To Be Announced — quantifying over information change

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What is epistemic logic good for?

Alice and Bob meet. Bob is a conference chair. Alice submitted a paper to that conference. They both don't know whether Alice's submission has been accepted. But Bob knows that rejections have already been sent out. They talk:

- Alice to Bob: "Is my submission accepted?"
- Bob to Alice: "Yes!"
- Alice to Bob: "I am glad to know that."

What is dynamic epistemic logic?

Alice and Bob meet. Bob is a conference chair. Alice submitted a paper to that conference. They both don't know whether Alice's submission has been accepted. But Bob knows that rejections have already been sent out. They talk:

rejected —*Alice*— <u>accepted</u> —*Bob*— rejected

Alice to Bob: "Is my submission accepted?" $\Rightarrow$

rejected —*Alice*— <u>accepted</u>

Bob to Alice: "Yes!" $\Rightarrow$

<u>accepted</u>

Alice to Bob: "I am glad to know that."

Quantifying over information change

- What is said in the example is called a public announcement.
- Public announcements are interpreted as model updates.
- Announcements may be false after the announcement.
- In many settings, such as epistemic planning, we wish to realize goal formulas.
- Announcing a goal may not make it true! How to make it true?
- Quantifying over information change:
  Is there an announcement after which φ is true?
  Is there an epistemic action after which φ is true?
  Is there an ontic action (factual change) after which φ is true?
- Quantifiers in dynamic epistemic logic are like temporal modalities in temporal epistemic logic: $[\varphi]K_a\psi$ is like $XK_a\psi$.
- **In this talk we present different ways to quantify over information change.**

Quantifying over information change

Different ways to quantify over information change:

- there is an announcement after which φ
- there is a boolean/positive/... announcement after which φ
- there is an announcement by the agents in G after which φ
- there is an announcement by the agents in G after which φ ... no matter what the other agents simultaneously announce
- there is an epistemic action (action model) after which φ
- there is an arrow update after which φ
- there is a refinement after which φ
- there is a simulation after which φ
- there is a model minus a state after which φ (sabotage logic)
- there is a resolution after which φ (resolving distr.knowledge)
- there is ... any other submodel operation after which φ.

[vD. To Be Announced. Information & Computation, 2023]

Language, structures and semantics

Languages. Countable set of *propositional variables* (*atoms*) P. Finite set of *agents* A. Below, $p \in P$, and $a \in A$:

$$
\begin{array}{llll}
\mathcal{L}(\emptyset) & \ni & \varphi & ::= \; p \mid \neg\varphi \mid (\varphi \wedge \varphi) \\
\mathcal{L}(\Diamond) & \ni & \varphi & ::= \; p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Diamond_a\varphi \\
\mathcal{L}(\Diamond, !) & \ni & \varphi & ::= \; p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Diamond_a\varphi \mid \langle\varphi\rangle\varphi
\end{array}
$$

Abbreviations: prop. connectives, $\Box_a\varphi := \neg\Diamond_a\neg\varphi$, $[\varphi]\psi := \neg\langle\varphi\rangle\neg\psi$.

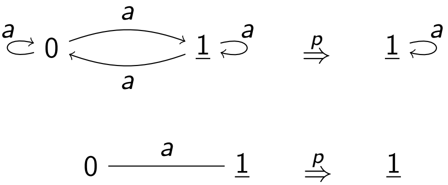
Structures. *Epistemic model* $M = (S, R, V)$ with non-empty *domain* S of *states*, *accessibility function* $R : A \to \mathcal{P}(S \times S)$ (*accessibility relation* R_a), and *valuation* $V : P \to \mathcal{P}(S)$. Pointed models M_s, multi-pointed models M_T (where $T \subseteq S$). If R_a is an equivalence relation we write $\sim_a$ (*indistinguishability relation*).

Semantics.
$M_s \models \Diamond_a\varphi$ iff there is a $t \in S$ such that $R_a st$ and $M_t \models \varphi$

Public Announcement

$M_s \models \langle\psi\rangle\varphi$ iff $M_s \models \psi$ and $(M|\psi)_s \models \varphi$

where $M|\psi$ is the restriction of the model to the states satisfying ψ



$$\langle p\rangle\Box_a p \qquad \Box_a p$$

$$\langle p \land \neg\Box_a p\rangle\neg(p \land \neg\Box_a p) \qquad \neg(p \land \neg\Box_a p)$$

Announcing a formula may not make it true.

[Plaza 1989]
[Wang, Cao. On axiomatizations of public ann. logics. Synthese 2013]

Arbitrary Announcement

Add $\langle ! \rangle \varphi$ to the BNF.

$M_s \models \langle ! \rangle \varphi$ iff there is a quantifier-free ψ such that $M_s \models \langle \psi \rangle \varphi$

$0 \overset{a}{\text{———}} \underline{1} \quad \overset{\top}{\Leftarrow} \quad 0 \overset{a}{\text{———}} \underline{1} \quad \overset{p}{\Rightarrow} \quad \underline{1}$

$\neg \Box_a p \qquad\qquad \langle ! \rangle \neg \Box_a p \wedge \langle ! \rangle \Box_a p \qquad\qquad \Box_a p$

A (boring) validity: $(p \wedge \neg \Box_a p) \rightarrow \langle ! \rangle \neg \Box_a p \wedge \langle ! \rangle \Box_a p$.

$0 \overset{a}{\text{———}} \underline{1} \quad \overset{p}{\Rightarrow} \quad \underline{1} \qquad\qquad \underline{0} \overset{a}{\text{———}} 1 \quad \overset{\neg p}{\Rightarrow} \quad \underline{0}$

A (not so boring) validity: $\langle ! \rangle (\Box_a p \vee \Box_a \neg p)$.
An interesting validity: $\langle ! \rangle (\Box_a \varphi \vee \Box_a \neg \varphi)$.

[van Benthem. What one may come to know. Analysis 2004]
[Balbiani et al. Knowable as known after an announcement. RSL 2008]
[vD, van der Hoek, Iliev. Everything is knowable. Theoria 2012]

Arbitrary Announcement

$M_s \models \langle ! \rangle \varphi$ iff there is a quantifier-free ψ such that $M_s \models \langle \psi \rangle \varphi$

- $[!]\varphi \rightarrow [!][!]\varphi$ is valid — 'first ψ then χ' is ann. $\psi \wedge [\psi]\chi$
- $[!]\varphi \rightarrow \varphi$ is valid — if true after any ann. then already true
- $\langle ! \rangle [!]\varphi \rightarrow [!]\langle ! \rangle \varphi$ (CR) is valid — closing the update diamond
- $[!]\langle ! \rangle \varphi \rightarrow \langle ! \rangle [!]\varphi$ (MK) — most informative ann. given φ
- APAL is not a normal modal logic! $p \rightarrow \langle ! \rangle \Box_a p$ is valid, but $(p \wedge \neg \Box_a p) \rightarrow \langle ! \rangle \Box_a (p \wedge \neg \Box_a p)$ is invalid (on $\mathcal{S}5$ models)
- complete infinitary axiomatization: PAL +
  derivation rule: from $\xi([\psi]\varphi)$ for all ψ, infer $\xi([!]\varphi)$
  axiom: $[!]\varphi \rightarrow [\psi]\varphi$ (quantifier-free ψ in both)
- more expressive than PAL, because $[!]$ quantifies over (i) all atoms and over (ii) formulas of arbitrarily large modal depth
- non-compact, undecidable satisfiability, model ch. PSPACE-c.

[French, vD. Undecidability for APAL. AiML 2008]
[Balbiani, vD. A simple proof of the completeness of APAL. 2015]

Arbitrary Announcement — open questions

$M_s \models \langle ! \rangle \varphi$ iff there is a quantifier-free ψ such that $M_s \models \langle \psi \rangle \varphi$

- Is there a finitary axiomatization for APAL?
- Alternative semantics for arbitrary announcement:
  $M_s \models \langle ! \rangle \varphi$ iff there is a qu.fr. ψ such that $(M|\psi)_s \models \varphi$
  Axiomatization of the language without public announcement $\langle \psi \rangle \varphi$ but with arbitrary announcem. $\langle ! \rangle \varphi$?
- Axiomatization of knowability logic with bundled $\langle ! \rangle \Box_a \varphi$?

[Wang. Beyond knowing that: a new generation . . . 2018]
[Galimullin, Kuijer. Satisfiability of Arbitrary Public Announcement Logic with Common Knowledge is Σ^1_1-hard. TARK 2023]
[Liu, Fan, vD, Kuijer. Logics for knowability. Logic and Log. Ph. 2022]
[Christensen. Logics of knowability. AiML 2024]

Variations of APAL

Why are there variations of APAL?
APAL has a (known) infinitary axiomatization, and is undecidable.
We want finitary axiomatizations, decidable logics. But why? (...)

$M_s \models \langle ! \rangle \varphi$ iff there is a qu.fr. ψ such that $M_s \models \langle \psi \rangle \varphi$ (APAL)
$M_s \models \langle ! \rangle \varphi$ iff there is a ψ such that $M_s \models \langle \psi \rangle \varphi$ (full APAL)
$M_s \models \langle ! \rangle \varphi$ iff there is a positive ψ such that $M_s \models \langle \psi \rangle \varphi$ (APAL$^+$)
$M_s \models \langle ! \rangle \varphi$ iff there is a boolean ψ such that $M_s \models \langle \psi \rangle \varphi$ (BAPAL)

full APAL: $\langle ! \rangle \varphi$ iff there is ordinal α with $\langle !_\alpha \rangle \varphi$. Just $\langle !_\omega \rangle$ is enough?
APAL$^+$: infinitary axiomatization and conjectured decidable
BAPAL: finitary axiomatization and conjectured decidable ~~arXiv~~
$\psi \rightarrow [\chi][p]\varphi$ implies $\psi \rightarrow [\chi][!]\varphi$, with p fresh

[vD, van der Hoek, Kuijer. Fully Arbitrary Pub. Announcem. AiML 2016]
[vD, French, Hales. Positive Announcements. Studia Logica 2021]
[vD, French. Quantifying over Boolean Announcements. LMCS 2022]

More variations of APAL

XAPAL, YAPAL, ZAPAL, ...

- APAL with memory: 1. current & initial domain; 2. φ^0 for 'φ *was* true'; 3. in $\langle\varphi\rangle\psi$, φ is qu.free; 4. finitary axiomatization
- SCAPAL: $M_s \models \langle!\rangle\varphi$ iff there is a qu.fr. ψ with only atoms from φ such that $M_s \models \langle\psi\rangle\varphi$ ($\not\models \langle!\rangle\varphi \vee \langle!\rangle\psi \leftrightarrow \langle!\rangle(\varphi \vee \psi)$)
- FSAPAL: $M_s \models \langle!_Q\rangle\varphi$ iff there is a qu.fr. ψ with only atoms in Q such that $M_s \models \langle\psi\rangle\varphi$
- **Novel**: $M_s \models \langle!^n\rangle\varphi$ iff there is a $\psi \in \mathcal{L}(\Diamond, !)$ with $d(\psi) \leq n$ and such that $M_s \models \langle\psi\rangle\varphi$. **Anyone?** *PS: restrict atoms too?*

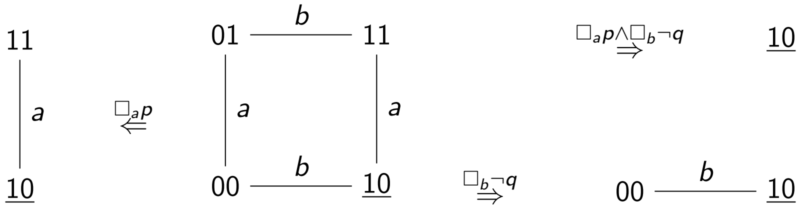
[Baltag, Özgün, Vargas Sandoval. APAL with memory. JPL 2022.]
[vD, Liu, Kuijer, Sedlár. Almost APAL. JLC 2023.]

IPAL — relation to dynamic consequence: $\varphi, \psi \Rightarrow \chi$ iff $\models [\varphi][\psi]\chi$

- $M_s \models \langle\chi^{\downarrow}\rangle\varphi$ iff there is qu.fr. ψ *implying* χ s.t. $M_s \models \langle\psi\rangle\varphi$ quantify over restrictions of M *contained in* $M|\chi$
- $M_s \models \langle\chi^{\uparrow}\rangle\varphi$ iff there is qu.fr. ψ *implied by* χ s.t. $M_s \models \langle\psi\rangle\varphi$ quantify over restrictions of M *containing* $M|\chi$

Group Announcement

$M_s \models \langle !_G \rangle \varphi$ iff there is a qu.fr. $\{\psi_a \mid a \in G\}$ s.t. $M_s \models \langle \bigwedge_{a \in G} \Box_a \psi_a \rangle \varphi$



- $M_{10} \models \langle !_a \rangle \Box_b p$ but $M_{10} \not\models \langle !_b \rangle \Box_b p$
- $M_{10} \models \langle !_b \rangle \Box_a \neg q$ but $M_{10} \not\models \langle !_a \rangle \Box_a \neg q$
- $M_{10} \models \langle !_{ab} \rangle (\Box_b p \wedge \Box_a \neg q)$ but $M_{10} \not\models \langle !_a \rangle (\Box_b p \wedge \Box_a \neg q)$ and $M_{10} \not\models \langle !_b \rangle (\Box_b p \wedge \Box_a \neg q)$

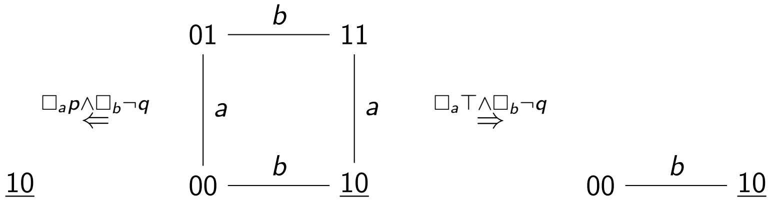
- $\langle !_G \rangle \langle !_H \rangle \varphi \rightarrow \langle !_{G \cup H} \rangle \varphi$ and so $\langle !_G \rangle \langle !_G \rangle \varphi \rightarrow \langle !_G \rangle \varphi$

[Ågotnes, vD. *Coalitions and Announcements*, AAMAS 2008]
[Ågotnes, Balbiani, vD, Seban, *Group Announcement Logic*. JAL 2010]

Coalition Announcement

$M_s \models \langle\!\langle !_G \rangle\!\rangle \varphi$ iff there is a qu.fr. $\{\psi_a \mid a \in G\}$ s.t. $M_s \models \bigwedge_{a \in G} \Box_a \psi_a$ and for all qu.-fr. $\{\psi_a \mid a \in A \setminus G\}$, $M_s \models [\bigwedge_{a \in A} \Box_a \psi_a]\varphi$

$M_{10} \not\models \langle\!\langle !_a \rangle\!\rangle \Diamond_a q$: b can prevent a from remaining ignorant about q by announcing $\neg q$. No matter whether a announces p or $\top$.



$M_{10} \models \langle\!\langle !_{ab} \rangle\!\rangle (\Box_b p \wedge \Box_a \neg q)$: trivial, as $\langle !_A \rangle \varphi \leftrightarrow \langle\!\langle !_A \rangle\!\rangle \varphi$.

Embeds *Coalition Logic*: $\langle\!\langle !_G \rangle\!\rangle \varphi \wedge \langle\!\langle !_H \rangle\!\rangle \varphi' \to \langle\!\langle !_{G \cup H} \rangle\!\rangle (\varphi \wedge \varphi')$ $G \cap H = \emptyset$

[Ågotnes, vD. *Coalitions and Announcements*, AAMAS 2008.]
[Galimullin. *Coalition Announcements*. Ph.D. Uni Nottingham, 2019]

Quantifying over announcements and over actions

We are playing cards. Let p be the proposition 'I hold the red queen'.

Putting the red queen open on the table is a public announcement of p. A public announcement is observed the same by all agents.

Showing the red queen to my right neighbour without the other players seeing which card it it, is a private announcement of p. A private announcement is not observed the same by all agents. Some players consider it possible you showed the black ace.

I am even more private when I sneakily show my right neighbour the red queen while the other players did not notice me doing so.

Such generalizations of public announcements are described with **action models** [Baltag, Moss, Solecki. TARK 1998]

Quantifying over action models is **easier** than quantifying over public announcements. Easy?

Action models, arrow updates, and refinements

Let a and b be ignorant about p. Different non-public updates where a but not b is informed about p are (assume transitivity):

$$0 \overset{b}{\text{———}} \underline{1} \quad \overset{*}{\Leftarrow} \quad 0 \overset{ab}{\text{———}} \underline{1} \quad \Rightarrow \quad 0 \overset{ab}{\text{———}} 1 \overset{b}{\text{———}} \underline{1}$$

Updates can be represented in very different ways. Update $(*)$ is an:

Action model consisting of two announcements p and $\neg p$ that a observes but b is uncertain about. We compute the restricted modal product of the epistemic model and the action model.

Arrow update consisting of arrows $p \rightarrow_a p, \neg p \rightarrow_a \neg p$, and $\top \rightarrow_b \top$. We restrict the initial model to the arrows (links) satisfying the source and target conditions.

Refinement pruning the tree representation of the (pointed) initial model resulting in the updated model.

Action models, arrow updates, and refinements

We now quantify over action models, arrow updates, refinements.

$M_s \models \langle \otimes \rangle \varphi$	iff	there is a qu.fr. action model E_e s.t. $M_s \models \langle E_e \rangle \varphi$
$M_s \models \langle \uparrow \rangle \varphi$	iff	there is a qu.fr. arrow update U_o s.t. $M_s \models \langle U_o \rangle \varphi$
$M_s \models \langle \succeq \rangle \varphi$	iff	there is a refinement $M'_{s'}$ of M_s s.t. $M'_{s'} \models \varphi$

In all these logics the quantifiers can be eliminated. Not in APAL. These logics are decidable. Not APAL. The restriction to qu.free formulas is not necessary, because the logics permit synthesis: for all φ there is a (unique!) E_T with $\models \langle \otimes \rangle \varphi \leftrightarrow \langle E_T \rangle \varphi$. Not APAL.

Furthermore: $\langle \otimes \rangle \varphi$ is equivalent to $\langle \uparrow \rangle \varphi$ is equivalent to $\langle \succeq \rangle \varphi$

[Hales. *Arbitrary Action Model Logic and Action Model Synthesis*, 2013]
[vD, van der Hoek, Kooi, Kuijer. *Arrow Update Synthesis*, 2020]
[Bozzelli, vD, French, Hales, Pinchinat. *Refinement Modal Logic*, 2014]

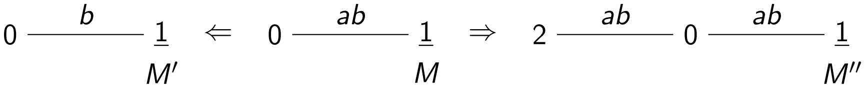
PS Some results are for arbitrary frames ($\mathcal{K}$), not equiv. relations ($\mathcal{S}5$).

Bisimulation, refinement, simulation

To compare the information content of epistemic models binary relations between their domains may satisfy different properties.

Given $M = (S, R, V)$ and $M' = (S', R', V')$, $Z \subseteq S \times S'$, $(s, s') \in Z$:
— **atoms.** $s \in V(p)$ iff $s' \in V'(p)$ for all $p \in P$
— **forth.** if $R_a st$, then there is a $t' \in S'$ such that $R'_a s' t'$ and Ztt'
— **back.** if $R'_a s' t'$, then there is a $t \in S$ such that $R_a st$ and Ztt'

- bisimulation: **atoms**, **forth**, **back** $M_s \simeq M'_{s'}$
- refinement: **atoms**, **back** $M_s \succeq M'_{s'}$
- simulation: **atoms**, **forth** $M_s \preceq M'_{s'}$



M' is a refinement of M and M'' is a simulation of M. Vice versa:
M is a simulation of M' and M is a refinement of M''.
A submodel is a refinement. A supermodel is a simulation. More or less.

Refinement Modal Logic (RML)

$M_s \models \langle\succeq\rangle\varphi$ iff there is a $M'_{s'}$ with $M_s \succeq M'_{s'}$ such that $M'_{s'} \models \varphi$

- refining a model is like pruning a tree
- therefore, validities : $[\succeq]\varphi \to [\succeq][\succeq]\varphi$ (4), $[\succeq]\varphi \to \varphi$ (T), $\langle\succeq\rangle[\succeq]\varphi \to [\succeq]\langle\succeq\rangle\varphi$ (CR), $[\succeq]\langle\succeq\rangle\varphi \to \langle\succeq\rangle[\succeq]\varphi$ (MK)
- action model execution is refinement & v.v. $(\langle\succeq\rangle\varphi \leftrightarrow \langle\otimes\rangle\varphi)$
- refinement is bisimulation quantification plus relativization: $\langle\succeq\rangle\varphi$ is equivalent to $\exists p \varphi^p$
- $\langle\succeq\rangle$ can be eliminated and RML is decidable, crucially: $\langle\succeq\rangle \bigwedge_{a\in A}(\bigwedge_{\varphi_a\in\Phi_a} \Diamond_a\varphi_a \wedge \Box_a\psi_a) \leftrightarrow \bigwedge_{a\in A}\bigwedge_{\varphi_a\in\Phi_a} \Diamond_a\langle\succeq\rangle(\varphi_a \wedge \psi_a)$
- axiomatization for $\mathcal{KD}45, \mathcal{S}5 \dots$: not a conservative extension! e.g., $\langle\succeq\rangle\Box_a\bot$ (remove all arrows) is valid on $\mathcal{K}$ but not on $\mathcal{S}5$.

[Bozzelli, vD, French, Hales, Pinchinat. *Refinement Modal Logic*, 2014]
[Hales. *Quantifying over epistemic updates.* Ph.D. 2016]

Simulation Modal Logic (SML)

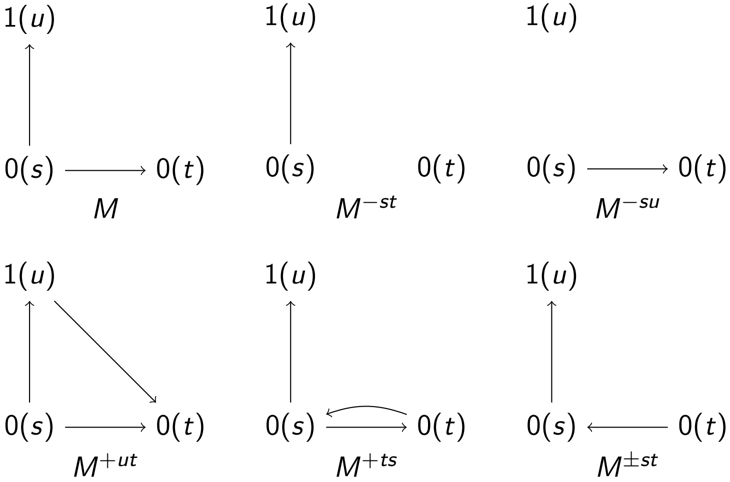
Refinement quantifier (pro memori):
$M_s \models \langle\succeq\rangle\varphi$ iff there is a $M'_{s'}$ with $M_s \succeq M'_{s'}$ such that $M'_{s'} \models \varphi$

Simulation quantifier:
$M_s \models \langle\preceq\rangle\varphi$ iff there is a $M'_{s'}$ with $M_s \preceq M'_{s'}$ such that $M'_{s'} \models \varphi$

- simulating a model is like growing a tree
  we recall that: refining a model is like pruning a tree
  terminology is not ideal, but 'enforced' by the community
- there are more ways to grow trees than to prune trees:
  SML is less straightforward (no reduction) than RML:
  $\langle\preceq\rangle \bigwedge_{a\in A}(\bigwedge_{\varphi_a\in\Phi_a} \Diamond_a\varphi_a \wedge \Box_a\psi_a) \leftrightarrow \bigwedge_{a\in A} \Box_a \bigvee_{\varphi_a\in\Phi_a}\langle\preceq\rangle(\varphi_a \wedge \psi_a)$
  provided all sets Φ_a are consistent
- an open question is simulation epistemic logic

[Xing, Zhu, Zhang. *Covariant-Contravariant Ref. Modal Logic*. 2019]
[Xing. Covar.-Contravar. Refinement Modal μ-calculus. arXiv 2022]
[vD, French, Galimullin, Kuijer. Manuscript involving Simulation ML]

Removing, adding, swapping arrows



Highly expressive, complex, and undecidable.

[van Benthem. *An Essay on Sabotage and Obstruction*. 2005]

[Areces, Fervari, Hoffmann. *Relation-changing modal operators.* 2015]

Anything else?

- iterating announcements instead of quantifying over them
- iterating actions instead of quantifying over them
- factual change (epist. planning) [vB . . . 2009],[vD, Kooi. 2008]
- common knowledge, distributed knowledge
- quantifying over what subgroups know (& distr. knowledge)
  [Åg., Wang][Baltag, Smets][Castañeda. . .][Cachin. . .][dos Santos Gomes]
- propositionally quantified modal logics [Li, Ding. AiML 2024]
- . . . *open questions:* [vD. To Be Announced. Inf.&Comp. 2023]

Literature additions and omissions and resolved open questions are most welcome. The arXiv version will be updated next year.

Thank you!