Discourse (In)consistency in Commitment Space Semantics

Nihil Seminar

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Epistemic contradictions (Yalcin 2007; Mandelkern 2019):

(1) a. # It is not raining and it might be raining. $\neg p \land \Diamond p$ b. # It might be raining and it is not raining. $\Diamond p \land \neg p$

Problematic for standard contextualism (e.g., Kratzer 1981, 2012):

- (2) $[\![\diamondsuit p]\!]^{c,w} = 1$ iff there is a world w' compatible with the *c*-determined information at w s.t. $[\![p]\!]^{c,w'} = 1$.
- There is nothing inconsistent between $\llbracket p \rrbracket^{c,w} = 0$ and $\llbracket \diamondsuit p \rrbracket^{c,w} = 1$.

Update semantics (Veltman 1996):

 The meaning of a sentence is conceived of as a function from input to output contexts/information states, standardly modeled as sets of worlds.

$$c[p] = \{w \in c \mid w \in \mathcal{I}(p)\}$$
$$c[\neg \varphi] = c - c[\varphi]$$

 ◇p is construed as a test that checks whether the input state is compatible with p.

$$c[\Diamond p] = \{w \in c \mid c[p] \neq \emptyset\}$$

Discourse consistency in update semantics:

- **Consistence:** φ is consistent iff $\exists c : c[\varphi] \neq \emptyset$.
 - (1) a. # It is not raining and it might be raining. $\neg p \land \Diamond p$
 - b. # It might be raining and it is not raining. $\Diamond p \land \neg p$

Assume a dynamic conjunction (Heim 1983): c[φ ∧ ψ] = c[φ][ψ], (1a) is correctly predicted to be inconsistent; but (1b) is incorrectly predicted to be consistent.

Discourse consistency in update semantics:

- Coherence: φ is coherent iff $\exists c : c \neq \emptyset \& c[\varphi] = c$.
 - φ is coherent iff there exists a non-absurd *fixed point* for the update.
- Given a notion of support (c ⊨ φ iff c[φ] = c), φ is coherent iff there exists a non-absurd state that supports φ.
 - (1) a. # It is not raining and it might be raining. $\neg p \land \Diamond p$ b. # It might be raining and it is not raining. $\Diamond p \land \neg p$
- Both (1a) and (1b) are incoherent regardless how conjunction is interpreted:
 - Dynamic (Heim 1983): $c[\varphi \land \psi] = c[\varphi][\psi]$
 - Static (Veltman 1996): $c[\varphi \land \psi] = c[\varphi] \cap c[\psi]$

There are intuitively contradictory sentences that coherence fails to account for (Aloni 2001; Mandelkern 2019; Yalcin 2015):

(2) #It might be raining and it is not raining, or it might not be raining and it is raining.

$$(\Diamond p \land \neg p) \lor (\Diamond \neg p \land p)$$

• Assume Veltman's (1996) definition for disjunction:

$$\boldsymbol{c}[\varphi \lor \psi] = \boldsymbol{c}[\varphi] \cup \boldsymbol{c}[\psi]$$

 Any c that contains both some p-world and some ¬p-world will serve as a fixed point for this update:

$$(3) \quad \{p,\bar{p}\}[\Diamond p \land \neg p] \cup \{p,\bar{p}\}[\Diamond \neg p \land p] = \{\bar{p}\} \cup \{p\} = \{p,\bar{p}\}$$

(◇p ∧ ¬p) ∨ (◇¬p ∧ p) can be rendered incoherent with a dynamic disjunction:

$$\boldsymbol{c}[\varphi \lor \psi] = \boldsymbol{c}[\varphi] \cup \boldsymbol{c}[\neg \varphi][\psi]$$

• However, this solution violates other classically valid principles (Mandelkern, 2020a):

Law of Excluded Middle (LEM): $\varphi \lor \neg \varphi$

Validity: φ is valid iff $\forall c : c[\varphi] = c$

- The update with $[\neg(\Diamond p \land \neg p) \lor \neg \neg(\Diamond p \land \neg p)]$ does not always return the input state.
- Let $c = \{p, \overline{p}\}$

•
$$c[\neg(\Diamond p \land \neg p)] = \{p\}$$

•
$$c[\neg\neg(\Diamond p \land \neg p)][\neg\neg(\Diamond p \land \neg p)] = c[\Diamond p \land \neg p][\Diamond p \land \neg p] = \emptyset$$

Fixed-Point Update

Klinedinst and Rothschild (2014) imposes a local coherence check which requires every update to always be repeated until it reaches a fixed point.

Fixed-point update c[φ]* := c[φ]...[φ] where the output state c' will always be a fixed point such that c'[φ] = c'.

Normal updates $[\cdot]$ are then defined in terms of fixed-point updates $[\cdot]^*$.

•
$$c[\neg \varphi] = c - c[\varphi]^*$$

• $c[\varphi \land \psi] = c[\varphi]^* \cap c[\psi]^*.$
• $c[\varphi \lor \psi] = c[\varphi]^* \cup c[\psi]^*.$

Fixed-point updates correctly predict that

- $(\Diamond p \land \neg p) \lor (\Diamond \neg p \land p)$ is inconsistent;
- $[\neg(\Diamond p \land \neg p) \lor \neg \neg(\Diamond p \land \neg p)]$ is valid.

Fixed-Point Update

However, the fixed-point update is too strong as a general update operation in the case of *standoffs* (Bennett 2003; Goldstein 2022).

• Suppose there are two levers which together control a water gate:

Configuration	Outcome
1 down; 2 up	Top Gate is open and the water flows left
1 up; 2 down	Top Gate is open and the water flows right
1 down; 2 down	Top Gate is closed

Problem with the Fixed-Point Update

- (4) Suppose Ann and Bob each can only see the position of one lever and nothing more: Ann sees that Lever 1 is down, and Bob sees that Lever 2 is down. They then each pass a note to Carl:
 - a. Ann: Top Gate might be open, and if it is open the water is flowing left.

 $\Diamond O \land (O \to L)$

b. Bob: Top Gate might be open, and if it is open the water is flowing right.

 $\Diamond O \land (O \rightarrow \neg L)$

- Carl should be able to infer that Top Gate is closed.
- But the fixed-point update c[◊O ∧ (O → L)]*[◊O ∧ (O → ¬L)]* necessarily leads to the absurd state Ø.

Problem with the Fixed-Point Update

The two sentences in (4) are indeed contradictory if they are uttered by a single speaker as in (5) or when they are reproduced as consecutive assertions by Ann and Bob in a single conversation as in (6).

- (5) # Top Gate might be open, and if it is open the water is flowing left, and if it is open the water is flowing right.
- (6) Ann: Top Gate might be open, and if it is open the water is flowing left.Bob: #Yes, and if it is open the water is flowing right.

Desiderata

We want an update system that

- captures standoffs,
 - If p then q and if p then ¬q can lead to consistent updates but only when they are presented as separate private information.
- accounts for cases of epistemic contradictions,

•
$$\Diamond p \land \neg p$$

• $(\Diamond p \land \neg p) \lor (\Diamond \neg p \land p)$

and does so without affecting other classically valid principles.

• The Law of Excluded Middle: $\varphi \vee \neg \varphi$

Commitment Space Semantics

Proposal: Invoking Commitments

- (4) a. Ann: Top Gate might be open, and if it is open the water is flowing left.
 - b. Bob: Top Gate might be open, and if it is open the water is flowing right.
- (5) #Top Gate might be open, and if it is open the water is flowing left, and if it is open the water is flowing right.

An intuitive explanation of the contrast between (4) and (5):

- (5) is inconsistent because the assertion commits the speaker to:
 - Top Gate might be open.
 - If Top Gate is open the water is flowing left.
 - If Top Gate is open the water is flowing right.
- In (4) because as Bob is ignorant of what Ann knows, he is not committed to *If Top Gate is open the water is flowing left*.

Proposal: Invoking Commitments

- (6) Ann: Top Gate might be open, and if it is open the water is flowing left.Bob: #Yes, and if it is open the water is flowing right.
- In (6), Bob's affirmation commits him to *If Top Gate is open the water is flowing left*, which is then in conflict with his other commitment that *If Top Gate is open the water is flowing right*.

Commitment States

A commitment state c is defined as a set of worlds. It embodies Stalnakerian common ground information as well as each discourse participant's commitment.

Assertions express commitments by the speaker (Krifka 2015, 2021, 2023).

• A new type of formulas: $S \vdash \varphi$ (the speaker S is committed to φ).

•
$$c[S \vdash \varphi] = \{ w \in c \mid w \in \mathcal{I}(S \vdash \varphi) \}$$

• To simplify, I will set aside the performative aspect of the update with commitment.

The update with $[\varphi]$ is a result of the following pragmatic inference.

 Commitment Closure of c: If s is a participant in the conversation that is trustworthy, and s ⊢ φ holds at every w in c, and the other participants in conversation do not object, then c[φ] thereby making φ common ground.

Standoffs

- (4) a. Ann: Top Gate might be open, and if it is open the water is flowing left.
 - b. Bob: Top Gate might be open, and if it is open the water is flowing right.
- (4) is consistent:
 - From Carl's perspective:
 - $c[Ann \vdash \Diamond O][Ann \vdash O \rightarrow L][Bob \vdash \Diamond O][Bob \vdash O \rightarrow \neg L]$
 - If Carl chooses to subsequently update c with $[O \rightarrow L]$ and $[O \rightarrow \neg L]$, then he is able to infer $\neg O$.
 - Additionally, we can construe Ann's and Bob's private commitment as a commitment state that only involves one discourse participant.
 - $C_{Ann}[Ann \vdash \Diamond O][Ann \vdash O \rightarrow L].$
 - By commitment closure, we have $C_{Ann}[Ann \vdash \Diamond O][Ann \vdash O \rightarrow L][\Diamond O][O \rightarrow L].$
 - The result is again consistent.

Standoffs

(5) #Top Gate might be open, and if it is open the water is flowing left, and if it is open the water is flowing right.

(5) is inconsistent:

- Updating on the speaker's commitment state: $c[S \vdash \Diamond O][S \vdash O \rightarrow L][S \vdash O \rightarrow \neg L] = c'$
- Given commitment closure, we have $c'[\diamondsuit O][O \rightarrow L][O \rightarrow \neg L]$, which will be inconsistent.

That being said, we still need to show that an update with $[\diamondsuit O][\neg O]$ is inconsistent in CSS.

Commitment Spaces

I define the update effects of modals at the level of commitment spaces.

A commitment space C is a set of commitment states. It contains the commitment state at the current stage of discourse (the *root*) along with all of its possible developments.

• \sqrt{C} , the root, is defined as $\{c \in C \mid \neg \exists c' \in C : c \subset c'\}$

Updating with a speech act \mathfrak{A} : $C[\mathfrak{A}] = \{ c \in C \mid c \subseteq \sqrt{C[\mathfrak{A}]} \}$

Speech act operations on commitment spaces:

•
$$C[\sim \mathfrak{A}] = C - C[\mathfrak{A}]$$

•
$$C[\mathfrak{A} \wedge \mathfrak{B}] = C[\mathfrak{A}] \cap C[\mathfrak{B}]$$

• $C[\mathfrak{A} \otimes \mathfrak{B}] = C[\mathfrak{A}] \cup C[\mathfrak{B}]$

Examples

 $C[p \lor q] := C[p] \cup C[q]$



 $C[p \lor q] := \{c \in C \mid c \subseteq \sqrt{C[p \lor q]}\}$



 To improve readability, I have ignored two commitment states in these figures, namely those that embody the information p ↔ q and p ↔ ¬q.

Meta Speech Act

A meta speech act restricts possible future developments of a commitment space without altering its root. Consider *denegation* (\sim) for example.

- (6) a. I don't promise to come.
 - b. I promise not to come.
- (7) a. I don't claim that Paul was at the party.
 - b. I claim that Paul wasn't at the party.



Dynamic Modals in CSS

The dynamic modal $\Diamond p$ is treated as the denegation of $\neg p$:

- $C[\Diamond p] := C[\sim ASSERT(\neg p)]$
- To further simplify, we can view an update with [◊p] as having the following effect: C[◊p] := C[∼¬p] = C − C[¬p]



 It delimits the commitment space by eliminating all commitment states that consist exclusively of ¬p-worlds.

Returning to the Desiderata

• The figure shows that $C[\Diamond p \land \neg p] = C[\Diamond p] \cap C[\neg p] = \emptyset$. Hence, $\Diamond p \land \neg p$ is predicted to be inconsistent.



- Likewise, ◇¬p ∧ p and (◇p ∧ ¬p) ∨ (◇¬p ∧ p) are also predicted to be inconsistent.
- On the other hand, the speech act equivalent of LEM φ W ~φ is valid
 ∀C : C[φ W ~φ] = C[φ] ∪ (C − C[φ]) = C

Fine-Tuning

Modal Commitments

Commitment to *might*-claim: if \diamond is conceived of as a meta speech act operator, then how can we interpret formulas of the form $S \vdash \diamond \varphi$?

- In CSS, the object of one's commitment needs to be a proposition.
- We may relax this condition and allow the objects of commitments to be context change potentials (i.e., functions from commitment states to commitment states).
- Are we forced to postulate additional semantic entities (i.e., functions from commitment spaces to commitment spaces) in order to make sense of modal commitments?
- I will argue that this is not needed.

- (8) It can't be raining.
- If <> is treated as a meta speech act operator, then the negation of
 >p will be ~<>p, which by definition is just ¬p.
- Not p and can't be p have different felicity conditions (von Fintel & Gillies, 2021):
- (9) (Billy looking out the window seeing brilliant sunshine)
 - a. It isn't raining.
 - b. # It can't be raining.
- (10) (Billy seeing people coming in with sunglasses and parasols and knowing sunshine is the only cause)
 - a. It isn't raining.
 - b. It can't be raining.

- This contrast is reminiscent of Karttunen's (1972) problem:
 - (11) (Billy is looking out the window seeing pouring rain.)
 - a. It is raining.
 - b. # It must be raining.
 - (12) (Billy sees people coming in with wet rain gear and knows rain is the only explanation.)
 - a. It is raining.
 - b. It must be raining.
 - von Fintel and Gillies (2010, 2021):
 - although must p (or can't be p) entails p (or $\neg p$),
 - must p (or can't be p) is felicitous only if evidence for p (or ¬p) is indirect.

However, there is evidence that the weakness of *must* cannot be fully attributed to evidentiality (Kibble 1994; Kendrick 2023).

- (13) a. John has a guitar. It's a Fender Stratocaster.
 - b. John must have a guitar. #It's a Fender Stratocaster. (Kibble 1994: 8)

The current framework enables an alternative way to cash out the discourse effect of necessity modals: they do not affect the root of a commitment space but only delimits future developments.

- $\Box p$ constrains future commitment states to only those where p holds.
- $\neg \Diamond p$ constrains future commitment states to only those where $\neg p$ holds.

 $C[\Box p] := \{\sqrt{C}\} \cup C[p]$





Source of the Meta Speech Act Effects

Epistemic modals can have meta speech act effects without themselves being meta speech act operators.

Potential source of the dynamic effects: a way to secure the following principles governing accruement of information.

• Idempotence: $c[\varphi] \models \varphi$

•
$$c \models \varphi$$
 iff $c[\varphi] = c$

- A successful update with φ should make an agent commit to $\varphi.$
- Persistence: If $c \models \varphi$ then $c[\psi] \models \varphi$
 - If an agent is already committed to something, then she continues to hold this commitment unless the commitment is retracted in light of new information.

Source of the Meta Speech Act Effects

In update semantics, idempotence and persistence hold for non-modal formulas.

But they fail when the test-modal \diamondsuit is involved.

- For Idempotence: $c[\Diamond p \land \neg p] \not\models \Diamond p \land \neg p$.
- For Persistence: Suppose $c \models \Diamond p$; then $c[\neg p] \not\models \Diamond p$.

These principles can be observed if we view \diamond not just as a test of its input state, but also as having a forward-looking effect on prospective contexts/commitment states.

The needs to satisfy these principles in turn manifests themselves as meta speech act effects on commitment spaces.

Question about modal commitments: given this way of deriving the meta speech act effects of modals, we do not need to introduce new semantic objects as the objects of commitments.

Source of the Meta Speech Act Effects

This idea that an update can place certain conditions on its output states is not completely alien.

Postsuppositions as delayed tests that check whether certain conditions are satisfied by output states (e.g., Brasoveanu 2012; Brasoveanu & Szabolsci 2013; Glass 2023; Henderson 2014).

- (14) A-mo B-mo hashitta.
 - 'A as well as B ran away.' (Brasoveanu & Szabolsci 2013)
 - B can serve as the antecedent for 'A too'.

Formal Details: Updates with Modals

Updates with Modals

Here is how the update will proceed if $C[can't \ be \ p]$ is treated not as $C[\sim \Diamond p]$ but instead as inducing the update $\{\sqrt{C}\} \cup C[\neg p]$.

• Goal: disentangle the meta speech act effects associated with modals from the rest of the update.

The update is first calculated and decomposed at the root as in standard update semantics with the exceptions that:

•
$$\sqrt{C}[\neg \Diamond \varphi] = \sqrt{C}[\Box \neg \varphi]$$

• $\sqrt{C}[\neg \Box \varphi] = \sqrt{C}[\Diamond \neg \varphi]$

This process continues until each constituent update is turned into (i) $[\Diamond \varphi]$, (ii) $[\Box \varphi]$, or (iii) $[\varphi]$ where φ is modal-free.

(15) a.
$$\sqrt{C[\Diamond p \land \neg p]} = \sqrt{C[\Diamond p]} \cap \sqrt{C[\neg p]}$$

b. $\sqrt{C[\neg \Diamond p \land \Box p]} = \sqrt{C[\Box \neg p]} \cap \sqrt{C[\Box p]}$

Updates with Modals

Lift the updates in (15) to commitment spaces in (16).

(15) a.
$$\sqrt{C[\Diamond p \land \neg p]} = \sqrt{C[\Diamond p]} \cap \sqrt{C[\neg p]}$$

b. $\sqrt{C[\neg \Diamond p \land \Box p]} = \sqrt{C[\Box \neg p]} \cap \sqrt{C[\Box p]}$
(16) a. $C[\Diamond p \land \neg p] = C[\Diamond p] \cap C[\neg p]$

b.
$$C[\neg \Diamond p \land \Box p] = C[\Box \neg p] \cap C[\Box p]$$

Updates on commitment spaces are then defined as follows:

•
$$C[\Diamond \varphi] := C - C[\neg \varphi]$$

•
$$C[\Box \varphi] := \{\sqrt{C}\} \cup C[\varphi]$$

• $C[\varphi] := \{ c \in C \mid c \subseteq \sqrt{C[\varphi]} \}$, where φ is modal-free.

(17) a.
$$C[\diamondsuit p \land \neg p] = (C - C[\neg p]) \cap C[\neg p] = \emptyset$$

b. $C[\neg \diamondsuit p \land \Box p] = (\{\sqrt{C}\} \cup C[\neg p]) \cap (\{\sqrt{C}\} \cup C[p]) = \{\sqrt{C}\}$

Strong Consistency

$$C[\neg \Diamond p \land \Box p] = \{\sqrt{C}\}$$



A stronger notion of consistency:

- **Strong Consistency:** φ is strongly consistent iff $\exists C : C[\varphi] \neq \emptyset \& C[\varphi] \neq \{\sqrt{C}\}.$
- For φ to be strongly consistent, it should be possible that an update with φ yields a commitment state that can be further developed.

Returning to the Desiderata

We correctly predict epistemic contradictions to be inconsistent:

(18) a.
$$\Diamond p \land \neg p$$

b. $(\Diamond p \land \neg p) \lor (\Diamond \neg p \land p)$

As for the Law of Excluded Middle:

(19) a. Decomposing at the root: $\sqrt{C}[\neg(\Diamond p \land \neg p) \lor \neg \neg(\Diamond p \land \neg p)] = (\sqrt{C} - (\sqrt{C}[\Diamond p] \cap \sqrt{C}[\neg p])) \cup (\sqrt{C}[\Diamond p] \cap \sqrt{C}[\neg p])$

b. Lifting to the commitment space:

$$C[\neg(\Diamond p \land \neg p) \lor \neg \neg(\Diamond p \land \neg p)] = (C - (C[\Diamond p] \cap C[\neg p])) \cup (C[\Diamond p] \cap C[\neg p]) = (C - (C[\neg \neg p] \cap C[\neg p])) \cup (C[\neg \neg p] \cap C[\neg p]) = (C - \emptyset) \cup \emptyset = C$$

Mandelkern (2020b) proposes a contextualist account where the domain of quantification for modals is the prospective common ground.

- 'Might *p*' means that *p* will be compatible with the future common ground,
- which in turn exerts a normative force that requires future common ground to be compatible with *p*.

Mandelkern draws on performatives like (20) to explain how the normative force arises.

(20) This afternoon, you will be cleaning the rabbit cage.

However, whereas the sincerity condition (Searle 1969) requires the speaker of (20) to want the address to clean the rabbit cage, the same does not seem to hold in general for epistemic modals.

(21) X: I can't find my phone!

Y: Have you checked under your bed? It might be there.

• The speaker may simply be drawing attention to a possibility (cf. Ciardelli et al. 2009; Roelofsen 2013; Bledin & Rawlins 2020; Zhang 2023) without wanting the prospective common ground to be compatible with the *might*-claim.

Incurvati & Scholöder (2017, 2019, 2022) proposes a logic that contains a speech act force marker which they call *weak rejection*.

- (22) a. Is it the case that X or Y will win the election? No, X or Y or Z will win.
 - b. The 'no' is a weak rejection: $\ominus(X \lor Y)$

They give a translation of this logic into S5 where \ominus is translated as:

•
$$\tau(\ominus \varphi) = \Diamond \neg \varphi$$

One main difference betwen \ominus and \sim is that unlike \sim , the force marker \ominus will always prefix a sentence and does not embed.

- Iterations of \ominus could turn out to be problematic.
- $\tau(\ominus \ominus \varphi) = \Diamond \neg \Diamond \neg \varphi = \Diamond \Box \varphi$, which is equivalent to $\Box \varphi$ in S5.

Logic of Commitment Space Semantics

Update-to-Test Consequence

Van Benthem's (2008) translation of the *update-to-test* consequence (\models_{ut}) into public announcement logic (\models_{pa}) :

(23) a.
$$\varphi_1, ..., \varphi_n \models_{ut} \psi$$

• iff $\forall c : c[\varphi_1]...[\varphi_n] \models \psi$
b. $\models_{pa} [!\varphi_1]...[!\varphi_n] K \psi$
• (K is a S5 modality, and we assume a single agent.)

• $\mathcal{M}, w \Vdash [!\varphi]\psi$ iff $\mathcal{M}, w \nvDash \varphi$ or $\mathcal{M} \mid \varphi, w \Vdash \psi$, where

- $W \mid \varphi := \{ v \in W : \mathcal{M}, v \Vdash \varphi \}$ eliminate all worlds where φ is false,
- leaves arrows between remaining worlds unchanged, and
- leave the valuation the same at remaining worlds.

Public Announcement Logic

PAL can be axiomatized as follows:

- Axioms and rules of S5
- $\ \, \textcircled{[} \varphi] p \leftrightarrow (\varphi \rightarrow p), \text{ where } p \text{ is atomic}$
- $\ \, \textbf{ [!}\varphi]\neg\psi\leftrightarrow(\varphi\rightarrow\neg[!\varphi]\psi)$
- $[!\varphi] K \psi \leftrightarrow (\varphi \to K[!\varphi] \psi)$
- **0** Necessitation for $[!\varphi]$

- We may follow the same method to translate the consequence in CSS into public announcement logic.
- But since the inputs and outputs are sets of context sets, the usual PAL based on Kripke semantics is not suitable. Instead, we could devise a PAL based on possibility semantics.
- Possibilities are partial states which can leave the truth value of a sentence unsettled.
 - A possibility y is a refinement of x just in case y settles as many truths as x (and possibly more).

Possibility Semantics for Classical Logic

Possibility Model: $\langle S, \sqsubseteq, V \rangle$ where

- S is a non-empty set;
- \sqsubseteq is a partial order on S (reflexive, transitive and anti-symmetric relation on S);
 - ' $y \sqsubseteq x$ ' reads "y is a refinement of x."
- V is a valuation that assigns to every atomic formula a nonempty subset U of S satisfying persistence and refinability:
 Persistence: if x ∈ U and y ⊑ x, then y ∈ U;
 Refinability: if x ∉ U, then ∃y ⊑ x ∀z ⊑ y : z ∉ U.
 - Converse of refinability: if $\forall y \sqsubseteq x \exists z \sqsubseteq y : z \in U$ then $x \in U$.

Possibility Semantics for Classical Logic

Let $\langle S, \sqsubseteq, V \rangle$ be a model with $x \in S$. Then $x \Vdash \varphi$ is defined as follows:

• Validity and entailment are defined in the usual way.

• Next, we add a binary relation *R* over possibilities to *M* so that we can translate the consequence relation of CSS as follows:

$$\varphi_1, ..., \varphi_n \models_{\mathit{css}} \psi \text{ iff } \models_{\mathit{pa*}} [!\varphi_1]...[!\varphi_n][R]\psi$$

Basic intuition:



- $[R] \sim p$ is true at \sqrt{C} iff $\sim p$ is true at all possibilities in the gray area.
- $x \Vdash \sim \varphi$ iff $x \not\Vdash \varphi$
- $x \Vdash [R]\varphi$ iff $\forall y \in R(x) : y \Vdash \varphi$
- Additionally, R should satisfy: if $y \in R(x)$ then $y \sqsubseteq x$.

• However, the usual method of generating submodels via world-elimination becomes problematic.



- This is no longer a possibility model as refinability fails.
- For example, $\forall y \sqsubseteq s_{p/q} \exists z \sqsubseteq y : z \Vdash q$, but $s_{p/q} \not\Vdash q$.

- In response, we could adopt a version of public announcement logic where submodels are generated via arrow-elimination while all possibilities and the valuation remain intact.
 - (e.g., Gerbrandy & Groeneveld 1997; Kooi & Renne 2001)



Should *R* be reflexive?

- If *R* is reflexive, then [!p][R]p, which is a theorem of PAL, is no longer valid.
- This is so because the reflexive arrow of a \bar{p} -possibility will make [R]p false when evaluated at that possibility.



- Let the model above be $\mathcal{M} \mid p$ where all possibilities are reflexive.
- Then $\mathcal{M} \mid p, s_{\bar{p}} \not\Vdash [R]p$ since $\mathcal{M} \mid p, s_{\bar{p}} \not\Vdash p$. Thus, $\mathcal{M} \not\Vdash [!p][R]p$.

Should R be reflexive?

• If we do not impose reflexivity, then $\neg \neg [R]p \not\models [R]p$



- Suppose the four terminal points don't see anything.
- Given that $x \Vdash \neg \neg [R]p$ iff $\forall x' \sqsubseteq x \exists x'' \sqsubseteq x' : x'' \Vdash [R]p$,
- $s_{\sqrt{C}} \Vdash \neg \neg [R]p$ since [R]p is vacuously true at all terminal points.
- However, $s_{\sqrt{C}} \not\Vdash [R]p$

A Simplification

As it turns out, \models_{css} is not in fact dynamic in the usual sense.

• Dynamic consequence usually fails to satisfy one of the following structural rules.

$$\begin{array}{ll} \bullet & \text{If } \Gamma, \varphi, \psi \vDash \chi, \text{ then } \Gamma, \psi, \varphi \vDash \chi \\ \bullet & \text{If } \Gamma, \varphi, \varphi \vDash \psi, \text{ then } \Gamma, \varphi \vDash \psi \\ \bullet & \varphi \vDash \varphi \\ \bullet & \text{If } \Gamma \vDash \varphi \text{ and } \Gamma, \varphi \vDash \psi, \text{ then } \Gamma \vDash \psi \\ \bullet & \text{If } \Gamma \vDash \psi, \text{ then } \Gamma, \varphi \vDash \psi \end{array}$$

For example:

$$\begin{array}{l} \bigcirc \neg p, \Diamond p \models \bot \text{ but } \Diamond p, \neg p \not\models \bot. \\ \bigcirc & \Diamond p \land \neg p, \Diamond p \land \neg p \models \bot \text{ but } \Diamond p \land \neg p \not\models \bot \end{array}$$

By contrast, with ⊨_{css}
¬p, ◇p ⊨_{css} ⊥ and ◇p, ¬p ⊨_{css} ⊥.
◇p ∧ ¬p, ◇p ∧ ¬p ⊨_{css} ⊥ and ◇p ∧ ¬p ⊨_{css} ⊥

A Simplification

- van Benthem (2008) shows that a consequence relation satisfies these rules iff there is a map from sentences φ to sets $Set(\varphi)$ such that $\varphi_1, ..., \varphi_n \models \psi$ iff $\bigcap_{1 \le i \le n} Set(\varphi)_i \subseteq Set(\psi)$.
 - This condition is satisfied by CSS. At the level of commitment spaces, each sentence can be mapped to a set of commitment states.
 - e.g., $Set(\sim p) = C_0 C_0[p]$, where C_0 is the initial commitment space that contains minimal information and has no restriction on future discourse.
 - This hints at a way to define \models_{css} without resorting to PAL.

- Since our language contains the speech act disjunction W, we begin with an axiomatization of inquisitive logic.
- The language is defined as follows:

$$\varphi ::= p \mid \varphi \land \varphi \mid \varphi \to \varphi \mid \varphi \lor \varphi \mid \bot$$

where $p \in Atoms$

• Classical negation is defined as:

$$\neg \varphi := \varphi \to \bot$$

• Classical disjunction is defined as:

$$\varphi \lor \psi := \neg (\neg \varphi \land \neg \psi)$$

• A natural deduction system for the standard inquisitive logic can be given as follows (Ciardelli 2022):

Conjunction Implication $\left[\varphi\right]$ $\begin{array}{ccc} \underline{\varphi} & \psi & & \underline{\varphi} \wedge \psi & \underline{\varphi} \wedge \psi \\ \overline{\varphi} \wedge \psi & & \overline{\varphi} & \overline{\psi} & & \overline{\psi} & & \underline{\varphi} & \overline{\psi} \\ \end{array} \end{array}$ Inquisitive disjunction Falsum $[\varphi]$ $[\psi]$ $\frac{\varphi}{\varphi \boxtimes \psi} \quad \frac{\psi}{\varphi \boxtimes \psi} \quad \frac{\varphi \boxtimes \psi}{\frac{\varphi}{\sqrt{\chi}}} \quad \frac{\vdots}{\chi}$ ⊥ 9 W-split $\neg \neg$ elimination $\frac{\alpha \to \psi \vee \chi}{(\alpha \to \psi) \vee (\alpha \to \gamma)}$ $\frac{\neg \neg \alpha}{\alpha}$

• where α is a classical formula which does not contain $\mathbb W.$

• We extend the language with denegation \sim :

$$\varphi ::= p \mid \varphi \land \varphi \mid \varphi \to \varphi \mid \varphi \lor \varphi \mid \bot \mid \sim \varphi$$

where $p \in Atoms$

- \bullet Classical negation \neg and disjunction \lor are still defined as before.
- Possibility modal is defined as:

$$\Diamond \varphi := \sim \neg \varphi$$

Introduction and elimination rules for $\sim:$ $[\varphi]$

 $\sim \varphi$

• \sim Introduction:

• ~ Elimination:
$$\frac{\varphi, \sim \varphi}{|}$$

• ~~ Elimination:
$$\frac{\sim \sim \varphi}{\varphi}$$

•
$$\neg \sim$$
 Elimination: $\frac{\neg \sim \varphi}{\varphi}$

• Soundness of this rule follows from the fact that \sqsubseteq is reflexive. Thus, if $\forall x' \sqsubseteq x : x \not\Vdash \sim \varphi$ which is $\forall x' \sqsubseteq x : x \Vdash \varphi$, then it follows that $x \Vdash \varphi$.

There is one important restriction on the use of subproofs for $\rightarrow \mathcal{I}$.

 For → Introduction, reiteration inside a subproof from a line outside the subproof is limited to only atoms and formulas whose main connective is → (which by definition also includes ¬ and ∨).

For example, $\Diamond p \not\vdash \neg p \rightarrow \bot$

- $\Diamond p$ cannot be reiterated inside the subproof introduced by $\neg p$.
- In contrast, $\Diamond p, \neg p \vdash \bot$
- Thus deduction theorem with side assumptions (i.e, $\chi, \varphi \vdash \psi$ iff $\chi \vdash \varphi \rightarrow \psi$) fails in general cases.
 - But it does obtain when χ is provably equivalent to either an atomic formula or a formula whose main connective is \rightarrow .

Some examples where reiteration is allowed:

•
$$\vdash p \rightarrow (q \rightarrow p)$$
 and $\vdash \neg p \rightarrow (q \rightarrow \neg p)$, though $\not\vdash \sim p \rightarrow (q \rightarrow \sim p)$

•
$$\vdash$$
 $(p \lor q) \rightarrow (p \lor q \lor r)$, though $\nvDash \sim p \rightarrow (\sim p \lor r)$

In contrast, reiteration remains unrestricted in subproofs for $\sim I$.



Likewise, reiteration is also unrestricted in subproofs of ${\mathbin{{\mathbb W}}{\mathcal E}}$

• For example,
$$p \lor q, \sim p \land \sim q \vdash \bot$$

The fact that reiteration is only restricted for $\rightarrow \mathcal{I}$ and only to certain formulas results from a combination of two things:

- In possibility semantics, classical conditional → requires quantifying over all refinements of the current point of evaluation.
- Atoms and formulas of certain types (e.g., →, ¬, ∨) are guaranteed to be persistent (i.e., if x ⊨ φ, then ∀x' ⊑ x : x' ⊨ φ) whereas others (e.g., ∧, ♥, ~) do not.

(24) a.
$$x \Vdash \varphi \to \psi$$
 iff $\forall y \sqsubseteq x$: if $y \Vdash \varphi$ then $y \Vdash \psi$

b.
$$x \Vdash \neg \varphi$$
 iff $\forall y \sqsubseteq x$: $y \not\Vdash \varphi$

c.
$$x \Vdash \varphi \lor \psi$$
 iff $\forall y \sqsubseteq x \exists z \sqsubseteq y : z \Vdash \varphi$ or $z \Vdash \psi$

(25) a.
$$x \Vdash \varphi \land \psi$$
 iff $x \Vdash \varphi$ and $x \Vdash \psi$

b.
$$x \Vdash \varphi \lor \psi$$
 iff $x \Vdash \varphi$ or $x \Vdash \psi$

c.
$$x \Vdash \sim \varphi$$
 iff $x \not\Vdash \varphi$

 Therefore, non-persistent formulas cannot be reiterated in a subproof for → *I* which assumes persistence.

Additional Results

 $\sim \Diamond p \lor \sim \Diamond q$ is incompatible with $\Diamond p \land \Diamond q$: • $C[\sim \Diamond p \lor \sim \Diamond q] = C[\neg p] \cup C[\neg q]$ \sqrt{C} p/\bar{q} \bar{p}/q p/q \bar{p}/\bar{q} ā а pą ρq pq • $C[\Diamond p \land \Diamond q] = (C - C[\neg p]) \cap (C - C[\neg q])$ \sqrt{C} p/\bar{q} \bar{p}/q p/q \bar{p}/\bar{q} ā q рa pą ρq pą

 Under the alternative definition for 'can't be p', the two sentences fail to be strongly consistent because their intersection is {√C}.

Thank you!

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