Connecting Different Logics: Translation, Reduction, Tracking

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This talk is not about proposing new logical systems, but about connecting or even 'identifying' existing systems.

The situation

1 **Landscape of logical systems**: endless proliferation?
One general perspective: study lattices of logics under extension (a 'surface connection').
Search for general results based on broad system features, e.g., about normal modal logics.

2 More intimate system connections. Illuminating **translations** are everywhere:
Standard translation ST, Gödel translation, Conditional Logic S4.3U, etc.
Neighborhood modal logics into bimodal logics, etc. (Kracht & Wolter).

3 These connections are a force for **unity** in the field of logic, and help mitigate the divergent tendency toward playing up (often small) differences between systems.
Today: discuss some general aspects/challenges of the translation perspective.
There are many concrete cases and claims, but no unified perspective.

4 What drives many translations: **formalizing compositional semantics**/metatheory.
Or the origin might be vice versa: Kripke semantics mirrors the Gödel translation.

5 Goes on all the time: One logic can contain other logics: cf. Dvorkin on normal modal logics in S4.

6 Discussion: Translatability study of propositional consequence relations in Brazilian school.
Jerabek 2012: Conservative translatability (what sense?) is the norm here rather than exception.

Basic technical notions and how they apply

7 Embeddings versus **faithful** ('conservative') **translations**.

8 General schema: correlated maps between (i) formulas, (ii) models,
Inverse correlation: $F(\mathbf{M}) \models \varphi$ iff $\mathbf{M} \models \tau(\varphi)$.

9 Galois connection, can (should?) make this very abstract in Category Theory.
Concrete instance: **Chu spaces**, infomorphisms (Barwise & Seligman, Zhong).
Characterization of first-order properties preserved by this (van Benthem 2000).

10 Extra conditions are needed in this setting to ensure faithful translation.

11 Does the abstract schema fit reality? LFD - GF as recent case study, needs further twists.

12 Issue: What (if any) are the extra benefits of **compositional** translation? Algebraic hom's?

13 Problem and technical desideratum: good criteria for **non-translatability**.

Landscapes with system connections

14 **SAT reductions** vs. translations. Often easier to get, but do not preserve 'meaning'.
Sometimes the border-line seems thin: see also point 12 above.

15 Landscapes of logical systems under translation and other reductions.
What do they look like? Which ordering? One mother system?

Concrete illustrations: information, dynamics, games

16 Logics for modeling **information structure at different levels**.
Landscape of systems at different grain/zoom levels, from syntax to set semantics.
Not linear sequence of stages: fit in qualitative and quantitative.

17 The levels are connected, e.g., by obvious **projection** or reduction maps on models.

18 Example: evidence models and plausibility models.
Syntactic counterpart in opposite direction: **Translation** between languages.

19 Aside: Travel to finer or coarser models equally valuable (think: topology).

20 Further connection issue. **Tracking the update dynamics** at different levels:
commuting diagram reduction map/update maps. When possible, when not?
Telling example: tracking Bayesian update in qualitative belief revision theory.
Connects to translating the dynamic extension of the static logical base language.

21 All this returns for games: translating and tracking at action/power/strategic form levels.
Open problems. E.g., PAL update on extensive games not trackable on power representations.

Practical and general issues: Field in broad themes

22 Truth definitions/formal **semantics is translation**? More case studies needed.

23 **Undercut current ideological differences**. Hyperintensionality and translating truthmaker logic. Three-Valued Logic, Binary Modalities. (Halvorsen program.)

24 **General mathematical theory of translations** as substantial agenda item for logic.

25 What is translation? Connecting models and languages, connecting **further practices**?
Compare translation between natural languages in richer 'process' vs. 'product' sense.

26 Issues of system identity. **When are two (logical) systems the same?**
Also issue in philosophy of science: reductions between theories.

Conclusion

Logic needs unifying themes across systems. Translation should be a prominent one.

References

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Various recent papers on translating relevant and truth maker logics into standard modal logics by Søren Brinck Knudstorp, ILLC Amsterdam.