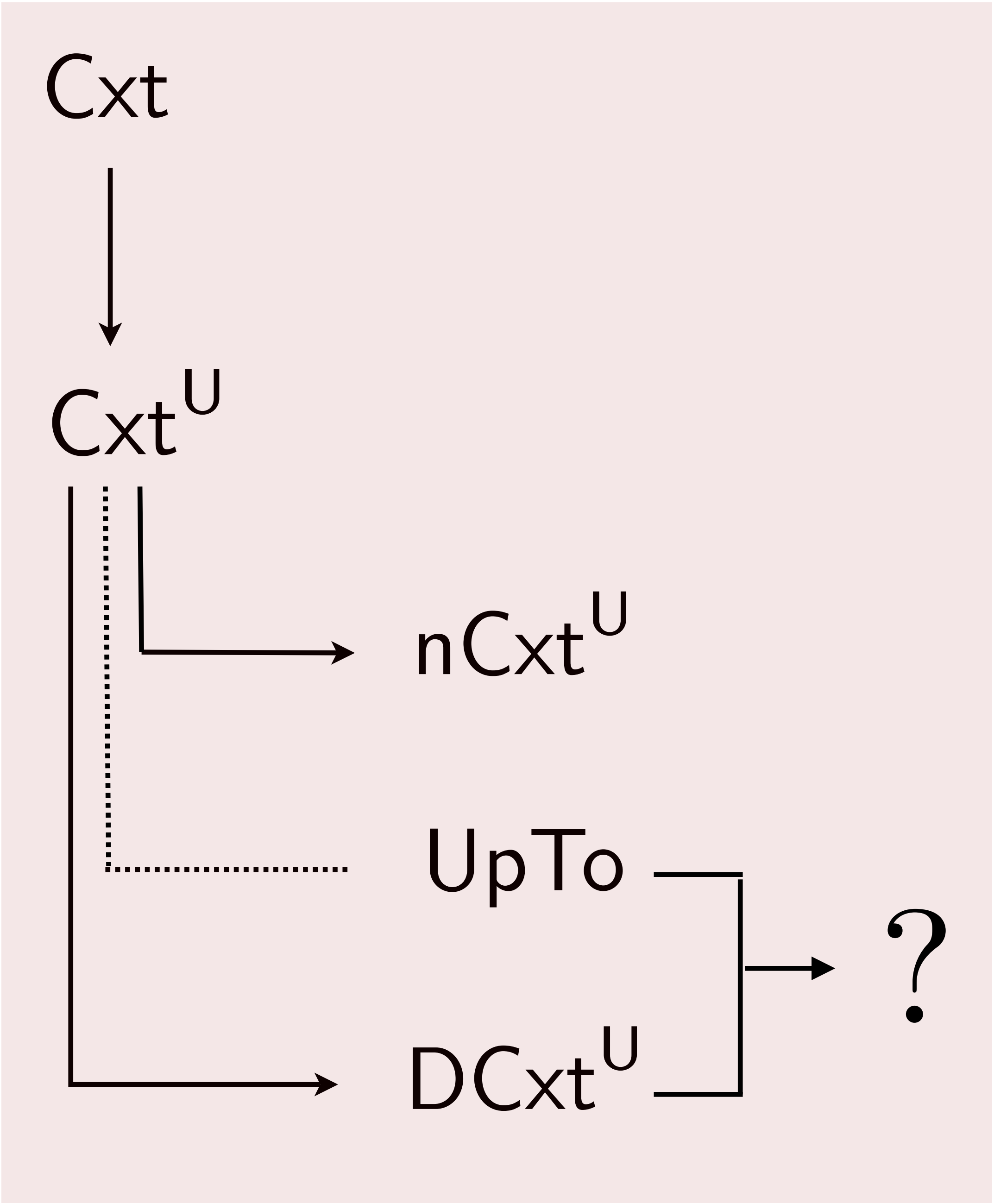
contextual classification

proper contextual classification

context definition

language creation

context dynamics



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Future work (ii)

- We have sophisticated languages to talk about norms as complex forms of classification/ascription of properties
- In a way, this is a toy-model of the legislative aspects of social design: "what should be the case"
- **AIM**: a similar toy-model of the executive aspects of social design: "how to make it happen as it should"
  - social software
  - game theory
  - mechanism design & social choice