Something from Nothing

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First theme: logic and change

- Many people think that logic matters for the study of what meaning is.
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- Many people think that logic matters for the study of what meaning is.
- Not many people think that logic matters for the study of how meaning changes.
First theme: logic and change

• Many people think that logic matters for the study of what meaning is.

• Not many people think that logic matters for the study of how meaning changes.

• (Of course, evolution has its own “logic”, but not in the sense of “valid arguments”.)
Second theme: fossils

“the main evidence I will adduce here comes from the structure of language as we see it today; I will look within modern language for traces of its past.” (Ray Jackendoff, *Foundations of Language*)
Second theme: fossils

“the main evidence I will adduce here comes from the structure of language as we see it today; I will look within modern language for traces of its past.” (Ray Jackendoff, *Foundations of Language*)

- Italian *né (nor)* derives from Latin *neque (nor)*: negation + *atque (and)*.
- English *nor* derives from negation + *or*. 
Second theme: fossils

“the main evidence I will adduce here comes from the structure of language as we see it today; I will look within modern language for traces of its past.” (Ray Jackendoff, *Foundations of Language*)

- Italian *né (nor)* derives from Latin *neque (nor):* negation + *atque (and).*
- English *nor* derives from negation + *or.*
- At lexical level we find a convergence to universal negatives, regardless of morphological history.
Third theme: negation

- Negation is not just Boolean negation.

\[ \models \neg (\varphi \land \neg \varphi) \]
\[ \models \varphi \lor \neg \varphi \]
Third theme:

- Negation is not just Boolean negation.
  \[\models \neg(\varphi \land \neg \varphi)\]
  \[\models \varphi \lor \neg \varphi\]

- Russell’s Thesis: “all forms of negation are reducible to a suitably placed it is not the case that”
  (Arthur Prior [16, 459]).
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  \[ \models \neg (\varphi \land \neg \varphi) \]
  \[ \models \varphi \lor \neg \varphi \]

- Russell’s Thesis: “all forms of negation are reducible to a suitably placed *it is not the case that*” (Arthur Prior [16, 459]).

- Today’s “Russellians”: Boolean negation + Scalar Implicatures (exh)
**First theme:** Logic explains aspects of language evolution.

**Second theme:** Fossil evidence of language change in language use.

**Third theme:** Negation is sometimes non-Boolean.
First theme: Logic explains aspects of language evolution.

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Third theme: Negation is sometimes non-Boolean.

- Homogeneity, NPIs, Lexical gaps: not separate phenomena but stages of a common history.
First theme: Logic explains aspects of language evolution.

Second theme: Fossil evidence of language change in language use.

Third theme: Negation is sometimes non-Boolean.

- Homogeneity, NPIs, Lexical gaps: not separate phenomena but stages of a common history.

- Different “grades” of cognitive involvement:

<table>
<thead>
<tr>
<th>BSML</th>
<th>Classical Logic</th>
</tr>
</thead>
<tbody>
<tr>
<td>BSML+</td>
<td>Aloni (2022)</td>
</tr>
<tr>
<td>BSML×</td>
<td></td>
</tr>
<tr>
<td>BSML+×</td>
<td>w/ T. Roberts</td>
</tr>
<tr>
<td>BSML*</td>
<td>w/ L. Incurvati w/ F. Carcassi</td>
</tr>
</tbody>
</table>
Overview

Background

Homogeneity

NPIs

Lexical gaps

Varieties of negation

Conclusions and outlook
Background
• We can add restrictions to BSML to study the effect of cognitive bias.
Neglect Zero

- We can add restrictions to BSML to study the effect of cognitive bias.
- NZ is the hypothesis that speakers have a tendency to disregard empty configurations in the interpretation of formulas. (Aloni, [1])
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NZ is a cognitive simplicity bias.
Neglect Zero

- We can add restrictions to BSML to study the effect of cognitive bias.
- NZ is the hypothesis that speakers have a tendency to disregard empty configurations in the interpretation of formulas. (Aloni, [1])
- NZ is a cognitive simplicity bias.

(1) All cats are black.
- On top of NZ we also assume One-and-Done Sampling (Availability heuristics) [22].
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• O&D-Sampling is the hypothesis that speakers may be sloppy in verification tasks that require a sampling method.

(1) All cats are black.
Homogeneity
Homogeneity inferences arise with definite plurals, conjunctions, and other constructions [3, 13, 14, 19, 21].

(2)  

a. Our questions were answered.  ⇒ Every question was answered.
b. Our questions were not answered.  ⇒ No question was answered.

(3)  

a. They saw Delta and Omicron coming.  ⇒ Delta was expected.
b. They didn’t see Delta and Omicron coming.  ⇒ Delta was not expected.

Homogeneity effects are “all or nothing” readings.

Not for today: Non-maximality.
⇒ Definite plurals have $\forall$ semantics.

Homogeneity effects arise whenever negation expresses contrariety ($\lnot \exists$), instead of contradiction ($\lnot \forall$): if so, negation is not Boolean.

Hom is modeled in BSML $\times$. 
NPIs
(4) a. Even John left. \[\begin{aligned}
\text{John left.} & \quad (i) \\
\text{John was the least likely to leave.} & \quad (ii) \\
\text{Someone else left.} & \quad (iii) \\
\text{Everyone else left.} & \quad (iv)
\end{aligned}\]
(4) a. Even John left. $\Rightarrow$ \\
\begin{align*}
\text{John left.} & \quad (i) \\
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\end{align*}

- Horn’s classic analysis [9]: (i)–(iii) are presuppositions.
- (iv) is not predicted but see [6, 8, 12], a.o.
(5) a. Even John left. $\models \begin{cases} \text{John left.} & (i) \\ \text{John was the least likely to leave.} & (ii) \\ \text{Someone else left.} & (iii) \\ \text{Everyone else left.} & (iv) \end{cases}$

b. Not even John left. $\models \begin{cases} \text{John did not leave.} & (i) \\ \text{John was the most likely to leave.} & (ii) \\ \text{Someone else did not leave.} & (iii) \\ \text{No one else left.} & (iv) \end{cases}$

- Horn’s classic analysis [9]: (i)–(iii) are presuppositions.
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(5)  

a. Even John left.  
\[ \begin{align*} 
\text{John left.} & \quad (i) \\
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\text{Everyone else left.} & \quad (iv) 
\end{align*} \]

b. Not even John left.  
\[ \begin{align*} 
\text{John did not leave.} & \quad (i) \\
\text{John was the most likely to leave.} & \quad (ii) \\
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\end{align*} \]

- Horn's classic analysis [9]: \((i)\)–\((iii)\) are presuppositions.
- \((iv)\) is not predicted but see \([6, 8, 12]\), a.o.
- None of the inferences in \((3b)\) are predicted.
Even

⇒ *Not even* is the (non-Boolean) negation of *even*.

- *Even* has ∀ “homogeneous” semantics, modeled in BSML\(+\times\).
- *Not even* is “on the way” to lexicalization (and it is lexicalized in other languages, such as Greek and Italian).
- The biases modeled in BSML\(+\times\) can be suspended, blocking universal and additive inferences. (Rullman’s data)
(6)  
   a. I didn’t see any cake.
   b. *I saw any cake.
(6)  
   a. I didn’t see any cake.
   b. *I saw any cake.

(7)  
   a. Any student can do it.  \[ \Rightarrow a \text{ can do it} \text{ and } b \text{ can do it} \ldots \]
   b. Any worker who arrived late was fired.
⇒ *any* ≈ *EVEN ONE* but *obligatorily* in BSML+× (cf. Lahiri [15], also: Dayal 1998, van Rooij 2008)

- Both *any* |= ∀ and ¬*any* |= ¬∃.
⇒ any ≈ EVEN ONE but **obligatorily** in BSML+× (cf. Lahiri [15], also: Dayal 1998, van Rooij 2008)

- Both any ⊨ ∀ and ¬any⊨ ¬∃.
- any is a “homogeneous” ∀, so what looks like FC is just downward monotonicity: any ∨□F ≈ ∀x□F ⊨ □Fa ∧ □Fb....
⇒ *any* ≈ EVEN ONE but **obligatorily** in BSML+ (cf. Lahiri [15], also: Dayal 1998, van Rooij 2008)

- Both *any* ⊨ ∀ and ¬*any* |= ¬∃.
- *any* is a “homogeneous” ∀, so what looks like FC is just downward monotonicity: *any* x◊F ∼ ∀x◊F |= ◊Fa ∧ ◊Fb....
- The logic is classical and ¬*any* |= ¬∃, hence ∃ |= *any*. Moreover, *any* ⊨ ∀. Therefore, *any* is not harmonious (Prior 1960, Belnap 1961, Dummett 1991).

ϕ ⊨ ϕ tonk ψ  \[ \varphi \text{ tonk } \psi \models \psi \]
⇒ any ≈ **EVEN ONE** but **obligatorily** in BSML+× (cf. Lahiri [15], also: Dayal 1998, van Rooij 2008)

- Both any ⪰ ∀ and ¬any ⪰ ¬∃.
- *any* is a “homogeneous” ∀, so what looks like FC is just downward monotonicity: any \(x\downarrow F \approx \forall x \downarrow F \models \downarrow Fa \land \downarrow Fb\).
- The logic is classical and ¬any ⪰ ¬∃, hence ∃ ⪰ any. Moreover, any ⪰ ∀. Therefore, *any* is not *harmonious* (Prior 1960, Belnap 1961, Dummett 1991).

\[
\phi \models \phi \text{ tonk } \psi \quad \phi \text{ tonk } \psi \models \psi
\]

- Disharmony is avoided by preventing any from being asserted, unless restricted (e.g. by a relative clause): any \(x\) is \(F \not\models \forall xFx\).
Lexical gaps
Lexical gaps

- **all**
- **always**
- **both**
- **and**
- **never**
- **never**
- **nor**
- **no**
- **some**
- **sometimes**
- **either**
- **or**

The diagram shows:

- Contraries: **∀, ∧**
- Contradictories: **¬∃, ¬∨**

* + and *nall* + *nalways* + *noth* + *nand*
Lexical gaps

- The Boolean negation of *some* can lexicalize as *no*, but the Boolean negation of *all* does not lexicalize as *nall*. Same for *or/nor* but not *and/nand*. (Horn, a.o.)
Lexical gaps

⇒ Lexical negation $n$- is not Boolean.

- In BSML*, the Boolean negations of every ($\forall$) and and ($\land$) cannot be expressed (but $\neg \exists$ and $\neg \lor$ can).

- Lexical gaps in the domain of connectives and quantifiers are explained by **undefinability** results in the (non-classical) logic of mental representations: the logic of the lexicon. [4, 10, 18]
An English fossil: *ever*.

1. Always, at all times; in all cases. Now mostly replaced by *forever adv.*

   i.1a. Throughout all time, eternally; throughout all past or all future time; forever; perpetually (often hyperbolically or in relative sense: throughout one’s life, etc.). Also emphatically *ever and ever*, †*ever ay*, †*ever and o*. Now archaic and literary.
1549 That wee may euer liue with thee in the worlde to come.

Booke of Common Prayer (STC 16267) Firste Daie of Lente f. xxxiii*
That wee may euer liue with thee in the worlde to come.

`Booke of Common Prayer (STC 16267) Firste Daie of Lente f. xxxiii`
Ever was a universal quantifier over times around 1550-1600.
A fossil

- **Ever** was a universal quantifier over times around 1550-1600.
- Traces of *ever* as a universal quantifier still exist (*forever*).
• Ever was a universal quantifier over times around 1550-1600.
• Traces of ever as a universal quantifier still exist (forever).
• Ever is now an NPI:
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\[ a. \text{No one would ever question my father.} \\
   b. \text{Have you ever met my father?} \]
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b. Have you ever met my father?
• Italian né (nor) derives from Latin neque (nor): negation + atque (and).
• English nor derives from negation + or.
• English never derives from negation + ever.
• **Ever** was a universal quantifier over times around 1550-1600.
• Traces of *ever* as a universal quantifier still exist (*forever*).
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• Italian *né* (*nor*) derives from Latin *neque* (*nor*): negation + *atque* (*and*).
• English *nor* derives from negation + *or*.
• English *never* derives from negation + *ever*.
• At lexical level we find a convergence to universal negatives, regardless of morphological history.
Varieties of negation
Bilateral State-Based Modal Logic

- Developed in Maria Aloni’s [1] account of Free Choice.
- Background: team logic [23], inquisitive semantics [5], alternative semantics [2], situation semantics [11], truth-maker semantics [7], bilateral logic [20].
- A classical modal logic as background, on top of which we may define different kinds of cognitive biases (here, two).
- A weak Neglect Zero bias (\(\text{NE}\)) accounts for Free Choice Disjunction.
Bilateral State-Based Modal Logic

Semantics

\[ M, s \models Pt \iff \forall i \in s : I_i(t) \in I_i(P, w_i) \]
\[ M, s \models Pt \iff \forall i \in s : I_i(t) \notin I_i(P, w_i) \]
\[ M, s \models \neg \varphi \iff M, s \not\models \varphi \]
\[ M, s \models \neg \varphi \iff M, s \not\models \varphi \]
\[ M, s \models \varphi \land \psi \iff M, s \models \varphi \land M, s \models \psi \]
\[ M, s \models \varphi \land \psi \iff \exists t, t' : s = t \cup t' \land M, t \models \varphi \land M, t' \models \psi \]
\[ M, s \models \varphi \lor \psi \iff \exists t, t' : s = t \cup t' \land M, t \models \varphi \land M, t' \models \psi \]
\[ M, s \models \varphi \lor \psi \iff M, s \models \varphi \land M, s \models \psi \]
\[ M, s \models \forall x \varphi \iff \forall d : M, s_d^x \models \varphi \]
\[ M, s \models \forall x \varphi \iff \exists S : s = \cup S \land \forall t \in S : \exists d : M, t_d^x \models \varphi \land \forall d' \exists t' \in S : t_d'^x \models \varphi \]
### Bilateral State-Based Modal Logic

#### Semantics

- $M, s \models Pt \iff \forall i \in s : l_i(t) \in l_i(P, w_i)$
- $M, s \not\models Pt \iff \forall i \in s : l_i(t) \notin l_i(P, w_i)$
- $M, s \models \neg \varphi \iff M, s \not\models \varphi$
- $M, s \models \neg \varphi \iff M, s \not\models \varphi$
- $M, s \models \varphi \land \psi \iff M, s \models \varphi \land M, s \models \psi$
- $M, s \models \varphi \land \psi \iff \exists t, t' : s = t \cup t' \land M, t \models \varphi \land M, t' \models \psi$
- $M, s \models \varphi \lor \psi \iff \exists t, t' : s = t \cup t' \land M, t \models \varphi \land M, t' \models \psi$
- $M, s \models \varphi \lor \psi \iff M, s \models \varphi \land M, s \models \psi$
- $M, s \models \forall x \varphi \iff \forall d : M, s^x_d \models \varphi$
- $M, s \models \forall x \varphi \iff \exists S : s = \cup S \land \forall t \in S : \exists d : M, t^x_d \models \varphi \land \forall d' \exists t' \in S : t'^x_{d'} \models \varphi$
• In BSML, negation is Boolean.
• BSML translates into Classical Modal Logic over the language of ML.
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• BSML translates into Classical Modal Logic over the language of ML.
• BSML+ is obtained as follows (Aloni 2022):

\[
M, s \models \text{NE} \iff s \neq \emptyset \\
M, s \models \text{NE} \iff s = \emptyset \\
M, s \models p^+ \iff M, s \models p \land \text{NE} \\
M, s \models p^+ \iff M, s \models p \land \text{NE} \\
M, s \models [O^n(\varphi_1, \ldots, \varphi_n)]^+ \iff M, s \models [O^n(\varphi_1^+, \ldots, \varphi_n^+)] \land \text{NE} \\
M, s \models [O^n(\varphi_1, \ldots, \varphi_n)]^+ \iff M, s \models [O^n(\varphi_1^+, \ldots, \varphi_n^+)] \land \text{NE}
\]

with $O \in \{\neg, \lor, \land, \forall, \exists\}$
- BSML is obtained as follows:

\[
M, s \models NS \iff s = \emptyset \& \forall t \subseteq s : M, t \models NS \rightarrow t = s
\]

\[
M, s \not\models NS \iff s \neq \emptyset \& \forall t \subseteq s : M, t \not\models NS \rightarrow t = s
\]

\[
M, s \models p^\times \iff M, s \models p \lor NS
\]

\[
M, s \models p^\times \iff M, s \models p \lor NS
\]

\[
M, s \models [O^n(\varphi_1, \ldots, \varphi_n)]^\times \iff M, s \models [O^n(\varphi_1^\times, \ldots, \varphi_n^\times)] \lor NS
\]

\[
M, s \not\models [O^n(\varphi_1, \ldots, \varphi_n)]^\times \iff M, s \models [O^n(\varphi_1^\times, \ldots, \varphi_n^\times)] \lor NS
\]

with \( O \in \{ \neg, \lor, \land, \forall, \exists \} \)
BSML× is obtained as follows:

\[ M, s \models \text{NS} \iff s = \emptyset \land \forall t \subseteq s : M, t \models \text{NS} \rightarrow t = s \]
\[ M, s \models \text{NS} \iff s \neq \emptyset \land \forall t \subseteq s : M, t \models \text{NS} \rightarrow t = s \]
\[ M, s \models p^\times \iff M, s \models p \lor \text{NS} \]
\[ M, s \models p^\times \iff M, s \models p \lor \text{NS} \]
\[ M, s \models [O^n(\varphi_1, \ldots, \varphi_n)]^\times \iff M, s \models [O^n(\varphi_1^\times, \ldots, \varphi_n^\times)] \lor \text{NS} \]
\[ M, s \models [O^n(\varphi_1, \ldots, \varphi_n)]^\times \iff M, s \models [O^n(\varphi_1^\times, \ldots, \varphi_n^\times)] \lor \text{NS} \]

with \( O \in \{ \lnot, \lor, \land, \forall, \exists \} \)

- **Neglect Zero**
- BSML$\times$ is obtained as follows:

$$M, s \models \text{NS} \iff s = \emptyset \& \forall t \subseteq s : M, t \models \text{NS} \rightarrow t = s$$

$$M, s \not\models \text{NS} \iff s \neq \emptyset \& \forall t \subseteq s : M, t \models \text{NS} \rightarrow t = s$$

$$M, s \models p^\times \iff M, s \not\models p \lor \text{NS}$$

$$M, s \not\models p^\times \iff M, s \not\models p \lor \text{NS}$$

$$M, s \models [O^n(\varphi_1, \ldots, \varphi_n)]^\times \iff M, s \models [O^n(\varphi_1^\times, \ldots, \varphi_n^\times)] \lor \text{NS}$$

$$M, s \not\models [O^n(\varphi_1, \ldots, \varphi_n)]^\times \iff M, s \not\models [O^n(\varphi_1^\times, \ldots, \varphi_n^\times)] \lor \text{NS}$$

with $O \in \{\neg, \lor, \land, \forall, \exists\}$

- Neglect Zero

- One-and-Done Sampling
BSML× implements NZ+O&D: if a state rejects something, it is equivalent to any of its (nonempty) substates that reject anything.

- Intuitively, states do not “split” under negation.
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- Intuitively, states do not “split” under negation.

\[
M, s \models \varphi \land \psi \text{ iff } M, s \models \varphi \land M, s \models \psi
\]
\[
M, s \models \varphi \land \psi \text{ iff } \exists t, t' : s = t \cup t' \land M, t \models \varphi \land M, t' \models \psi
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M, s \models \varphi \land \psi \text{ iff } \exists t, t' : s = t \cup t' \land M, t \models \varphi \land M, t' \models \psi
\]

\[
s \models - (p \land q) \text{ and } s \not\models [- (p \land q)]^* \\
s' \models - (p \land q) \text{ and } s' \not\models [- (p \land q)]^* \\
s'' \models - (p \land q) \text{ and } s'' \not\models [- (p \land q)]^*
\]
BSML× implements NZ+O&D: if a state rejects something, it is equivalent to any of its (nonempty) substates that reject anything.

- Intuitively, states do not “split” under negation.

\[
M, s \models \varphi \wedge \psi \iff M, s \models \varphi \land M, s \models \psi \\
M, s \models \varphi \wedge \psi \iff \exists t, t' : s = t \cup t' \land M, t \models \varphi \land M, t' \models \psi
\]

\[
s \models \neg(p \wedge q) \text{ and } s \not\models [\neg(p \wedge q)]^\times \\
s' \models \neg(p \wedge q) \text{ and } s' \models [\neg(p \wedge q)]^\times \\
s'' \models \neg(p \wedge q) \text{ and } s'' \not\models [\neg(p \wedge q)]^\times
\]

\[
[\neg(p \wedge q)]^\times \models \neg p \wedge \neg q
\]
$M, s \models \forall x \varphi$ iff $\forall d : M, s^d \models \varphi$

$M, s \models \forall x \varphi$ iff $\exists S : s = \cup S$ & $\forall t \in S : \exists d : M, t^d \models \varphi$ & $\forall d' \exists t' \in S : t'^d' \models \varphi$
BSML

$M, s \models \forall x \varphi \iff \forall d : M, s_d^x \models \varphi$

$M, s \models \forall x \varphi \iff \exists S : s = \cup S \& \forall t \in S : \exists d : M, t_d^x \models \varphi \& \forall d' \exists t'_{d'} : t_{d'}^x \models \varphi$

$\left[ \neg \forall x F : G \right]^x \models \neg \exists x F : G$
\[ M, s \models \forall x \varphi \ \text{iff} \ \forall d : M, s^d \models \varphi \]
\[ M, s \models \forall x \varphi \ \text{iff} \ \exists S : s = \cup S \ \& \ \forall t \in S : \exists d : M, t^d \models \varphi \ \& \ \forall d' \exists t' \in S : t'^{d'} \models \varphi \]

\[
[-\forall x F : G]^\times \models \neg \exists x F : G
\]

- BSML× accounts for Homogeneity assuming definite plurals have \( \forall \) semantics.
Lexicalizability

- BSML does not predict lexical gaps (*nand and *nall are definable);
- BSML is obtained as follows:
Lexicalizability

• BSML\(\times\) does not predict lexical gaps (*\textit{nand} and *\textit{nall} are definable);

• BSML\(\star\) is obtained as follows:

\[
M, s \models \text{LN} \iff s \neq \emptyset \& \forall t \subseteq s : M, t \models \text{LN} \rightarrow t = s
\]

\[
M, s \not\models \text{LN} \iff s \neq \emptyset \& \forall t \subseteq s : M, t \not\models \text{LN} \rightarrow t = s
\]

\[
M, s \not\models p^\star \iff M, s \not\models p \& M, s \models \text{LN}
\]

\[
M, s \models p^\star \iff M, s \models p \& M, s \models \text{LN}
\]

\[
M, s \models [O^n(\varphi_1, \ldots, \varphi_n)]^\star \iff M, s \models O^n(\varphi_1^\star, \ldots, \varphi_n^\star) \& M, s \models \text{LN}
\]

\[
M, s \not\models [O^n(\varphi_1, \ldots, \varphi_n)]^\star \iff M, s \not\models O^n(\varphi_1^\star, \ldots, \varphi_n^\star) \& M, s \not\models \text{LN}
\]

with \(O \in \{\neg, \lor, \land, \forall, \exists\}\)
Lexicalizability

The operators and relations highlighted in orange are definable in BSML*. The others are not definable.
Lexicalizability

The operators and relations highlighted in orange are definable in BSML*. The others are not definable.

\[
\begin{align*}
\text{all} & \quad \text{always} \\
\text{always} & \quad \text{both} \\
\text{both} & \quad \text{and} \\
\forall, \wedge & \quad \neg \exists, \neg \vee \\
\text{contraries} & \\
\text{contradictories} & \\
\text{A} & \quad \text{E} \\
\text{I} & \\
\text{O} & \neg \forall, \neg \wedge \\
\text{some} & \quad \text{sometimes} \\
\text{sometimes} & \quad \text{either} \\
\text{either} & \quad \text{or} \\
\exists, \vee & \quad \neg \forall, \neg \wedge \\
\text{no} & \quad \text{never} \\
\text{never} & \quad \text{neither} \\
\text{neither} & \quad \text{nor} \\
\text{r} & \quad \neg \text{p} \quad \neg \text{q} \\
\text{s} & \quad \text{p} \quad \text{q} \\
\text{r} & \quad \neg \text{p} \quad \neg \text{x} \neg \varphi \\
\text{O} & \quad \neg \forall \text{x} \varphi \\
\text{I} & \quad \neg \exists \text{x} \varphi \\
\text{contraries} & \\
\text{contradictories} &
\end{align*}
\]
Lexicalizability

- BSML* implements a very strong version of Neglect Zero + One-and-Done Sampling.
Lexicalizability

- BSML⋆ implements a very strong version of Neglect Zero + One-and-Done Sampling.

Negation in natural language is many things (contrary to Russell’s Thesis).
  - Lexical negatives are defined in BSML⋆.
  - Lexicalization goes by BSML⋆ rather than BSML because BSML⋆ is cognitively simpler.
**Varieties of negation**

### BSML+

| \(|s \models \text{NE} \iff s \neq \emptyset| \) | \(|s \models p^+ \iff s \models p \land \text{NE}| \) |
| --- | --- |
| \(|s \models \text{NE} \iff s = \emptyset| \) | \(|s \models p^+ \iff s \models p \land \text{NE}| \) |
| \(|s \models [O^n(\varphi_1, \ldots, \varphi_n)]^+ \iff s \models [O^n(\varphi_1^+, \ldots, \varphi_n^+)] \land \text{NE}| \) |
| \(|s \models [O^n(\varphi_1, \ldots, \varphi_n)]^+ \iff s \models [O^n(\varphi_1^+, \ldots, \varphi_n^+)] \land \text{NE}| \) |

### BSML×

| \(|s \models \text{NS} \iff s = \emptyset \land \forall t \subseteq s : t \models \text{NS} \rightarrow t = s| \) | \(|s \models p^\times \iff s \models p \lor \text{NS}| \) |
| --- | --- |
| \(|s \models \text{NS} \iff s \neq \emptyset \land \forall t \subseteq s : t \models \text{NS} \rightarrow t = s| \) | \(|s \models p^\times \iff s \models p \lor \text{NS}| \) |
| \(|s \models [O^n(\varphi_1, \ldots, \varphi_n)]^\times \iff s \models [O^n(\varphi_1^\times, \ldots, \varphi_n^\times)] \lor \text{NS}| \) |
| \(|s \models [O^n(\varphi_1, \ldots, \varphi_n)]^\times \iff s \models [O^n(\varphi_1^\times, \ldots, \varphi_n^\times)] \lor \text{NS}| \) |

### BSML⋆

| \(|s \models \text{LN} \iff s \neq \emptyset \land \forall t \subseteq s : t \models \text{LN} \rightarrow t = s| \) | \(|s \models p^* \iff s \models p \land s \models \text{LN}| \) |
| --- | --- |
| \(|s \models \text{LN} \iff s \neq \emptyset \land \forall t \subseteq s : t \models \text{LN} \rightarrow t = s| \) | \(|s \models p^* \iff s \models p \land s \models \text{LN}| \) |
| \(|s \models [O^n(\varphi_1, \ldots, \varphi_n)]^* \iff s \models O^n(\varphi_1^*, \ldots, \varphi_n^*) \land s \models \text{LN}| \) |
| \(|s \models [O^n(\varphi_1, \ldots, \varphi_n)]^* \iff s \models O^n(\varphi_1^*, \ldots, \varphi_n^*) \land s \models \text{LN}| \) |
• In BSML+$\times$, which is exactly what you think it is.
• In BSML+×, which is exactly what you think it is.
• Add to BSML+× an operator: \( \text{EVEN } a\varphi \) asserts that \( a\varphi \) and that \( a\varphi \) is at the bottom of the \( \leq \)-ordering of focus alternatives.

\[
\text{EVEN } a\varphi := a\varphi \land \forall x \in Alt(a) : a\varphi \leq a\varphi
\]
Even with T. Roberts (UU)

- In BSML+$\times$, which is exactly what you think it is.
- Add to BSML+$\times$ an operator: \( \text{EVEN } \varphi \) asserts that \( \varphi \) and that \( \varphi \) is at the bottom of the \( \leq \)-ordering of focus alternatives.

\[
\text{EVEN } \varphi := \varphi \land \forall x \in \text{Alt}(a) : \varphi a \leq \varphi x
\]

- Assume MEP: If \( A, B \in \text{Alt} \), \( A \) is true, and \( A \leq B \), then \( B \) is true.

For all \( A, B \in \text{Alt} : (A \leq B) \land A \models B \)
• In BSML+$\times$, which is exactly what you think it is.
• Add to BSML+$\times$ an operator: \textsc{even} $\varphi_a$ asserts that $\varphi_a$ and that $\varphi_a$ is at the bottom of the $\leq$-ordering of focus alternatives.

\[
\text{EVEN } \varphi_a := \varphi_a \land \forall x \in Alt(a) : \varphi_a \leq \varphi_x
\]

• Assume MEP: If $A, B \in Alt$, $A$ is true, and $A \leq B$, then $B$ is true.

For all $A, B \in Alt : (A \leq B) \land A \models B$

\[
[EVEN \varphi_a]^+ \models \varphi_a
\]  

(i)
Even with T. Roberts (UU)

- In BSML+×, which is exactly what you think it is.
- Add to BSML+× an operator: \( \text{EVEN} \ a \varphi \) asserts that \( \varphi a \) and that \( \varphi a \) is at the bottom of the \( \leq \)-ordering of focus alternatives.

\[
\text{EVEN} \ a \varphi := \varphi a \land \forall x \in \text{Alt}(a) : \varphi a \leq \varphi x
\]

- Assume MEP: If \( A, B \in \text{Alt} \), \( A \) is true, and \( A \leq B \), then \( B \) is true.

For all \( A, B \in \text{Alt} \) : \( (A \leq B) \land A \vdash B \)

\[
[\text{EVEN} \ a \varphi]^+ \vdash \varphi a
\]

\[
[\text{EVEN} \ a \varphi]^+ \vdash \forall x \in \text{Alt}(a) : \varphi a \leq \varphi x
\]
Even with T. Roberts (UU)

- In BSML $+\times$, which is exactly what you think it is.
- Add to BSML $+\times$ an operator: $\text{EVEN } a \varphi$ asserts that $\varphi a$ and that $\varphi a$ is at the bottom of the $\leq$-ordering of focus alternatives.

$$\text{EVEN } a \varphi := \varphi a \land \forall x \in \text{Alt}(a) : \varphi a \leq \varphi x$$

- Assume MEP: If $A, B \in \text{Alt}$, $A$ is true, and $A \leq B$, then $B$ is true.

For all $A, B \in \text{Alt} : (A \leq B) \land A \models B$

1. $[\text{EVEN } a \varphi]^+ \times \models \varphi a$  
2. $[\text{EVEN } a \varphi]^+ \times \models \forall x \in \text{Alt}(a) : \varphi a \leq \varphi x$  
3. $[\text{EVEN } a \varphi]^+ \times \models \exists x \in \text{Alt}(a) : \varphi x$ (by $+$)
Even with T. Roberts (UU)

- In BSML+×, which is exactly what you think it is.
- Add to BSML+× an operator: \( \text{EVEN} \, \varphi \) asserts that \( \varphi a \) and that \( \varphi a \) is at the bottom of the \( \leq \)-ordering of focus alternatives.

\[
\text{EVEN} \, \varphi := \varphi a \land \forall x \in \text{Alt}(a) : \varphi a \leq \varphi x
\]

- Assume MEP: If \( A, B \in \text{Alt} \), \( A \) is true, and \( A \leq B \), then \( B \) is true.

For all \( A, B \in \text{Alt} : (A \leq B) \land A \models B \\

\begin{align*}
\lbrack \text{EVEN} \, \varphi \rbrack^{+\times} & \models \varphi a \quad (i) \\
\lbrack \text{EVEN} \, \varphi \rbrack^{+\times} & \models \forall x \in \text{Alt}(a) : \varphi a \leq \varphi x \quad (ii) \\
\lbrack \text{EVEN} \, \varphi \rbrack^{+\times} & \models \exists x \in \text{Alt}(a) : \varphi x \quad (\text{by +}) \quad (iii) \\
\lbrack \text{EVEN} \, \varphi \rbrack^{+\times} & \models \forall x \in \text{Alt}(a) : \varphi x \quad (\text{by + and MEP}) \quad (iv)
\end{align*}

• In BSML+$\times$, which is exactly what you think it is.

• Add to BSML+$\times$ an operator: EVEN $a\varphi$ asserts that $\varphi a$ and that $\varphi a$ is at the bottom of the $\leq$-ordering of focus alternatives.

$$\text{EVEN } a\varphi := \varphi a \land \forall x \in Alt(a) : \varphi a \leq \varphi x$$

• Assume MEP: If $A, B \in Alt$, $A$ is true, and $A \leq B$, then $B$ is true.

For all $A, B \in Alt : (A \leq B) \land A \models B$

\begin{align*}
[EVEN a\varphi]^+ \models & \varphi a & \text{(i)} \\
[EVEN a\varphi]^+ \models & \forall x \in Alt(a) : \varphi a \leq \varphi x & \text{(ii)} \\
[EVEN a\varphi]^+ \models & \exists x \in Alt(a) : \varphi x & \text{(by +)} & \text{(iii)} \\
[EVEN a\varphi]^+ \models & \forall x \in Alt(a) : \varphi x & \text{(by + and MEP)} & \text{(iv)}
\end{align*}

(2) a. Even John left. $\models$

$$\begin{cases}
\text{John left.} \quad & \text{(i)} \\
\text{John was the least likely to leave.} \quad & \text{(ii)} \\
\text{Someone else left.} \quad & \text{(iii)} \\
\text{Everyone else left.} \quad & \text{(iv)}
\end{cases}$$
• In BSML+$\times$, which is exactly what you think it is.

• Add to BSML+$\times$ an operator: $\text{EVEN } \varphi$ asserts that $\varphi a$ and that $\varphi a$ is at the bottom of the $\leq$-ordering of focus alternatives.

$$\text{EVEN } \varphi := \varphi a \land \forall x \in \text{Alt}(a) : \varphi a \leq \varphi x$$

• Assume MEP: If $A, B \in \text{Alt}$, $A$ is true, and $A \leq B$, then $B$ is true.

For all $A, B \in \text{Alt}$ : $(A \leq B) \land A \models B$

$[\text{EVEN } \varphi]^{+\times} \models \varphi a$ ............................................... (i)

$[\text{EVEN } \varphi]^{+\times} \models \forall x \in \text{Alt}(a) : \varphi a \leq \varphi x$ .............................................. (ii)

$[\text{EVEN } \varphi]^{+\times} \models \exists x \in \text{Alt}(a) : \varphi x$ (by $+\times$) .............................................. (iii)

$[\text{EVEN } \varphi]^{+\times} \models \forall x \in \text{Alt}(a) : \varphi x$ (by $+\times$ and MEP) .............................................. (iv)

(2) a. Even John left. $\models$

$$\begin{cases} 
\text{John left.} & (i) \\
\text{John was the least likely to leave.} & (ii) \\
\text{Someone else left.} & (iii) \\
\text{Everyone else left.} & (iv)
\end{cases}$$

• $+\times$ is a bias: if absent, (iii) and (iv) do not follow (cf. Rullmann [17], a.o.)
• Moreover, in BSML+$\times$ we obtain \([\neg \text{EVEN } a \varphi]^+\times \equiv [\text{EVEN } a \neg \varphi]^+\times\).
• Moreover, in BSML+× we obtain $[\neg \text{EVEN } a \varphi]^+\times \equiv [\text{EVEN } a \neg \varphi]^+\times$. 
• Intuitively, $[\neg \forall x \varphi]^+\times \equiv [\neg \exists x \varphi]^+\times \equiv [\forall x \neg \varphi]^+\times$.
Even with T. Roberts (UU)

• Moreover, in BSML $\times$ we obtain $[\neg \text{EVEN } a\varphi]^x \equiv [\text{EVEN } a\neg \varphi]^x$.

• Intuitively, $[\neg \forall x \varphi]^x \equiv [\neg \exists x \varphi]^x \equiv [\forall x \neg \varphi]^x$

(2) b. Not even John left. $\models \begin{cases} 
\text{John did not leave.} \quad (i) \\
\text{John was the most likely to leave.} \quad (ii) \\
\text{Someone else did not leave.} \quad (iii) \\
\text{No one else left.} \quad (iv) 
\end{cases}$
• In BSML+× *even* may be under NZ+O&D but doesn’t have to. What happens if an operator **lexicalizes** NZ+O&D?

\[ M, s \models \text{ANY } \forall x \phi \text{ iff } \forall d : M, s^x_d \models \phi^{+x} \]

\[ M, s \models \text{ANY } \exists S : S = \bigcup S \text{ & } \forall t \in S : \exists d : M, t^x_d \models \phi^{+x} \text{ & } \forall d' \exists t' \in S : t'^{l^x}_{d'} \models \phi^{+x} \]

\[ M, s \models \forall x \phi \text{ iff } \forall d : M, s^x_d \models \phi \]

\[ M, s \models \forall x \phi \text{ iff } \exists S : S = \bigcup S \text{ & } \forall t \in S : \exists d : M, t^x_d \models \phi \text{ & } \forall d' \exists t' \in S : t'^{l^x}_{d'} \models \phi \]
• In BSML+× even may be under NZ+O&D but doesn’t have to. What happens if an operator lexicalizes NZ+O&D?

\[ M, s \models \text{ANY } x \varphi \text{ iff } \forall d : M, s_d^x \models \varphi^{+×} \]

\[ M, s \models \text{ANY } x \varphi \text{ iff } \exists S : s = \cup S \text{ and } \forall t \in S : \exists d : M, t^x_d \models \varphi^{+×} \text{ and } \forall d' \exists t' \in S : t'^{t_x}_d' \models \varphi^{+×} \]

\[ M, s \models \forall x \varphi \text{ iff } \forall d : M, s_d^x \models \varphi \]

\[ M, s \models \forall x \varphi \text{ iff } \exists S : s = \cup S \text{ and } \forall t \in S : \exists d : M, t^x_d \models \varphi \text{ and } \forall d' \exists t' \in S : t'^{t_x}_d' \models \varphi \]

• Consequently, both \text{ANY } x \varphi \models \forall x \varphi \text{ and } \lnot \text{ANY } x \varphi \models \lnot \exists x \varphi
• In BSML+× even may be under NZ+O&D but doesn’t have to. What happens if an operator lexicalizes NZ+O&D?

\[ M, s \models \text{ANY } \forall x \varphi \iff \forall d : M, s^x_d \models \varphi^{+\times} \]

\[ M, s \models \text{ANY } \forall x \varphi \iff \exists S : s = \cup S \land \forall t \in S : \exists d : M, t^x_d \models \varphi^{+\times} \land \forall d' \exists t' \in S : t'^x_{d'} \models \varphi \]

\[ M, s \models \forall x \varphi \iff \forall d : M, s^x_d \models \varphi \]

\[ M, s \models \forall x \varphi \iff \exists S : s = \cup S \land \forall t \in S : \exists d : M, t^x_d \models \varphi \land \forall d' \exists t' \in S : t'^x_{d'} \models \varphi \]

• Consequently, both \( \text{ANY } \forall x \varphi \models \forall x \varphi \) and \( \neg \text{ANY } \forall x \varphi \models \neg \exists x \varphi \)

• But the speaker is not biased! The system is still classical, hence CONTRAPOSITION holds \( (\neg \varphi \models \neg \psi \implies \psi \models \varphi) \). Therefore:

\[ \exists x \varphi \models \text{ANY } x \varphi \models \forall x \varphi \]
• In BSML+$\times$ even may be under NZ+O&D but doesn’t have to. What happens if an operator **lexicalizes** NZ+O&D?

\[ M, s \models \text{ANY } x \varphi \iff \forall d : M, s_d^x \models \varphi^+ \times \]

\[ M, s \models \text{ANY } x \varphi \iff \exists S : s = \cup S \land \forall t \in S : \exists d : M, t_d^x \models \varphi^+ \times \land \forall d' \exists t' \in S : t'_d^x \models \varphi^+ \times \]

\[ M, s \models \forall x \varphi \iff \forall d : M, s_d^x \models \varphi \]

\[ M, s \models \forall x \varphi \iff \exists S : s = \cup S \land \forall t \in S : \exists d : M, t_d^x \models \varphi \land \forall d' \exists t' \in S : t'_d^x \models \varphi \]

• Consequently, both $\text{ANY } x \varphi \models \forall x \varphi$ and $\neg \text{ANY } x \varphi \models \neg \exists x \varphi$

• But the speaker is not biased! The system is still classical, hence **contraposition** holds ($\neg \varphi \models \neg \psi \implies \psi \models \varphi$). Therefore:

\[ \exists x \varphi \models \text{ANY } x \varphi \models \forall x \varphi \]

• *any* violates **harmony** (Prior 1960, Dummett 1991).
In BSML+$\times$ even may be under NZ+O&D but doesn’t have to. What happens if an operator \textit{lexicalizes} NZ+O&D?

\[ M, s \models \text{ANY } x \varphi \text{ iff } \forall d : M, s^x_d \models \varphi^{+\times} \]

\[ M, s \models \text{ANY } x \varphi \text{ iff } \exists S : S = \bigcup_S \text{ and } \forall t \in S : \exists d : M, t^x_d \models \varphi^{+\times} \text{ and } \forall d' : t'^t_{xS} : t'^t_{xS} \models \varphi^{+\times} \]

\[ M, s \models \forall x \varphi \text{ iff } \forall d : M, s^x_d \models \varphi \]

\[ M, s \models \forall x \varphi \text{ iff } \exists S : S = \bigcup_S \text{ and } \forall t \in S : \exists d : M, t^x_d \models \varphi \text{ and } \forall d' : t'^t_{xS} : t'^t_{xS} \models \varphi \]

Consequently, both \text{ANY } x \varphi \models \forall x \varphi \text{ and } \neg \text{ANY } x \varphi \models \neg \exists x \varphi

But the speaker is not biased! The system is still classical, hence \textit{contraposition} holds ($\neg \varphi \models \neg \psi \implies \psi \models \varphi$). Therefore:

\[ \exists x \varphi \models \text{ANY } x \varphi \models \forall x \varphi \]


Harmony is kept by preventing \textit{any} from occurring unrestricted in an assertion (upward monotonic contexts).
- In BSML+$\times$ even may be under NZ+O&D but doesn’t have to. What happens if an operator lexicalizes NZ+O&D?

\[ M, s \models \text{ANY } x \varphi \text{ iff } \forall d : M, s_d^x \models \varphi^{+\times} \]

\[ M, s \models \text{ANY } x \varphi \text{ iff } \exists S : s = \cup S \& \forall t \in S : \exists d : M, t_d^x \models \varphi^{+\times} \& \forall d' \exists t' \in S : t_{d'}^x \models \varphi^{+\times} \]

\[ M, s \models \forall x \varphi \text{ iff } \forall d : M, s_d^x \models \varphi \]

\[ M, s \models \forall x \varphi \text{ iff } \exists S : s = \cup S \& \forall t \in S : \exists d : M, t_d^x \models \varphi \& \forall d' \exists t' \in S : t_{d'}^x \models \varphi \]

- Consequently, both \text{ANY } x \varphi \models \forall x \varphi \text{ and } \neg \text{ANY } x \varphi \models \neg \exists x \varphi

- But the speaker is not biased! The system is still classical, hence CONTRAPOSITION holds (\neg \varphi \models \neg \psi \implies \psi \models \varphi). Therefore:

\[ \exists x \varphi \models \text{ANY } x \varphi \models \forall x \varphi \]


- Harmony is kept by preventing \textit{any} from occurring unrestricted in an assertion (upward monotonic contexts).

- \textit{any} in assertion is only possible if \text{ANY } x \varphi \not\models \forall x \varphi (restricted “universal” \textit{any})
Conclusions and outlook
Conclusion

First theme: Logic and change.
Conclusion

First theme: Logic and change.
Second theme: Linguistic fossils.
Conclusion

First theme: Logic and change.

Second theme: Linguistic fossils.

Third theme: Non-Boolean negation.
Conclusion

First theme: Logic and change.

Second theme: Linguistic fossils.

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- We begin with a classical modal logic as background: BSML.
Conclusion

First theme: Logic and change.

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Third theme: Non-Boolean negation.

- We begin with a classical modal logic as background: BSML.
- BSML accounts for Homogeneity inferences.

Conclusion

*First theme:* Logic and change.

*Second theme:* Linguistic fossils.

*Third theme:* Non-Boolean negation.

- We begin with a classical modal logic as background: BSML.
- BSML× accounts for Homogeneity inferences.
- BSML+×, if +× is not obligatory, accounts for *even*/not *even*.
Conclusion

*First theme:* Logic and change.

*Second theme:* Linguistic fossils.

*Third theme:* Non-Boolean negation.

- We begin with a classical modal logic as background: BSML.
- BSML× accounts for Homogeneity inferences.
- BSML+×, if +× is not obligatory, accounts for *even/not even*.
- BSML+×, if +× is obligatory, accounts for *any* (universal, NPI, and FC *any*).
First theme: Logic and change.

Second theme: Linguistic fossils.

Third theme: Non-Boolean negation.

- We begin with a classical modal logic as background: BSML.
- BSML\(\times\) accounts for Homogeneity inferences.
- BSML\(+\times\), if \(\times\) is not obligatory, accounts for even/not even.
- BSML\(+\times\), if \(\times\) is obligatory, accounts for any (universal, NPI, and FC any).
- BSML\(\ast\) accounts for lexical gaps.
Conclusion

First theme: Logic and change.

Second theme: Linguistic fossils.

Third theme: Non-Boolean negation.

- We begin with a classical modal logic as background: BSML.
- BSML accounts for Homogeneity inferences.
- BSML, if + × is not obligatory, accounts for even/not even.
- BSML, if + × is obligatory, accounts for any (universal, NPI, and FC any).
- BSML accounts for lexical gaps.
- Homogeneity is the manifestation of a cognitive simplicity bias.
Conclusion

First theme: Logic and change.

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• We begin with a classical modal logic as background: BSML.
• BSML× accounts for Homogeneity inferences.
• BSML+×, if +× is not obligatory, accounts for even/not even.
• BSML+×, if +× is obligatory, accounts for any (universal, NPI, and FC any).
• BSML× accounts for lexical gaps.
• Homogeneity is the manifestation of a cognitive simplicity bias.
• In the long (evolutionary) run, simplicity pressure explains the lexicalization path of even/not even, and then of any: lexical negatives “in the making”
Conclusion

First theme: Logic and change.

Second theme: Linguistic fossils.

Third theme: Non-Boolean negation.

- We begin with a classical modal logic as background: BSML.
- BSML accounts for Homogeneity inferences.
- BSML, if + is not obligatory, accounts for even/not even.
- BSML, if + is obligatory, accounts for any (universal, NPI, and FC any).
- BSML accounts for lexical gaps.
- Homogeneity is the manifestation of a cognitive simplicity bias.
- In the long (evolutionary) run, simplicity pressure explains the lexicalization path of even/not even, and then of any: lexical negatives “in the making”
- and eventually, the same constraint explains lexical gaps.
## Varieties of negation

### BSML+

- \( s \models \text{NE} \iff s \neq \emptyset \)  
  \( s \models p^+ \iff s \models p \land \text{NE} \)
- \( s \models \text{NE} \iff s = \emptyset \)  
  \( s \models p^+ \iff s \models p \land \text{NE} \)
- \( s \models [O^n(\varphi_1, \ldots, \varphi_n)]^+ \iff s \models [O^n(\varphi_1^+, \ldots, \varphi_n^+)] \land \text{NE} \)
- \( s \models [O^n(\varphi_1, \ldots, \varphi_n)]^+ \iff s \models [O^n(\varphi_1^+, \ldots, \varphi_n^+)] \land \text{NE} \)

### BSML×

- \( s \models \text{NS} \iff s = \emptyset \land \forall t \subseteq s : t \models \text{NS} \rightarrow t = s \)  
  \( s \models p^\times \iff s \models p \lor \text{NS} \)
- \( s \models \text{NS} \iff s \neq \emptyset \land \forall t \subseteq s : t \models \text{NS} \rightarrow t = s \)  
  \( s \models p^\times \iff s \models p \lor \text{NS} \)
- \( s \models [O^n(\varphi_1, \ldots, \varphi_n)]^\times \iff s \models [O^n(\varphi_1^\times, \ldots, \varphi_n^\times)] \lor \text{NS} \)
- \( s \models [O^n(\varphi_1, \ldots, \varphi_n)]^\times \iff s \models [O^n(\varphi_1^\times, \ldots, \varphi_n^\times)] \lor \text{NS} \)

### BSML∗

- \( s \models \text{LN} \iff s \neq \emptyset \land \forall t \subseteq s : t \models \text{LN} \rightarrow t = s \)  
  \( s \models p^\ast \iff s \models p \land s \models \text{LN} \)
- \( s \models \text{LN} \iff s \neq \emptyset \land \forall t \subseteq s : t \models \text{LN} \rightarrow t = s \)  
  \( s \models p^\ast \iff s \models p \land s \models \text{LN} \)
- \( s \models [O^n(\varphi_1, \ldots, \varphi_n)]^\ast \iff s \models O^n(\varphi_1^\ast, \ldots, \varphi_n^\ast) \land s \models \text{LN} \)
- \( s \models [O^n(\varphi_1, \ldots, \varphi_n)]^\ast \iff s \models O^n(\varphi_1^\ast, \ldots, \varphi_n^\ast) \land s \models \text{LN} \)
References


Homogeneity cancellation

- Cancellation effects with *all* and *both*:

(9)  
\begin{align*}
  \textbf{a.} & \quad \text{Bianca likes the girls. } \approx \text{Bianca likes every girl.} \\
  \textbf{b.} & \quad \text{Bianca does not like all the girls. } \not\approx \text{Bianca likes no girl.}
\end{align*}

(10)  
\begin{align*}
  \textbf{a.} & \quad \text{Mia bought a and b. } \approx \text{Mia bought both a and b.} \\
  \textbf{b.} & \quad \text{Mia did not buy a and b. } \not\approx \text{Mia did not buy both a and b.}
\end{align*}

\begin{align*}
  \textit{both}(\varphi \land \psi) & \equiv \varphi \land \psi \\
  \textit{all}(\forall x \varphi) & \equiv \forall x \varphi \\
  \neg \textit{both}(\varphi \land \psi) & \not\equiv \neg \varphi \\
  \neg \textit{all}(\forall x \varphi) & \not\equiv \neg \varphi
\end{align*}

\begin{align*}
  [\textit{both}(\varphi \land \psi)]^x & \equiv \varphi^x \land \psi^x \\
  [\textit{all}(\forall x \varphi)]^x & \equiv \forall x \varphi^x \\
  [\neg \textit{both}(\varphi \land \psi)]^x & \not\equiv \neg \varphi \\
  [\neg \textit{all}(\forall x \varphi)]^x & \not\equiv \neg \varphi
\end{align*}
There is a “homogeneous” universal $\forall$ and then there is $\text{EVERY}$. The latter always expresses *not every* under negation even under cognitive bias.

\[
M, s \models \forall x \varphi \iff \forall d : M, s^x_d \models \varphi
\]

\[
M, s \models \forall x \varphi \iff \exists S : s = \cup S \& \forall t \in S : \exists d : M, t^x_d \models \varphi \& \forall d' \exists t' \in S : t'^x_d \models \varphi
\]

\[
M, s \models \text{EVERY} x \varphi \iff \forall d : M, s^x_d \models \varphi
\]

\[
M, s \models \text{EVERY} x \varphi \iff \exists S : s = \cup S \& \forall t \in S : \exists d : M, t^x_d \models \varphi
\]