Exclusive Disjunction in BSML

Hurford Disjunctions as evidence for split $\vee$-support and split $\wedge$-rejection

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Acknowledgments

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Overview

Introduction

Let’s start from puzzling data!

Hurford Disjunctions are disjunctions in which one of the disjuncts entails the other, as in the following classical example:

(1) # Pierre lives in Paris or France.

Hurford Disjunctions have been a crucial case of study to understand the role of disjunctions and logical triviality/redundancy in natural language (Hurford, 1974; Singh, 2008; Schlenker, 2009; Anvari, 2018; a.o.)

A new case of Hurford Disjunction

Consider now the following example:

(2) # Pierre lives in Paris and he's married or he lives in France and he's single.

(Amir Anvari, p.c.)

- We cannot strictly talk about Hurford disjunction, since **no disjunct entails the other**.

In fact, *Paris* $\wedge$ *married* does not entail *France* $\wedge$ *single*, and viceversa.

- However, the sentence reminds us of the ungrammaticality of Hurford disjunctions, so I dub sentences like (2) **Disjoint Hurford Disjunctions (DHDs)**.

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'Short-term' desiderata

Two objectives:

- Find a way to capture Standard Hurford Disjunctions (SHD) and Disjoint Hurford Disjunctions (DHD) **via the same mechanism**;
- Reshaping the Hurford Disjunction Puzzle: **deriving ungrammaticality by the semantics itself**, without assuming any additional constraint (such as 'Hurford' constraint).

The solution

We can show that all ungrammaticalities can be predicted by the following two assumptions:

- Natural language semantics is modelled by BSML and neglect-$\emptyset$;
- Disjunction in natural language is intrinsically exclusive.

The patterns captured by these assumptions might indeed provide further evidence for Bilateral State-based Modal Logic and its definitions of

- split disjunction support
- split conjunction rejection

The core features

Our objectives:

- disjuncts have to be 'live *independent* possibilities';
- conjuncts need to be individually seen by elements outside the disjunction.

(2) $(\alpha \wedge \beta) \vee (\gamma \wedge \delta)$

As a result...

Provide further evidence for:

- neglect-zero effects
- split definitions

The core features

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As a result...

Provide further evidence for:

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‘Long-term’ desiderata

The solution will hopefully give us...

A notion of disjunction that is a good predictor for standard assertability conditions of natural language disjunction:

- Independence/ignorance of disjuncts;
- Uniqueness interpretation of multiple disjuncts;
- Disjunctive reasoning fallacies.

Exclusive Disjunction in BSML

Why BSML?

Bilateral systems, team logics and neglect-zero effects have recently been proven to be useful in accounting for a number of different linguistic phenomena:

- free choice (Aloni, 2022);
- epistemic modals (Hawke & Steinert-Threlkeld, 2020; Aloni, 2023);
- homogeneity (Sbardolini, 2023);
- connectives (Carcassi & Sbardolini, 2022; Sbardolini, 2023);
- numerals (Aloni & van Ormondt, 2023);
- indefinites (Aloni & Degano, 2022; Cao, 2023);
- conditionals (Flachs, 2023)
- a.o.;
- and possibly many others!

The ‘meta-logical’ argument

When non-classical logic are introduced to modal something in our head (our linguistics competence or reasoning ability), the following concern might arise:

The ‘meta-logical’ concern:

- If this logic is in our head, how come logicians took so long to find it and defined classical logic first?

There are two possible answers to this question.

The ‘meta-logical’ argument

- **First possible answer**:
  Well, logicians found classical logic first because that is the part of language that is observable. (Amir Anvari, p.c.)

- **Second possible answer**: That is because classical (modal) logic is exactly the logic people use in logico-mathematical reasoning: $\text{BSML}^{\emptyset}$

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The second answer

When there are mismatches between mathematical reasoning and natural language:

- the first answer needs additional assumptions;
- while the second answer gets them for free (once the can be connected to neglect zero effects

(4) Is 4 smaller or equal to 5?↑

Engineers and math people: Yes.
'Common' people: ?/No.

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Bilateral State-based Modal Logic

BSML (henceforth 'BeShaMeL') from Aloni (2022):

$M, s \vDash p$ iff $\forall w \in s : V(w, p) = 1$

$M, s \dashv p$ iff $\forall w \in s : V(w, p) = 0$

$M, s \vDash \neg\phi$ iff $M, s \dashv \phi$

$M, s \dashv \neg\phi$ iff $M, s \vDash \phi$

$M, s \vDash \phi \vee \psi$ iff $\exists t, t' : t \cup t' = s \;\&\; M, t \vDash \phi \;\&\; M, t' \vDash \psi$

$M, s \dashv \phi \vee \psi$ iff $M, s \dashv \phi \;\&\; M, s \dashv \psi$

$M, s \vDash \phi \wedge \psi$ iff $M, s \vDash \phi \;\&\; M, s \vDash \psi$

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$M, s \vDash \Diamond\phi$ iff $\forall w \in s : \exists t \subseteq R[w] : t \neq \emptyset \;\&\; M, s \vDash \phi$

$M, s \dashv \Diamond\phi$ iff $\forall w \in s : M, R[w] \dashv \phi$

$M, s \vDash \text{NE}$ iff $s \neq \emptyset$

$M, s \dashv \text{NE}$ iff $s = \emptyset$

Bilateral State-based Modal Logic

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$$M, s \vDash p \text{ iff } \forall w \in s : V(w,p) = 1$$
$$M, s \dashv p \text{ iff } \forall w \in s : V(w,p) = 0$$
$$M, s \vDash \neg\phi \text{ iff } M, s \dashv \phi$$
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Exclusive disjunction

Let's extend BSML with an exclusive disjunction ∇:

$$M, s \vDash \phi \nabla \psi \text{ iff } M, s \vDash \phi \vee \psi \ \& \ M, s \vDash \neg(\phi \wedge \psi)$$

This amounts to the following:

$M, s \vDash \phi \nabla \psi$ iff

1. $\exists t, t' : t \cup t' = s \ \& \ M, t \vDash \phi \ \& \ M, t' \vDash \psi$
2. $\exists r, r' : r \cup r' = s \ \& \ M, r \dashv \phi \ \& \ M, r' \dashv \psi$

If we neglect $\emptyset$, t has to be the same as r' and t' the same as r, this leads to the following compact and intuitive definition:

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Standard vs exclusive disjunction

From Aloni (2022):

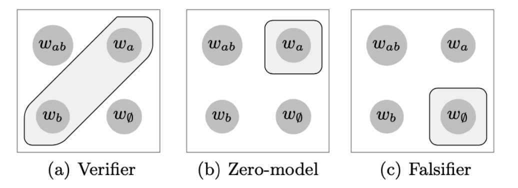

Figure 1: Models for both $(a \vee b)$ and $(a \triangledown b)$

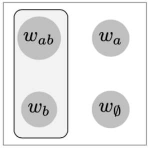

Figure 2: Model for $(a \vee b)$, but not for $(a \triangledown b)$

Exclusivity as presupposition

The negation of the previous definition gives us the following bizarre meaning:

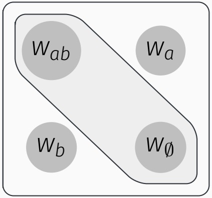

Figure 3: Model for $M, s \dashv (a \wedge \neg b) \vee (b \wedge \neg a)$

However, this is hardly what happens to disjunction after negation.

(5) I didn't see John or Mary.
$\rightsquigarrow$ I didn't see any of the two.

Exclusive presupposition

Indeed, it seems that the conjunctive possibility is not really there, in other words:

- **exclusivity** $(\neg(a \wedge b))$ **'projects' under negation**.

$M, s \vDash \phi \nabla \psi$ iff $M, s \vDash (\phi \vee \psi)_{[\neg(\phi \wedge \psi)]}$

See Bassi et al. (2021) for a similar proposal.

(6) I didn't see John OR_F Mary. I saw both!

(7) He doesn't KNOW_F that the earth is flat, he merely believes it.

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BSML* or BSML$^+$

There are various ways to implement **neglect-zero effects** in BSML.

Two possible ways for ED:

- Global ban on zero-models through BSML*;
- Local pragmatic enrichment (obligatory) through BSML$^+$.

- If we define exclusivity as an *at issue* content of ED, then we need to go for BSML*;
- but if we treat it as a presupposition, we can also go for the pragmatic enrichment version BSML$^+$.

TODAY: it doesn't matter.

A viable alternative?

BSML allows for various possible definitions and it seems we don't have to get as extreme as making disjunction exclusive:

$M, s \vDash \phi \vee \psi \ \& \ M, s \nvDash \phi \ \& \ M, s \nvDash \psi$

While this might work for atomic formulas, it won't give us the desired prediction for complex formulas:

- Assume one of the disjuncts to be $a \wedge b$:

$M, s \nvDash (a \wedge b)$ $\qquad\qquad$ $M, s \dashv (a \wedge b)$

We will get back to this at the end, stay tuned!

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Neglect-zero and split definitions in Hurford Disjunctions

Resolving SHDs: (independent) Split Disjunction Support

Consider the following classical case of Hurford disjunction:

(8) # Pierre lives in Paris or France.

Traditional frameworks need to stipulate an *ad hoc* ban on disjuncts related by entailment (*Hurford constraint*).

However, Standard Hurford Disjunctions (henceforth SHDs) are predicted to be bad directly from the BSML exclusive split disjunction we just defined, **if zero-models are neglected**.

Resolving SHDs: (independent) Split Disjunction Support

(9) # Pierre lives in Paris or France.

In fact, for M, s to support (19) we need to find two substates t, t' such that

> t supports *Pierre lives in Paris but not France*
>
> t' supports *Pierre lives in France but not Paris*.

However, the only state that could support *Pierre lives in Paris but not France* is **the empty state** $\emptyset$, since it is a contradiction. But pragmatic enrichment forbids support through the empty state.

Resolving SHDs: (independent) Split Disjunction Support

Note:

- This makes (9) ungrammatical on a semantic ground (plus pragmatic enrichment) with no need for additional constraints that explicitly forbid entailment between disjuncts;
- in classical logic, even assuming disjunction to be exclusive, (9) would just mean *Pierre lives in France but not Paris.*

Resolving DHDs: Split Conjunction Rejection

Consider now the following sentence (Anvari, p.c.):

(10) #Either Pierre lives in Paris and he's married or he lives in France and he's single.

As said before, *Paris* $\wedge$ *married* does not entail *France* $\wedge$ *single*, and viceversa.

No analysis of Hurford disjunctions is capable of explaining this fact (Hurford, 1974; Singh, 2008; Schlenker, 2009; Anvari, 2018; a.o.).

(11) #Either Pierre lives in France and he's single, or, if he's married, he lives in Paris.

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Interestingly, the ungrammaticality of (12) can also be directly predicted by ∇ in BSML, if the sentence is pragmatically enriched.

(12) # Either Pierre lives in Paris and he's married or he lives in France and he's single.

If we interpret disjunction as a BSML exclusive disjunction, then:

- we need to find a state t that supports *Paris and married* and rejects *France and single*;
- this means that we need to find a state $r \subseteq t$ that supports *Paris and married* and rejects *France*;
- but this is a contradiction, as every state supporting *Paris* also supports *France*.

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Resolving DHDs

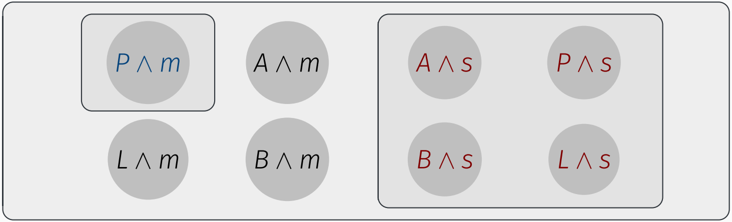
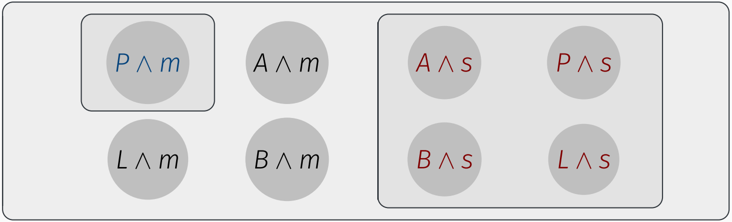


Figure 4: $M, t \vDash$ (*Paris and married*) and $M, t' \vDash$ (*France and single*)

$M, s \Dashv (Paris \wedge married) \wedge (France \wedge single)$
$\exists t, t'$ s.t. $t \cup t' = s$:

- $M, t \Dashv Paris \wedge married$
  $\exists u, u'$ s.t. $u \cup u' = t$:
  - $M, u \Dashv Paris$
  - $M, u' \Dashv married$
- $M, t' \Dashv France \wedge single$
  $\exists r, r'$ s.t. $r \cup r' = t'$:
  - $M, r \Dashv France$
  - $M, r' \Dashv single$

Asymmetric cases



We cannot find a non-empty substate r of the state t supporting *Paris* that rejects *France*. This explains also the following pattern with ‘asymmetric’ disjuncts:

(13) # Either Pierre lives in Paris or he lives in France and he’s single.

(14) Either Pierre is married or he lives in France and he’s single.

Short-term desiderata accomplished

Hurford Disjuntions:

- Exclusive disjunction in BSML predicts both SHDs and DHDs to be bad on semantic grounds and ungrammaticality is captured by the same mechanism.
- Thanks to split conjunction rejection we were able to sort out a specific conjunct and reduce the complex disjoint case of Hurford Disjunction to the standard case.

Hurford Disjunctions as contradictions?

Under this analysis HDs are **contradictions** (that lead to presupposition failure).

HDs feel different from classic L-triviality examples (von Fintel, 1993; Gajewski, 2009; Del Pinal, 2018; 2021; a.o.)

(15) a. *There is every curious student.
b. *Some student but Mary smoke.

However, HDs are also very different from other contradictions:

(16) a. It is raining and it isn't raining.
b. ?If I have a car, then I don't have a car.

Hurford Disjunctions as contradictions?

Consider the following contradiction:

(17) #This is not a story where the protagonist is a knight or is not a knight.

At least to me, Hurford Disjunctions feel very close to this type of contradictions, which are almost *ungrammatical.

Additional work is needed to understand the nature of this ungrammaticality:

- rejection?
- disjunction?
- modulation?

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Hurford disjunctions in mathematical reasoning

If we are dealing with neglect-zero effects we should expect to observe different patterns in mathematical contexts.

SCENARIO: Alice is trying to solve a mathematical problem and Bob asks her whether she found the solution. Alice replies:

(18) The solution is a power of 2 or an even number.

Hurford disjunctions as exclusive disjunctions

(19) # Pierre lives in Paris or France.

Why Hurford constraint was formulated in the first place?

Again, if we assumed disjunction to be exclusive in classical logic, (19) would just mean *Pierre lives in France but not Paris*.

The idea behind Hurford (1974) is that disjunction has a dual behaviour, and examples like (19) display the exclusive behaviour (with an additional constraint).

Is disjunction exclusive?

Exclusive disjunction seems indeed to account for our natural language intuitions:

(20) a. John ate pasta or pizza.
b. Mary or Sue competed in the race today.
c. Alice, Bob, or Carl will adopt a cat.

Question:

- Why was the idea of disjunction being exclusive abandoned later?

Predictions of ED

The strongest argument: the odd-disjuncts problem

Answer:

- Because in classical logic 'exclusive disjunction' is not exclusive at all when we introduce more disjuncts.

p	∇	q	∇	r
1	0	1	1	1
1	0	1	0	0
1	1	0	0	1
0	1	1	0	1
1	1	0	1	0
0	1	1	1	0
0	0	0	1	1
0	0	0	0	0

Table 1: Truth table for $(p \nabla q) \nabla r$ in classical logic

Odd-disjuncts problem

Exclusive disjunction in BSML solves the issue!

PROOF: Assume $M, s \vDash (p \nabla q) \nabla r$, then there exist t, t' such that $t \cup t' = s$ and $M, t \vDash (p \nabla q) \wedge \neg r$ and $M, t' \vDash r \wedge \neg (p \nabla q)$.

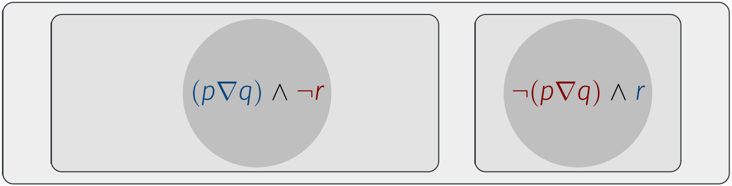

Figure 5: $t \cup t' = s$ and $M, t \vDash (p \nabla q) \wedge \neg r$ and $M, t' \vDash r \wedge \neg (p \nabla q)$

Odd-disjuncts problem

1. Consider $M, t \vDash (p \nabla q) \land \neg r$:
    - since $M, t \vDash (p \nabla q)$, then there exist u, u' such that $u \cup u' = t$ and $M, u \vDash p \land \neg q$ and $M, u' \vDash q \land \neg p$, meaning $\forall w \in t : V(w, p) = 1$ and $V(w, q) = 0$, or $V(w, p) = 0$ and $V(w, q) = 1$;
    - since $M, t \vDash \neg r$, then $\forall w \in t : V(w, r) = 0$.

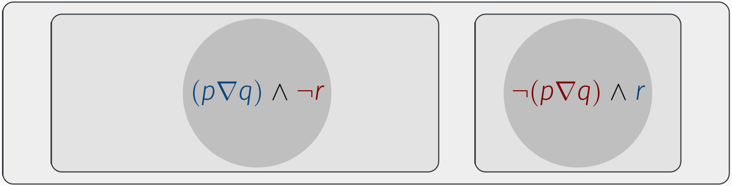

Figure 6: $t \cup t' = s$ and $M, t \vDash (p \nabla q) \land \neg r$ and $M, t' \vDash r \land \neg(p \nabla q)$

Odd-disjuncts problem

1. Consider $M, t \vDash (p \nabla q) \wedge \neg r$:
    - since $M, t \vDash (p \nabla q)$, then there exist u, u' such that $u \cup u' = t$ and $M, u \vDash p \wedge \neg q$ and $M, u' \vDash q \wedge \neg p$, meaning $\forall w \in t : V(w, p) = 1$ and $V(w, q) = 0$, or $V(w, p) = 0$ and $V(w, q) = 1$;
    - since $M, t \vDash \neg r$, then $\forall w \in t : V(w, r) = 0$.

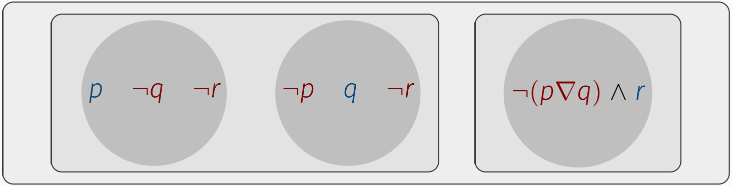

Figure 7: $t \cup t' = s$ and $M, t \vDash (p \nabla q) \wedge \neg r$ and $M, t' \vDash r \wedge \neg(p \nabla q)$

Odd-disjuncts problem

2. Consider $M, t' \vDash r \wedge \neg(p \nabla q)$:
    - since $M, t' \vDash r$ then $\forall w \in t : V(w, r) = 1$;
    - since $M, t' \vDash \neg(p \nabla q)$, then $M, t' \vDash \neg p$ and $M, t' \vDash \neg q$, meaning $\forall w \in t' : V(w, p) = 0$ and $V(w, q) = 0$.

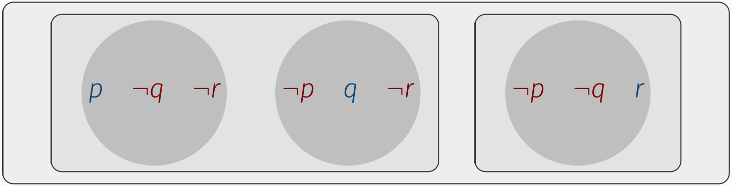

Figure 8: $t \cup t' = s$ and $M, t \vDash (p \nabla q) \wedge \neg r$ and $M, t' \vDash r \wedge \neg(p \nabla q)$

Odd-disjuncts problem

The uniqueness interpretation:

Since $t \cup t' = s$, then for $\forall w \in s$:

- either $w \in u \subset t$, therefore
  $V(w,p) = 1$, $V(w,q) = 0$ and $V(w,r) = 0$;
- or $w \in u' \subset t$, therefore
  $V(w,p) = 0$, $V(w,q) = 1$ and $V(w,r) = 0$;
- or $w \in t'$, therefore
  $V(w,p) = 0$, $V(w,q) = 0$ and $V(w,r) = 1$;

Disjunctive reasoning fallacy

A common fallacy in reasoning:

(21) a. A or (B and C),
b. B,
c. therefore not A.

(Johnson-Laird et al., 1992; and Mascarenhas, 2014)

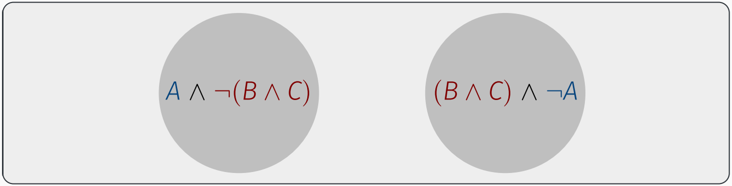

Figure 9: $t \cup t' = s$ and $M, t \vDash A \wedge \neg(B \wedge C)$ and $M, t' \vDash A \wedge \neg(B \wedge C)$

Disjunctive reasoning fallacy

If $M, s \vDash A \nabla (B \wedge C)$, then $\exists t, t'$ s.t. $t \cup t' = s$

$M, t \vDash A \wedge \neg(B \wedge C)$ $\quad$ $M, t' \vDash (B \wedge C) \wedge \neg A$

If $M, s \vDash B$, then for every $s' \subset s, M, s' \vDash B$.

- Therefore $\nexists t \subset s$ s.t. $M, t \vDash A \wedge \neg(B \wedge C)$, since we should be able to find $r \subset t$ s.t. $M, r \vDash \neg B$.

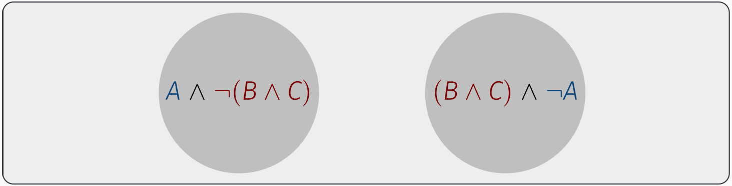

Figure 10: $t \cup t' = s$ and $M, t \vDash A \wedge \neg(B \wedge C)$ and $M, t' \vDash A \wedge \neg(B \wedge C)$

Disjunctive reasoning fallacy

Crucially however, this solution presupposes that the two alternatives generated by $A \nabla (B \wedge C)$ are somehow 'frozen':

- it is not possible to shrink the state s s.t. $M, s \vDash A \wedge \neg(B \wedge C)$ to its substate s' s.t. $M, s' \vDash A \wedge \neg C$.

Further inquiry is needed to determine whether this is indeed the dynamics of updates.

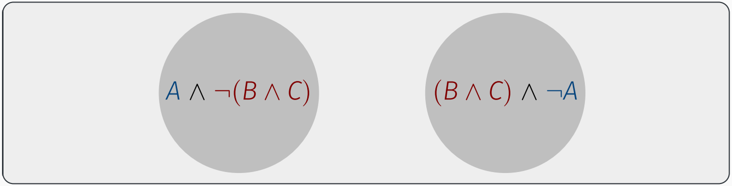

Figure 11: $t \cup t' = s$ and $M, t \vDash A \wedge \neg(B \wedge C)$ and $M, t' \vDash A \wedge \neg(B \wedge C)$

Future Work and Conclusions

Paris or France but not Paris

Consider the following **felicitous** sentence:

(22) John lives in Paris or in France but not in Paris.

At first glance, it seems good for ED as we have two mutually incompatible disjuncts…

- However, in this case we cannot treat France but not Paris as a normal conjunction that will be rejected in a split manner.

Scalar Hurford Disjunctions

A similar case arises with certain **scalar Hurford Disjunctions** (Gazdar, 1979; Singh, 2008; Fox and Spector, 2018; a.o.):

(23) (Only) Alice or Alice and Bob came to the party.

Recall that we already excluded this as a possible model for $a \nabla b$:

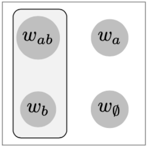

Figure 12: Model for $(a \vee b)$, but not for $(a \nabla b)$

Two types of conjunction?

It seems that we have to introduce a special rule that treats certain (explicit) conjunctions as atomic formulas:

- If ϕ and ψ in $\phi \wedge \psi$ share the same 'topic', the conjunction can be interpreted as a new meaning-preserving atomic formula ξ.

Clearly, to enrich our semantics with such distinction we need to introduce something along the lines of *subject matter* from truth-maker semantics (Fine, 2017).

Absence of Exclusivity

Using the mystery box paradigm, Degano et al. (2023), observed a lack of exclusivity in a production task for the prompt

The box contains a yellow ball or a blue ball

when it's possible that the box contains both.

This is unpredicted under the present view, which says that disjunction is feliciotus only if the conjunction of the disjuncts is rejected.

- Crucially, this data directly contrasts with Hurford Disjunctions which require two disjuncts to be (possibly) incompatible in order for the disjunction to be uttered.

Wide scope FC

One of the reasons for the introduction of BSML was the possibility of deriving wide scope Free Choice (Aloni, 2022):

(24) You may buy a newspaper, or you may rent a movie.
$\Diamond a \vee \Diamond b$
$\rightsquigarrow \Diamond a \wedge \Diamond b$

This is directly in contrasts with an exclusive disjunction, which would predict the opposite:

(25) $\Diamond a \triangledown \Diamond b$
$\rightsquigarrow (\Diamond a \vee \Diamond b) \wedge \neg(\Diamond a \wedge \Diamond b)$

Possible solution, dynamics of disjunction

We already know that disjunction has certain dynamic effects:

- Partee's bathroom sentences:

(26) a. Either the bathroom is in a weird place or this house has no bathroom.
b. Either this house has no bathroom or the bathroom is in a weird place.

- scalar Hurford disjunctions :

(27) a. John ate some or all of the cookies.
b. #John ate all or some of the cookies.

Possible solution, dynamics of disjunction

We could derive exclusivity from conversational principles operating in the dynamics of update semantics (Aloni, 2023):

$$s[p] = s \cap \{w \in W | V(p, w) = 1\}$$
$$s[\phi \wedge \psi] = s[\phi] \cap s[\psi]$$
$$s[\phi \vee \psi] = s[\phi] \cup s[\psi]$$
$$s[\neg\phi] = s - s[\phi]$$
$$s[\Diamond\phi] = s, \text{ if } s[\phi] \neq \emptyset; = \emptyset, \text{ otherwise.}$$

Possible solution, dynamics of disjunction

Conversational principles:

- $s[\phi]$ is felicitous iff $s \not\models \phi$,
  i.e. $\exists s' \subset s, s' \Dashv \phi$;
- $s[\phi \vee \psi] = s[\phi] \cup s[\psi]$ means that $s \not\models \phi$ and $s \not\models \psi$,
  i.e. $\exists s' \subset s, s' \Dashv \phi$ and $\exists s'' \subset s, s'' \Dashv \psi$;
- as a consequence, $s[\psi] \not\models \phi$ and $s[\phi] \not\models \psi$,
  modeled with $\exists s' \subset s[\psi], s' \Dashv \phi$ and $\exists s'' \subset s[\phi], s'' \Dashv \psi$.

Possible solution, dynamics of disjunction

Assuming that we treat all modals as the epistemic modals in Aloni (2023):

$$s[\phi \vee \psi] = s[\phi] \cup s[\psi]$$

$$s[\Diamond\phi \vee \Diamond\psi] = s[\Diamond\phi] \cup s[\Diamond\psi] = s \cup s$$

- This strategy confines exclusiveness to disjunctions of literals.

Possible solution, dynamics of disjunction

On the other hand, if exclusiveness is a property of discourse dynamics, we might expect to not find it in the setting of Degano et al. (2023), in which participants needed to evaluate a sentences on the foreground of a scenario.

How to test this?

- interpretation task proposed in Degano et al. (2023);
- with an experiment in which participants read a story about a person living in Paris and are asked to reply to some questions on the story.

One of the questions would be:

(28) The protagonist of the story lives in Paris or France.

Conclusions

In this talk I have presented a notion of Exclusive Disjunction in BSML, that interacts with neglect-zero in interesting ways:

- it gives a semantic explanation to Standard and Disjoint Hurford Disjunctions, reducing them to contradictions thanks to the notions of split disjunction support and split conjunction rejection;
- it constitutes a real exclusive disjunction, which - in contrast with classical exclusive disjunction - avoids the odd-disjuncts problem;
- it explains a common fallacy in reasoning which is not a valid reasoning if we suspend neglect-zero.

Future Work

On the other hand, we are left with several open questions:

- what is the interaction of ED with scalar item and how can we define a notion of 'compact' conjunction?
- how can we model exclusiveness in such a way that it does not apply to modalized statements (Aloni, 2022) and reflects the findings of Degano et al. (2023)?

Thank you for your attention!

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