Suggestions for Strategies in Modeling the Role of Reasoning in Ensemble Coordination

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Overview

1. A coordination problem in a music ensemble analysed in terms of a multi-agent system
2. Differing views of the rules guiding performance described in terms of variable frame theory
3. Trying to incorporate the idea of individual interpretations and revision of rules

A Possible Role for Common Knowledge in Ensemble Coordination

An idealized passage from a fictitious score

	Bar 1	Bar 2	Bar 3	Bar 4	Bar 5
oboe	Phrase1	Phrase1	Phrase2	Phrase1	Phrase1
violin	Phrase3	Phrase3	Phrase3	Phrase4	Phrase3
cello	Phrase5	Phrase5	Phrase5	Phrase5	Phrase6

A Possible Role for Common Knowledge in Ensemble Coordination

- In the spirit of Fagin et al: *Reasoning About Knowledge*, MIT Press 1995, let us model the performance situation as a multi-agent system, more specifically a system of information states developing over time.
- Intuitively, we let the label of the musical phrase denote the information state of a player playing that phrase.
- We model time as a stepwise development where each step is the length of an arbitrary bar in the score.

A Possible Role for Common Knowledge in Ensemble Coordination

- We call the information state of a given player i, the local state of i. For each i, we have a set of local states L_i, such that

$$L_{\text{oboe}} = \{\text{phrase1}, \text{phrase2}\}$$

$$L_{\text{violin}} = \{\text{phrase3}, \text{phrase4}\}$$

$$L_{\text{cello}} = \{\text{phrase 5}, \text{phrase6}\}$$

(We could also add a state Λ to each set L_i, denoting that nothing is played, but we choose to omit this here for clarity.)

A Possible Role for Common Knowledge in Ensemble Coordination

- We define a *global state* as a set of the local states of each player at a given time point m, $m \in \{0,1\ldots.\}$:
- $r(m) = (s_{oboe}, s_{violin}, s_{cello})$, where s_i is the local state of player i.
- The function r is called *a run* and describes a development of the global state over time.
- The multi-agent system can be described as a set of runs over the set of possible global states.

A Possible Role for Common Knowledge in Ensemble Coordination

- In our system, we can think of the runs as different performances
- We define a player i's local state at a given time m in a given run r as $r_i(m)$.
- We say that i cannot distinguish between two global states r(m) and r´(m), if i has the same local (information!) state at both of these global states, $r(m) \sim_i r'(m)$, if $r_i(m) = r'_i(m)$

Examples of different possible runs (performances)

r^{score}

	m=1	m=2	m=3	m=4	m=5
oboe	Phrase1	Phrase1	Phrase2	Phrase1	Phrase1
violin	Phrase3	Phrase3	Phrase3	Phrase4	Phrase3
cello	Phrase5	Phrase5	Phrase5	Phrase5	Phrase6

Examples of different possible runs (performances)

rlateoboe

	m=1	m=2	m=3	m=4	m=5
oboe	Phrase1	Phrase1	Phrase1	Phrase2	Phrase1
violin	Phrase3	Phrase3	Phrase3	Phrase4	Phrase3
cello	Phrase5	Phrase5	Phrase5	Phrase5	Phrase6

Examples of different possible runs (performances)

rlateviolin

	m=1	m=2	m=3	m=4	m=5
oboe	Phrase1	Phrase1	Phrase1	Phrase2	Phrase1
violin	Phrase3	Phrase3	Phrase3	Phrase3	Phrase4
cello	Phrase5	Phrase5	Phrase5	Phrase5	Phrase6

Examples of different possible runs (performances)

rscorevar1

	m=1	m=2	m=3	m=4	m=5
oboe	Phrase1	Phrase1	Phrase1	Phrase1	Phrase1
violin	Phrase3	Phrase3	Phrase3	Phrase4	Phrase3
cello	Phrase5	Phrase5	Phrase5	Phrase5	Phrase6

Examples of different possible runs (performances)

rscorevar2

	m=1	m=2	m=3	m=4	m=5
oboe	Phrase1	Phrase1	Phrase1	Phrase1	Phrase1
violin	Phrase3	Phrase3	Phrase3	Phrase3	Phrase3
cello	Phrase5	Phrase5	Phrase5	Phrase5	Phrase6

Examples of different possible runs (performances)

$r^{violinwaits}$, $t > 3$

	m=1	m=2	m=3	m=t	m=t+1	m=t+2
oboe	Phrase1	Phrase1	Phrase1	Phrase2	Phrase1	Phrase1
violin	Phrase3	Phrase3	Phrase3	Phrase3	Phrase4	Phrase3
cello	Phrase5	Phrase5	Phrase5	Phrase5	Phrase5	Phrase6

Examples of different possible runs (performances)

$r^{lateoboe}$

	m=1	m=2	**m=3**	m=4	m=5
oboe	Phrase1	Phrase1	**Phrase1**	Phrase2	Phrase1
violin	Phrase3	Phrase3	**Phrase3**	Phrase4	Phrase3
cello	Phrase5	Phrase5	**Phrase5**	Phrase5	Phrase6

$r^{lateoboe}(3) \sim_i r^{lateviolin}(3)$; $r^{lateoboe}(3) \sim_i r^{violinwaits}(3)$, $r^{lateoboe}(3) \sim_i r^{scorevar1}(3)$, $r^{lateoboe}(3) \sim_i r^{scorevar2}(3)$ and ($r^{lateoboe}(3) \sim_i r^{violinwaits}(3)$

Examples of different possible runs (performances)

$r^{lateviolin}$

	m=1	m=2	**m=3**	m=4	m=5
oboe	Phrase1	Phrase1	**Phrase1**	Phrase2	Phrase1
violin	Phrase3	Phrase3	**Phrase3**	Phrase3	Phrase4
cello	Phrase5	Phrase5	**Phrase5**	Phrase5	Phrase6

$r^{lateoboe}(3) \sim_i r^{lateviolin}(3)$; $r^{lateoboe}(3) \sim_i r^{violinwaits}(3)$, $r^{lateoboe}(3) \sim_i r^{scorevar1}(3)$, $r^{lateoboe}(3) \sim_i r^{scorevar2}(3)$ and $(r^{lateoboe}(3) \sim_i r^{violinwaits}(3)$

Examples of different possible runs (performances)

$r^{violinwaits}$

	m=1	m=2	**m=3**	m=t	m=t+1
oboe	Phrase1	Phrase1	**Phrase1**	Phrase2	Phrase1
violin	Phrase3	Phrase3	**Phrase3**	Phrase3	Phrase4
cello	Phrase5	Phrase5	**Phrase5**	Phrase5	Phrase5

$r^{lateoboe}(3) \sim_i r^{lateviolin}(3)$; $r^{lateoboe}(3) \sim_i r^{violinwaits}(3)$, $r^{lateoboe}(3) \sim_i r^{scorevar1}(3)$, $r^{lateoboe}(3) \sim_i r^{scorevar2}(3)$ and $(r^{lateoboe}(3) \sim_i r^{violinwaits}(3)$

Examples of different possible runs (performances)

$r^{scorevar1}$

	m=1	m=2	**m=3**	m=4	m=5
oboe	Phrase1	Phrase1	**Phrase1**	Phrase1	Phrase1
violin	Phrase3	Phrase3	**Phrase3**	Phrase4	Phrase3
cello	Phrase5	Phrase5	**Phrase5**	Phrase5	Phrase6

$r^{lateoboe}(3) \sim_i r^{lateviolin}(3)$; $r^{lateoboe}(3) \sim_i r^{violinwaits}(3)$, $r^{lateoboe}(3) \sim_i r^{scorevar1}(3)$, $r^{lateoboe}(3) \sim_i r^{scorevar2}(3)$ and $(r^{lateoboe}(3) \sim_i r^{violinwaits}(3)$

Examples of different possible runs (performances)

$r^{scorevar2}$

	m=1	m=2	**m=3**	m=4	m=5
oboe	Phrase1	Phrase1	**Phrase1**	Phrase1	Phrase1
violin	Phrase3	Phrase3	**Phrase3**	Phrase3	Phrase3
cello	Phrase5	Phrase5	**Phrase5**	Phrase5	Phrase6

$r^{lateoboe}(3) \sim_i r^{lateviolin}(3)$; $r^{lateoboe}(3) \sim_i r^{violinwaits}(3)$, $r^{lateoboe}(3) \sim_i r^{scorevar1}(3)$, $r^{lateoboe}(3) \sim_i r^{scorevar2}(3)$ and $(r^{lateoboe}(3) \sim_i r^{violinwaits}(3)$

- The only way in which the players can distinguish the runs from each other at m=3, is if they have **common knowledge** of a rule p (broadly, a consciousness that p is known by everyone and known to be known by everyone), where p determines which run is being executed if deviations from r^{score} occur.
- We take for granted that by knowing p, and that p is common knowledge, a player i will follow p.
- Problem: This does not allow for *any* disagreement on the content of p. Intuitively, everyone must have the same idea of the central rules of the composition.

Variable Frame Theory

- Example: ”Three Cubes and a Pyramid”
- Objectively a 0.25 chance of coordinating on the same object – but experimental results show that people tend to coordinate on the green pyramid.
- Define a set of objects *S*, a set of predicates *P* and a function *E* assigning subsets of *S* to members of *P*

Variable Frame Theory

- If φ is a predicate, $E(\varphi)$ is the set of objects φ describes (the extension of φ) Examples:
- $S_{\text{objects}}=\{x_1, x_2, x_3, x_4\}$
- $E_{\text{objects}}(\text{cube})=\{x_1, x_2, x_4\}$
- $E_{\text{objects}}(\text{pyramid})=\{x_3\}$
- Singling out one cube = **picking** a cube
- Singling out the pyramid = **choosing** the pyramid

Variable Frame Theory

- Predicates can be arranged in *families*, sets of predicates where if one predicate comes to mind, so does the other members of the family. Examples:
- F_{shape} = {cube, pyramid…}
- F_{colour}={blue, red, yellow, green…}
- F_{thing}={thing}, where E_{objects}(thing)={x_1, x_2, x_3, x_4}

Variable Frame Theory

- For each player, we define a set of families that come to mind for that player – a *frame.*
- Such a set is a subset of the universal frame F, containing all families that can be taken into consideration in the example (in this case $F = \{F_{\text{thing}}, F_{\text{shape}}, F_{\text{colour}} \ldots\}$)
- Each player assigns to his opponent (we are assuming a game of two players) a probability $v(F_i)$ that the opponent has a family F_i in his frame (called the *availability* of the frame)

Variable Frame Theory

- A predicate defines an act-description, so if a player *frames* the problem according to F_{shape}, he can either “pick a cube” or “choose the pyramid”
- E.g. a player may think that $v(F_{\text{thing}})=1$, $v(F_{\text{colour}})=0.6$ and $v(F_{\text{shape}})=0.8$ for his opponent.
- Assuming he is right, the possibility that the players coordinate on a cube is 0.33*0.8 = 0.26
- The possibility that they coordinate on the pyramid is, however, 1*0.8 = 0.8

Variable Frame Theory

- According to Michael Bacharach: *Beyond Individual Choice: Teams and Frames in Game Theory*, Princeton 2006, we tend to reason in a way where we find it rational to choose the action that we *think* will most likely lead to coordination (if coordination is the object of the game), even if we are strictly speaking *not sure* that coordination will take place.

How Do Musicians With Different Views on the Composition Coordinate?

- To simplify our initial example, let us consider two possible strategies:
- ”Wait” – corresponding to $r^{violinwaits}$
- ”Don’t wait” – where everyone, including the player with the erroneous phrase continues according to the score
- The object of the ”game” is coordination on either of the two strategies

How do Musicians with Different Views on the Composition…

- In ”the objective game” the players have a 0.25 chance of coordinating on the same strategy. But this is assuming that the players choose at random.
- ”Wait” could for instance be described as ”more melodic” and ”Don’t Wait” as ”more rhythmical”
- Let ”Wait” be symbolized by x_1 and “Don’t Wait” by x_2

How do Musicians with Different Views on the Composition…

- F_{rhythm}={keeps the piece going rhythmically,…}, where E(keeps the piece going rhythmically) = {x_2}
- F_{melody}={melodic,…}, where E(melodic) = {x_1}
- F_{thing}={thing} where E(thing) = {x_1, x_2}
- We have the universal frame for the coordination game: F={F_{thing}, F_{rhythm}, F_{melody}…}

How do Musicians with Different Views on the Composition…

- Three different act-descriptions: ”pick a thing (something)”, “choose the option that keeps the piece going rhythmically” or “choose the melodic”
- A complication compared to ”Three Cubes and a Pyramid”: three players instead of two
- Each player may assign different availabilities to the same frame for each of the two co-players

How do Musicians with Different Views on the Composition…

- Example: The violin assigns $v_{\text{oboe}}(F_{\text{melody}}) = 0.7$, $v_{\text{oboe}}(F_{\text{rhythm}}) = 0.3$, $v_{\text{cello}}(F_{\text{melody}}) = 0.6$ and $v_{\text{cello}}(F_{\text{rhythm}}) = 0.5$.
- If the violin is right, the chance of coordinating with the others on “choose the option that keeps the piece going rhythmically” is 0.3*0.5*1 = 0.15.
- His chances of coordinating with the others on ”choose the melodic” would be 0.7*0.6*1 = 0.42

How do Musicians with Different Views on the Composition…

- It seems that given his expectations of how the other players frame the situation, it would be rational for the violin to ”choose the melodic”.
- Of course, the idea of possibility assessments is an idealized model of considerations musicians make while playing, but the idea captures important insights.

Integrating Individual Interpretations of a Composition

If we view the composition as a set of rules guiding performance, not as a specific sonic outcome, we could define interpretations (made by musicians) of a composition in the following way:

Integrating Individual Interpretations of a Composition

Rule 1
Rule 2
Rule 3
Rule 4
Rule 5
Rule 6
Rule 7
Rule 8
…

The score or model performance

Rule 2
Rule 3
Rule 6
Rule 7
Rule 8

The conventional or subjective view of which rules are more important (constitutes the composition)

1. Rule 3
2. Rule 2
3. Rule 8
4. Rule 6
5. Rule 7

The individual musician's ranking of these rules = the musician's interpretation

Integrating Individual Interpretations of a Composition

- We could for instance describe the rankings of the interpretation in terms of a preference ordering for each player that determines which rules he/she will try to follow when having to choose between two or more different actions

- This could also amount to modeling an extensive strategy for each player, that is, a description of what the player should do in any situation that might occur

Integrating Individual Interpretations of a Composition

- The possible benefit of such an approach would be that it allows a musician to reason about the strategies of other musicians without considering them capable of describing the different choices to themselves (as in variable frame theory)

Revision of Strategies

- It seems to be a consequence of wanting to achieve coordination that some musicians – simply by being outnumbered – must occassionally give up their own strategy and adjust to those of other musicians (if we are looking at an improvisation context, revising ones own strategy is a constantly reoccuring phenomenon).

Revision of Strategies

- A model for a less drastic revision process could also be imagined in which a musician adds, removes or moves a rule to/from/in his or her interpretation (still viewed as a ranking of rules) as the performance moves along, in order to either adjust to the other musicians or simply changing his or her mind about the relevant aspects of the music.
- Rules added might include musical ideas external to the score

Revision of Strategies

- Of course, the boundaries between small changes in the strategy of the agent and a complete replacement with a new strategy are fluent.
- It is a point of further discussion how much an interpretation can be altered before it constitutes a new (or radically new view of the) composition. (This is a problem because of the possibility of adding new elements external to the initial composition)

Conclusion

- In order for perfect coordination to be *certain* to take place, some rule determining the prioritized ranking of instructions in the composition must be common knowledge
- Even if not everyone agrees on the ranking of instructions in the composition, coordination is still possible, because musicians reason according to their expectations of the actions of their co-players