Games with Different Decision Criteria

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Intro

- Michael Franke
- Johannes Marti
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Outline

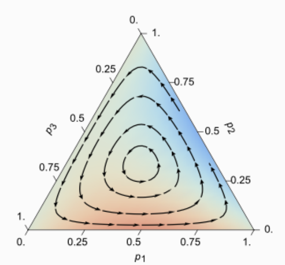
General idea: Games with different decision criteria

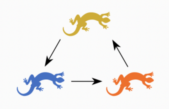
- From an evolutionary point of view
- From a game-theoretic point of view
- From an epistemic point of view

From an evolutionary point of view

- Single-game model
- Evolution of simple behavior

	r	p	s
r	0,0	-1,1	1,-1
p	1,-1	0,0	-1,1
s	-1,1	1,-1	0,0





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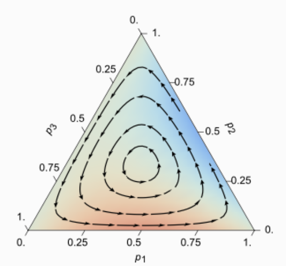

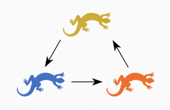

Replicator dynamics [Taylor and Jonker, 1978]:

$$\dot{p}_i = p_i \left(u(a_i, \sigma) - u(\sigma, \sigma) \right)$$

where:

- u utility/payoff/fitness
- a_i action of type i
- σ action mix in the population

From an evolutionary point of view

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Natural environments are so complex, dynamic, and unpredictable that natural selection cannot possibly furnish an animal with an appropriate, specific behavior pattern for every conceivable situation it might encounter. Instead, we should expect animals to have evolved a set of psychological mechanisms which enable them to perform well on average across a range of different circumstances.

[Fawcett et al., 2012]

From an evolutionary point of view

The model:

- The environment: a set $\mathcal{G}$ of symmetric two-player games
- The types: decision criteria, mechanisms that produce behavior in each game

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Example:

- The environment:

PD	I	II
I	2,2	0,3
II	3,0	1,1

SH	I	II
I	3,3	0,2
II	2,0	2,2

- The types: $\mathcal{T} \subseteq \{I, II\}^{|\mathcal{G}|} = \{I, II\} \times \{I, II\}$

From an evolutionary point of view

Decision criteria as types:

Decision criteria:

$$d\colon \text{Utilities} \times \text{Beliefs} \to \text{Actions}$$

where

- $u : Z \to \mathbb{R}$ utility function
- $B \subseteq \Delta(S)$ belief

Decision problem:

$$(S, A, Z, c)$$

where

- S set of states of the world
- A set of actions
- Z set of outcomes
- $c : S \times A \to Z$ outcome function

From an evolutionary point of view

	$1/3$	$2/3$
	0	1
SH	I	II
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A game is a decision problem:

- $S = A_{-i} = \{I, II\}$
- $A_i = \{I, II\}$
- $Z = \{(3,3), (0,2), (2,0), (2,2)\}$
- $c((I,I)) = (3,3)$, $c((I,II)) = (0,2)$, $c((II,I)) = (2,0)$, $c((II,II)) = (2,2)$
- $u_i((3,3)) = 3$, $u_i((0,2)) = 0$, $u_i((2,0)) = 2$, $u_i((2,2)) = 2$
- $B = \{p \in \Delta(S) : 0 \leq p(I) \leq 1/3\}$

From an evolutionary point of view

Decision criteria as types:

Classic decision criteria:

- Expected utility maximization

$$a^* \in \mathrm{argmax}_{a \in A} E_p[u|a]$$

- Maxmin expected utility

$$a^* \in \mathrm{argmax}_{a \in A} \min_{p \in B} E_p[u|a]$$

- Realization-regret minimization

$$a^* \in \mathrm{argmin}_{a \in A} \max_{p \in B} r_R(a, p)$$

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Where

$$E_p[u|a] := \sum_{s \in S} u(c(s, a))p(s)$$

and

$$r_R(a, p) := E_p\left[\max_{a' \in A} u(c(s, a')) - u(c(s, a))\right] = \sum_{s \in S} p(s) \left(\max_{a' \in A} u(c(s, a')) - u(c(s, a))\right)$$

From an evolutionary point of view

Decision criteria as types: example.

	1/3	2/3
	0	1
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- Expected utility maximization (+ principle of insufficient reason):

$$\begin{aligned} E[u|I] &= 3 \cdot 1/6 = 0.5 \\ E[u|II] &= 2 \cdot 1/6 + 2 \cdot 5/6 = 2 \end{aligned}$$

- Maxmin expected utility:

$$\begin{aligned} \min_{p \in B} E_p[u|I] &= \min\{1, 0\} = 0 \\ \min_{p \in B} E_p[u|II] &= \min\{2, 2\} = 2 \end{aligned}$$

From an evolutionary point of view

Decision criteria as types:

Less classic decision criteria:

- Distribution-regret minimization

$$a^* \in \operatorname{argmin}_{a\in A} \max_{p\in B} r_D(a,p)$$

- Altruistic expected utility maximization

$$a^* \in \operatorname{argmax}_{a\in A} E_p[\tilde{u}|a]$$

- Random type: pick an action at random
- Action-l type: $d(u,B) = l$ for all u and B
- ...

From an evolutionary point of view

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Where

$$r_D(a, p) := \max_{a' \in A} E_p\left[u|a'\right] - E_p[u|a] = \max_{a' \in A} \sum_{s \in S} p(s)u(c(s, a')) - \sum_{s \in S} p(s)u(c(s, a))$$

and

$$\tilde{u}(a, a') := u_{\mathbf{1}}(a, a') + u_{\mathbf{2}}(a, a') = u(a, a') + u(a', a)$$

From an evolutionary point of view

What we have done so far:

- Enrich the environment: from single-game to multi-game model

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Not investigated very much in the literature in evolutionary game theory:

- [Zollman, 2008]: multi-game with three games only (Nash bargaining game, ultimatum game, a hybrid of the two), no full-fledged decision criteria

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- Evolution of preferences (i.e., utility functions): preferences as underlying mechanisms, but no multi-game environment (e.g., [Dekel et al., 2007, Robson and Samuelson, 2011])

From an evolutionary point of view

Analytic results:

- **Proposition 1**. Let $\mathcal{G}$ be the class of 2×2 symmetric games $G = (a, b, c, d)$ generated by i.i.d. sampling a, b, c, d from a set of values with at least three elements in the support. Then, distribution-regret minimization strictly dominates maxmin in the resulting multi-game.

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Analytic results:

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- **Corollary 1**. Let $\mathcal{G}$ be as in Proposition 1. The unique evolutionarily stable state in a population of maximinimizers and distribution-regret minimizers is a monomorphic population of regret minimizers.

[Galeazzi and Franke, 2017]

From an evolutionary point of view

Simulation results:

Table 1 Number of times EU strictly dominates all other criteria for all combinations of m (in the rows), n (in the columns), and $\overline{v}$ ($\overline{v} = 50$ in Table 1a, and $\overline{v} = 100$ in Table 1b)

(a)

	2	3	5	7	9	12	15
2	0	0	4	8	7	8	8
3	0	1	10	10	10	10	10
7	0	7	10	10	10	10	10

(b)

	2	3	5	7	9	12	15
2	0	0	5	5	7	6	5
3	0	0	9	10	10	10	10
7	0	10	10	10	10	10	10

- $\{0, \dots, \overline{v}\}$ the set of randomly drawn fitness values
- n the number of actions in the games
- m the number of points in the beief set B

[Galeazzi and Galeazzi, 2020]

From a game-theoretic point of view

- Gap in the literature: games with homogeneous vs heterogeneous criteria
- Homogeneous criteria:
  - [Lo, 1996]: maxmin expected utility
  - [Klibanoff, 1996]: maxmin expected utility
  - [Marinacci, 2000]: Choquet expected utility
  - [Kajii and Ui, 2005]: maxmin expected utility
  - [Renou and Schlag, 2010]: realization-regret minimization
  - [Halpern and Pass, 2012]: realization-regret minimization
  - ...

From a game-theoretic point of view

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- Heterogeneous criteria:
    - [Epstein, 1997]: introduces the concepts of rationalizability, (iterated) dominance and equilibrium for general preferences (i.e., utility functions) on acts. Based on [Epstein and Wang, 1996], more on this later.

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- Heterogeneous criteria:
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  - Team reasoning: [Bacharach, 1999, Lecouteux, 2018]

From a game-theoretic point of view

Two-player Bertrand competition. Each firm i produces the same commodity at a cost c_i, and sell it at a price of p_i. The firm that chooses the lowest price captures the whole market, pocketing a profit of $p_i - c_i$, or $(p_i - c_i)/2$ in the case of a tie. Hence:

$$u_1(c_1, c_2, p_1, p_2) = \begin{cases} p_1 - c_1 & p_1 < p_2 \\ (p_1 - c_1)/2 & p_1 = p_2 \\ 0 & p_1 > p_2 \end{cases}$$

where c_1, c_2 are the types/private information of the players and p_1, p_2 are the actions of the players, with $c_i \in C_i := \{0, 0.1, ..., 1\}$ and $p_i \in P_i := \{0, 0.1, ...\}$.

From a game-theoretic point of view

Suppose firm 1 is regret minimizer and firm 2 is maxmin, and $B_i|c_i = \Delta(C_{-i})$ for all c_i.

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Equilibrium. An equilibrium is then a pair of strategies (σ_1, σ_2) with $\sigma_i : C_i \to P_i$ such that:

1. $\forall c_1 \in C_1$, $\sigma_1(c_1) \in \arg\min_{p_1} \max_{q \in B_1|c_1} r_R(p_1, c_1, q)$
2. $\forall c_2 \in C_2$, $\sigma_2(c_2) \in \arg\max_{p_2} \min_{q \in B_2|c_2} E_q[u_2|p_2, c_2]$

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Where

$$E_q[u_2|p_2, c_2] := \sum_{c_1 \in C_1} q(c_1) u_2(c_1, c_2, \sigma_1(c_1), p_2))$$

and

$$r_R(p_1, c_1, q) := E_q\left[\max_{p_1'} u_1(c_1, c_2, p_1', \sigma(c_2)) - u_1(c_1, c_2, p_1, \sigma(c_2))\right]$$

$$= \sum_{c_2 \in C_2} q(c_2)\left(\max_{p_1'} u_1(c_1, c_2, p_1', \sigma(c_2)) - u_1(c_1, c_2, p_1, \sigma(c_2))\right)$$

From a game-theoretic point of view

The pair of strategies

$$\sigma_R(c_1) = \begin{cases} \frac{1.1+c_1}{2} & c_1 \text{ odd} \\ \frac{1.1+c_1}{2} - 0.05 & c_1 \text{ even} \end{cases}$$

and

$$\sigma_M(c_2) = \begin{cases} 0.4 & c_2 < 0.4 \\ 0.5 & c_2 = 0.4 \\ c_2 + 0.1 & 0.4 < c_2 < 1 \\ 1 & c_2 = 1 \end{cases}$$

constitute an equilibrium of the game.

- In equilibrium, firm 1 or firm 2 gets positive profit
- The same pricing strategies do not constitute a single-criterion equilibrium
  - (σ_R, σ_R) is a regret-equilibrium, but (σ_M, σ_M) is not a maxmin-equilibrium

From an epistemic point of view

- [Epstein and Wang, 1996]
- [Di Tillio, 2008]
- [Bjorndahl et al., 2017]

From an epistemic point of view

[Epstein and Wang, 1996, Di Tillio, 2008]: Inconsistency between probabilistic type spaces and Savage approach.

Savage approach (single-agent decision problem):

- Primitives: states of the world S, outcomes Z, acts $f \in Z^S$, preferences over acts

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Savage approach (single-agent decision problem):

- Primitives: states of the world S, outcomes Z, acts $f \in Z^S$, preferences over acts
- Utility and probabilistic beliefs are derived from preferences

From an epistemic point of view

Type spaces (interactive decision problem):

- Primitives: states of nature A_{-i} (i.e., parameters of the game and actions of the opponents), player i's types T_i, player i's belief function $\beta_i : T_i \to \Delta(A_{-i} \times T_{-i})$

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- Type spaces generate hierarchies of interactive probabilistic beliefs, e.g.

$$t_i^I \mapsto (1/3 t_j^I, 2/3 t_j^{II}) \qquad t_j^I \mapsto (1 t_i^I)$$
$$t_i^{II} \mapsto (3/4 t_j^I, 1/4 t_j^{II}) \qquad t_j^{II} \mapsto (2/5 t_i^I, 3/5 t_i^{II})$$

$$(t_i^I, t_j^I) \models B_i^{1/3} I_j \qquad (t_i^I, t_j^I) \models B_i^{2/3}(II_j \wedge B_j^{3/5} II_i)$$

SH	I	II
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	t_j^I	t_j^{II}
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- Probabilistic beliefs are taken as primitive and not derived from behavior, i.e., preferences

From an epistemic point of view

[Epstein and Wang, 1996] and [Di Tillio, 2008] preference structures:

- Quite similar, technical differences not relevant for our purposes here
- Savage approach into type spaces

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- Primitives: states of nature A_{-i} (i.e., parameters of the game and actions of the opponent), player i's types T_i, player i's "preference" function $\theta_i : T_i \to \Pi(A_{-i} \times T_{-i})$

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- Π vs Δ: $\Pi(X) = \mathcal{P}(F(X))$
  - $F(X) := Z^X$ is the set of acts over X for given outcome set Z

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  - $\mathcal{P}(F(X))$ is the set of all preference relations over acts over X

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- Preference structures generate hierarchies of interactive preference relations (see next slide)

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- $S = A_j = \{I, II\}$
- $Z = \{(3,3),(0,2),(2,0),(2,2)\}$

Hierarchies of preference relations:

1. $\pi_1 \in \Pi(A_j)$, i.e., preference relation over acts Z^{A_j}
2. $\pi_2 \in \Pi(A_j \times \Pi(A_i))$, i.e., preference relation over acts $Z^{A_j \times \Pi(A_i)}$
3. ...

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3. ...

Remark. But where are all those acts? E.g., the act $f(x) = (2,2)$ for all $x \in A_j$

From an epistemic point of view

- Preference relations are transitive, regret is not

	s	s'	s''
I	1	0	2
II	2	1	0
III	0	2	1

Table: Regret is not transitive

$$I \succ II \quad II \succ III \quad III \succ I$$

- Choice structues: from hierarchies of interactive preference relations to hierarchies of interactive choice functions ($C : 2^X \to 2^X$ s.t. $C(Y) \subseteq Y$ for all $Y \subseteq X$)

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I	1	0	2
II	2	1	0
III	0	2	1

Table: Regret is not transitive

$$I \succ II \quad II \succ III \quad III \succ I$$

- Choice structues: from hierarchies of interactive preference relations to hierarchies of interactive choice functions ($C : 2^X \to 2^X$ s.t. $C(Y) \subseteq Y$ for all $Y \subseteq X$)
- Primitives: states of nature A_{-i} (i.e., parameters of the game and actions of the opponent), player i's types T_i, player i's "choice" function $\theta_i : T_i \to \Gamma(A_{-i} \times T_{-i})$

From an epistemic point of view

- Preference relations are transitive, regret is not

	s	s'	s''
I	1	0	2
II	2	1	0
III	0	2	1

Table: Regret is not transitive

$$I \succ II \quad II \succ III \quad III \succ I$$

- Choice structues: from hierarchies of interactive preference relations to hierarchies of interactive choice functions ($C : 2^X \to 2^X$ s.t. $C(Y) \subseteq Y$ for all $Y \subseteq X$)
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- $\Gamma(X) := \mathcal{C}(F(X))$
  - $F(X) := Z^X$ is the set of acts over X for given outcome set Z
  - $\mathcal{C}(F(X))$ is the set of all choice functions over acts over X

From an epistemic point of view

Results:

Theorem 1. $\Omega_i \simeq \Gamma(A_j \times \Omega_j)$ and $\Omega_j \simeq \Gamma(A_i \times \Omega_i)$.

Theorem 2. For every choice structure $\mathcal{X}$ there is a unique morphism of choice structures $v : \mathcal{X} \to \mathcal{U}$ from $\mathcal{X}$ to the universal choice structure $\mathcal{U}$.

- which is a complicated way to say that there exists a universal choice structure.

From an epistemic point of view

Results:

Theorem 1. $\Omega_i \simeq \Gamma(A_j \times \Omega_j)$ and $\Omega_j \simeq \Gamma(A_i \times \Omega_i)$.

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Theorem 3. Assume that A_i and A_j are finite. Then the uncertainty spaces Ω_i' and Ω_j' of all preference hierarchies are isomorphic to two subspaces of the spaces of all choice hierarchies Ω_i and Ω_j respectively.

- which is a complicated way to say that choice structures embed preference structures.

Conclusion

Thanks for your attention.

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