Free Choice With Arbitrary Objects

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Free Choice in natural language

- 'Free choice' *any*

(1) Any owl hunts mice.
$\rightsquigarrow$ *For all* x, if x is an owl, then x hunts mice.
(Kadmon & Landman 1993)

(2) I can catch any raven.
$\rightsquigarrow$ *For all* x, if x is a raven, then I can catch x.
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- 'Free choice' *or*

(3) I would dance with Mary or Sue.
$\rightsquigarrow$ I would dance with Mary *and* I would dance with Sue.
(Kadmon & Landman 1993)

(4) Alfonso or Claribel {is/would be} a good choice for chair.
$\rightsquigarrow$ Alfonso {is/would be} a good choice *and* Claribel {is/would be} a good choice.

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- *wh*-ever free relatives

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- Free choice effects with deontic modals (Kamp 1974):

(6) You may borrow any of my toys.
$\rightsquigarrow$ *For all* x, if x is one of my toys, you may borrow x.

(7) You may drink the whiskey or the gin.
$\rightsquigarrow$ You may have the whiskey *and* you may have the gin.

(8) You can't eat soup or salad.
$\rightsquigarrow$ You can't have soup *and* you can't have salad.

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- Cherchia's (2013) **Identity Thesis**: "free choice (FC) effects constitute a unitary phenomenon empirically, and call for a uniform explanation conceptually" (p. 50).
- A methodological implication of the **Identity Thesis** is that we should be suspicious of accounts of FC that seem tailor-made for a particular item or fail to generalize across different environments.

Proposal in brief

‘Any a’ denotes only one a, but it is wholly irrelevant which it denotes, and what is said will be equally true whichever it may be. Moreover, ‘any a’ denotes a variable a, that is, whatever particular a we may fasten upon, it is certain that ‘any a’ does not denote that one; and yet of that one any proposition is true which is true of any a.

(Russell in *Principles of Mathematics*, §60)

Proposal in brief

- FC uses of *any* and *or*, as well as *wh*-ever free relatives, denote *arbitrary objects* (entities, actions, times, etc.) that range over individual objects (entities, actions, times, etc.) and instantiate a property only when it is common to these *values* in their range.

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- *Any owl* in (1) denotes the arbitrary owl, owl*, which has a property only when it is instantiated by all the owls. So we may predicate of owl* the property of having binocular vision, of being zygodactyl, and, of course, the property of being an owl. However, owl* is not nocturnal since not all owl species hunt at night.

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- *Any owl* in (1) denotes the arbitrary owl, owl*, which has a property only when it is instantiated by all the owls. So we may predicate of owl* the property of having binocular vision, of being zygodactyl, and, of course, the property of being an owl. However, owl* is not nocturnal since not all owl species hunt at night.
- By deploying an arbitrary object, a speaker deemphasizes the individual identities of the values in its range and brings the common properties of these values to the foreground. We may think of owl* as functioning, so to speak, as a peg on which to hang properties common to all owls.

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- When a speaker predicates a new property of an arbitrary object, such as predicating of owl* the property of hunting mice in (1), this has the effect of adding the new property to the peg. Since arbitrary objects have properties common to their values, this implies that for all x, if x is an owl, then x hunts mice.

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- Crucially, however, the speaker is *not* saying of each individual owl that it hunts mice. Rather, the speaker conveys that all owls hunt mice by effectively setting up a *dependency* between the property of being an owl and the new property of hunting mice.

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- Crucially, however, the speaker is *not* saying of each individual owl that it hunts mice. Rather, the speaker conveys that all owls hunt mice by effectively setting up a *dependency* between the property of being an owl and the new property of hunting mice.
- This 'property-based' verification not only accounts for their universal or conjunctive force, but also explains other distinctive features of free choice items, such as why sentences like (1) have a non-accidental 'law-like' flavor (cf. Hale 2020 on "instantial" versus "generic" truthmakers, and Linnebo 2022 on "instance-based" versus "generic" explanations of universal generalizations).

Reference to arbitrary objects

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- While exactly analogous statements sound awkward in colloquial speech, similar instructions are common in general conditional proof (Barwise & Etchemendy 1999):

  ?Let Hedwig be an arbitrary owl.

  *Let's use the name 'Hedwig' to stand for {any/*every} owl.*

FC *any*: theoretical and empirical issues

- We would like an account of the universal force of FC *any*, especially since polarity sensitive *any* is a narrow-scope existential indefinite.

(11) a. I can catch any raven. (FC *any*)
b. I didn't see any pigs. (PS *any*) (Horn 2000)

(12) I wonder if Susan married anybody. (Fauconnier 1979)

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- Furthermore, FC *any* statements are 'law-like', supporting counterfactual inferences (Ryle 1949; Vendler 1962):

(14) Any owl hunts mice. So, if Tweety were an owl, Tweety would hunt mice.

FC *any*: theoretical and empirical issues

- Another law-like feature is that *any* statements, like non-episodic generalizations, do not carry existential commitments:

(15) She may never marry, but {anyone/*everyone/*the person} she does marry will be Jewish. (Horn 2000)

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- As emphasized by Kadmon & Landman (1990, 1993), *any* generally ranges over a broader domain than ordinary universal determiners:

(16) Context: We have just sat down at a family-style restaurant where everybody shares, but haven't looked over the menu.

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- In this talk, I focus on the meaning rather than distribution facts.

(17) Romeo danced with {?any woman./any woman who was receptive to his advances.}
('subtrigging'; LeGrand 1975; Dayal 1998)

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(19) A mosquito carries West Nile virus.

Difficulties with K&L's generic analysis

- Generics allow exceptions, however this is not evident for *any*:

(20) A: A large dog gives live birth.
B: What? ANY dog gives live birth.

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- *An* does not have generic readings in subtrigged cases (Dayal 1998):

(24) {Any/A} person who saw the fly in the food went hungry.

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- Within English itself, the existential-universal flip-flop with *any* also finds a parallel in a disjunctive-conjunctive flip-flop with *or* (Horn 1972; Kadmon & Landman 1993), however we presumably do not want to claim that *or* is lexically ambiguous.

Indefinites: variable reference to a thing

Menu semantics

- The formal treatment of FC will be implemented by grafting a version of arbitrary object theory (Fine 1983, 1985a,b; Horsten 2019) onto a compositional version of truthmaker semantics (Bledin 2024, drawing on Fine 2017a,b,c, Champollion & Bernard 2022, a.o.).

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- When $x \leqslant y$, I say that x **is a part of** y, or that y **contains** x.
- A state space contains a set $W \subseteq S$ of **world states**, where no world state is part of any other. A state is a **possible state** just when it is part of some world state.

Indefinite DPs: individual disjunctions

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(29) **Bind** ($\gg=$) $:= \lambda X \lambda f . \bigcup_{x \in X} f(x)$ :: Ma → (a → Mb) → Mb

(30)
$$
\begin{aligned}
& [\![[[[\text{Ag}]\textit{Hedwig or Archimedes} \text{ hooted}]\!] \\
= \;& ([\![\textit{Hedwig or Archimedes}]\!] \gg= [\![[\text{Ag}]]\!]) \cap [\![\textit{hoot}]\!] \\
= \;& (\textstyle\bigcup_{e \in \{\text{Hedwig},\text{Archimedes}\}} [\![[\text{Ag}]]\!](e)) \cap [\![\textit{hoot}]\!] \\
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Indefinite DPs: PS *any*

- I offer a basic treatment of *any* as an indefinite (Kadmon & Landman 1993; Horn 2000; among others):

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- Nor will I try to explain the NPI licensing behavior of PS *any*.

Free choice: reference to a variable thing

Introducing arbitrary objects

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(36) Any apple or any pear costs a dollar.

Conditions on the entity space

- To ensure that we have all the arbitrary objects we need, I impose two *existence* conditions (first is based on Fine 1985a, Chapter 3):

(37) For any function f from each $s \in S$ to a set $X \subseteq E_O$ of ordinary entities, there is an arbitrary entity $a \in E_A$ with $V(a) = f$.

(38) For any function f from each $s \in S$ to a *finite* set $X \subseteq E$ of entities, there is an arbitrary entity $a \in E_A$ with $V(a) = f$.

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- This ensures that the valuation V is invertible.

The FC operator

(40) **Free choice operator**

$$[\![\mathsf{FC}]\!] \quad = \quad \lambda f . V^{-1}(f) \quad :: \quad (\mathsf{s} \to \mathsf{Me}) \to \mathsf{e}$$

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(41) **FC *any***

$$\begin{aligned} & [\![\text{FC } any_{s^*}(\alpha)]\!] \\ = \quad & [\![\text{FC}]\!]([\![\lambda s^*.any_{s^*}(\alpha)]\!]) \\ = \quad & V^{-1}(\lambda s^*.\{e : \exists s \leqslant s^*(\text{Possessor}(s) = e \wedge s \in [\![\alpha]\!])\}) \ :: \ \text{e} \end{aligned}$$

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$$\begin{aligned} & [\![\text{FC } any_{s^*}(owl)]\!] \\ = \quad & V^{-1}(\lambda s^*.\{e : \exists s \leqslant s^*(\text{Possessor}(s) = e \wedge s \in [\![owl]\!])\}) \\ & (\text{the arbitrary owl}^*) \ :: \ \text{e} \end{aligned}$$

Arbitrary attribution, operationalized version

(43) **Principle of Arbitrary Attribution:** If an arbitrary object instantiates a property, then this property is common to all the values in its range.

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- One approach is to operationalize this principle:

(44) Where $X_1, X_2, ...$ are Ma-type sets,

$$\bigsqcup\{X_1, X_2, ...\} \quad := \quad \{x_1 \sqcup x_2 \sqcup ... : x_1 \in X_1, x_2 \in X_2, ...\}$$

(45) **Individuation operator**

$$\mathsf{I} \quad := \quad \lambda f \lambda e. \bigsqcup_{e' \in V(e)(w^*)} f(e') \quad :: \quad (\mathrm{e} \to \mathrm{Ms}) \to (\mathrm{e} \to \mathrm{Ms})$$

(46) **Arbitrary Attribution (operationalized version)**

To compose an expression denoting an arbitrary object with a verbal or other predicative projection, the arbitrary object-denoting expression must first be raised to a position above the projection, allowing this projection to combine with the individuation operator I before application.

Arbitrary attribution, modal version

- My own preference, however, is to let the truthmakers for FC *any* statements openly display their arbitrariness by allowing truthmakers involving arbitrary items:

(47)
$$\begin{aligned} & [\![[[\text{Pos}]\text{FC } \textit{any}_{s^*}(\textit{owl})] \textit{ hunts mice}]\!] \\ = \quad & \{s : \text{Possessor}(s) = \text{owl}^* \wedge \text{hunts-mice}(s)\} \quad :: \quad \text{Ms} \end{aligned}$$

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Given state s involving arbitrary object e_A, an individuation of s is a state $s_{e_A \rightarrow e'}$ obtained by substituting the value e' for e_A.

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Given state s involving arbitrary object e_A and world $w \in W$, if $s \leqslant W$ then there is some $s_{e_A \to e'} \leqslant W$ for each $e' \in V(e_A)(w)$.

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- A state of the arbitrary owl possessing the property of hunting mice should bring with it, as it were, states of each of the ordinary owls (in the world of evaluation) possessing this property.

Explaining the meaning facts

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- The account validates counterfactual entailments:

(50) Any match that I strike lights. So if I had struck this match instead of you, it would have lit.

Relative to an alternative circumstance s where the speaker struck the relevant match m, the arbitrary speaker-struck match would range over a set that includes m, so in combination with a state of this arbitrary match possessing the property of lighting, the Principle of Arbitrary Attribution would ensure a state of m lighting.

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- The value range of an arbitrary object may be empty relative to the actual world state, so FC *any* statements remain import-free.

Narrow FC permission with *any*

(51) You may borrow any of my toys.

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- Let $s_{\Diamond\{t:\text{Agent}(t)=X\wedge\varphi(t)\}}$ be a truthmaker for $\ulcorner X$ *may/can/might* $\varphi\urcorner$. I assume this involves the proposition $\{t : \text{Agent}(t) = X \wedge \varphi(t)\}$ in some way, but I would like to remain noncommittal about this.

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- A truthmaker for (51) involves the arbitrary toy of the speaker, toy*:

$s_{\Diamond\{t:\text{Agent}(t)=\text{Hearer}\wedge\text{borrow}(t)\wedge\text{Theme}(s)=\text{toy}^*\}}$

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- According to the Principle of Arbitrary Attribution, such a state obtains in a world only if its individuated states also obtain:

  $s_{\Diamond\{t:\text{Agent}(t)=\text{Hearer}\wedge\text{borrow}(t)\wedge\text{Theme}(s)=\text{Barbie}\}}$
  $s_{\Diamond\{t:\text{Agent}(t)=\text{Hearer}\wedge\text{borrow}(t)\wedge\text{Theme}(s)=\text{Power Ranger}\}}$ ...

Narrow FC permission with *or*

(52) You may drink the whiskey or the gin.

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(53) **FC *or*:** $[\![\text{FC } \alpha \text{ or } \beta]\!]$

$$= [\![\text{FC}]\!]([\![\lambda s^*.\alpha \text{ or } \beta]\!])$$

$$= V^{-1}(\lambda s^*.[\![\alpha]\!] \cup [\![\beta]\!]) \quad :: \quad \text{e}$$

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- A truthmaker for (52) involves the arbitrary liquor from among the whiskey and the gin, liquor*:

$s_{\Diamond\{t:\text{Agent}(t)=\text{Hearer}\wedge\text{drink}(t)\wedge\text{Theme}(s)=\text{liquor}^*\}}$

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- This state brings with it the following individuated states:

  $s_{\Diamond\{t:\text{Agent}(t)=\text{Hearer} \wedge \text{drink}(t) \wedge \text{Theme}(s)=\text{the whiskey}\}}$
  $s_{\Diamond\{t:\text{Agent}(t)=\text{Hearer} \wedge \text{drink}(t) \wedge \text{Theme}(s)=\text{the gin}\}}$

Dual Prohibition

(55) You can't eat soup or salad.

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- Applying the theory of negative states in Bledin (2024):

$\neg S_{\Diamond}\{t{:}\text{Agent}(t){=}\text{Hearer}\wedge\text{eat}(t)\wedge\text{Theme}(s){=}\text{soup-or-salad}^{*}\}$

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- Applying the Principle of Arbitrary Attribution:

$\neg S_{\Diamond}\{t\text{:Agent}(t)\text{=Hearer}\wedge\text{eat}(t)\wedge\text{Theme}(s)\text{=soup}\}$

$\neg S_{\Diamond}\{t\text{:Agent}(t)\text{=Hearer}\wedge\text{eat}(t)\wedge\text{Theme}(s)\text{=salad}\}$

Wide Scope FC

(56) Mrs. X might live in Victoria or she might live in Brixton.

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- It is less clear to me at the moment that arbitrary objects help with wide-scope free choice. We might try appealing to arbitrary modal states, but it might also be that the full range of free choice phenomena must be explained by an array of different mechanisms.

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